MATEMATIK
Chalmers tekniska högskola

Hjälpmedel: Inga hjälpmedel är tillåtna
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2004–12–14 kl. 8.30-13.30

TMA372/MAN660 Partial Differential Equations TM

OBS! Skriv namn och personnummer på samtliga inlämnade papper.

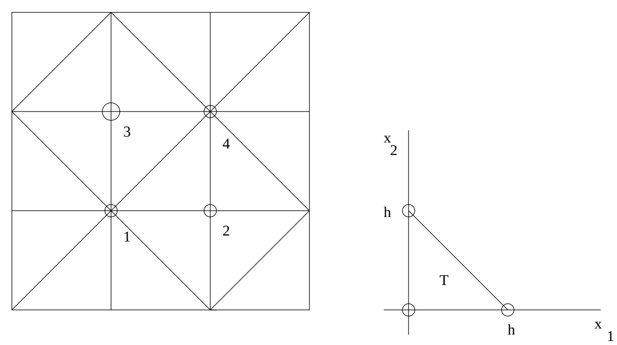
1. Let α and β be positive constants. Give the piecewise linear finite element approximation procedure, on the uniform mesh, for the problem

$$-u''(x) = 1, \quad 0 < x < 1; \qquad u(0) = \alpha, \quad u'(1) = \beta.$$

2. Formulate the $cG(1)$ method for the boundary value problem

$$-\Delta u + u = f, \quad x \in \Omega; \qquad u = 0, \quad x \in \partial\Omega.$$

Write down the matrix form of the resulting equation system using the following uniform mesh:



3. Consider the boundary value problem for the stationary heat flow in $1D$:

$$(BVP) \qquad -\left(a(x)u'(x)\right)' = f(x), \quad 0 < x < 1; \qquad u(0) = u(1) = 0.$$

Formulate the corresponding variational formulation (VF), minimization problem (MP) and show that: $\quad (BVP) \Longleftrightarrow (VF) \Longleftrightarrow (MP)$.

4. (a) Prove an a priori and an a posteriori error estimate for the finite element method for the problem

$$-u''(x) + u'(x) = f(x), \quad 0 < x < 1; \qquad u(0) = u(1) = 0.$$

(b) Give an adaptive algorithm based on a posteriori estimates.

5. Let Ω be a bounded domain in $\mathbb{R}^d$. Consider the initial-boundary value problem

$$\begin{cases} u_t - \Delta u = 0, & \text{in} \quad \Omega \times \mathbb{R}^+, \\ u = 0, & \text{on} \quad \partial\Omega \times \mathbb{R}^+, \\ u(\cdot, 0) = v, & \text{in} \quad \Omega. \end{cases}$$

Show the stability estimates

$$(i) \quad ||\nabla u(t)||^2 \le \frac{1}{2t}||v||^2, \quad \text{and} \qquad (ii) \quad \int_0^t s||\Delta u(s)||^2\, ds \le \frac{1}{4}||v||^2.$$

MA