Error Rates for Unvalidated Medical Age Assessment Procedures: Supplementary material

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This document contains a number of computations further strengthening the argument of our paper.

1 Initial models

In Section 3 of our main paper we defined two models where we use use the starting point age profile defined in the paper's Section 2.3 together with various fixed age indicator model parameters:

Model 0L fixes the teeth parameters at values estimated from the Lucas study and the knee parameters at values estimated from the Ottow study.

Model 0M is the same as model 0L except that the Lucas numbers are replaced with the Mincer numbers.

For both models, we estimated the remaining parameters using maximum likelihood, obtaining for model 0L $\hat{\theta}_{13} = 0.19$, $\hat{\theta}_{14} = 0.025$, $\hat{\theta}_{23} = 0.03$, and $\hat{\theta}_{24} = -0.003$, and for model 0M $\hat{\theta}_{13} = 0.18$, $\hat{\theta}_{14} = -0.013$, $\hat{\theta}_{23} = 0.03$, and $\hat{\theta}_{24} = -0.028$.

Predicting count data from models 0L and 0M we obtain the expected counts given in Table 1. These are very obviously different from the real data given in Table 1 of the paper. To illustrate this one may for example compute p-values testing whether each of the 9 counts in the real data (excluding summary counts) could come from model 0L. Except for the count of cases with no observed age indicators, all p-values are very small (less than $3 \cdot 10^{-6}$). An overall Chi squared test gives a p-value of less than $2.2 \cdot 10^{-16}$. Similar results are obtained for model 0M.

2 Results for additional models

We now present results for a number of models not considered in the main text. In the first nine we use the starting point age profile presented in Section 2.3 of

0L	Knees mature	Knees immature	No data knees	SUM
Teeth mature	2949	802	130	3881
Teeth immature	1035	2663	165	3863
No data teeth	947	533	55	1535
SUM	4931	3998	350	9279
0M	Knees mature	Knees immature	No data knees	SUM
Teeth mature	2115	762	77	2954
Teeth immature	1984	2239	345	4568
No data teeth	864	779	115	1758
SUM	4963	3780	537	9280

Table 1: Expected data under models 0L and 0M. In model 0L rounding prevents the numbers to sum to 9280.

our paper. We combine this with all possible combinations of the three teeth priors and three knee priors defined in the main text. The results can be seen in Table 2. We see immediately that the posteriors are much more similar than in Table 2 of the main paper. The reason is that the population profile is fixed. Thus, with a fixed population profile, we can obtain strong results about age indicator parameters. In particular, we see that with this age profile, knees need to mature one year before teeth in order to explain the data of Table 1 of the main paper.

The remaining models we consider use the hierarchical prior for the age profile presented in Section 2.3 of our paper. In six new models, we combine this prior with (more or less) fixed age indicator model parameters:

Model S10 combines Lucas parameters with Ottow parameters.

Model S11 combines Mincer parameters with Ottow parameters.

Model S12 combines an average of Lucas and Mincer parameters with Ottow parameters.

Model S13 combines Lucas parameters with adjusted Ottow parameters.

Model S14 combines Mincer parameters with adjusted Ottow parameters.

Model S15 combines an average of Lucas and Mincer parameters with adjusted Ottow parameters.

The resulting posterior population age profiles can be seen in Figure 2. We see that most are widely unrealistic in that they contain very sharp steps. This means that the combinations of parameter estimates of the corresponding models can be seen as incompatible with the data of Table 1 of the main paper.

In the main paper, we consider three priors for teeth and three priors for knees. However, we report results for only 5 out of the 9 possible combinaitons of these priors. We now look at results for the remaining 4 combinations:

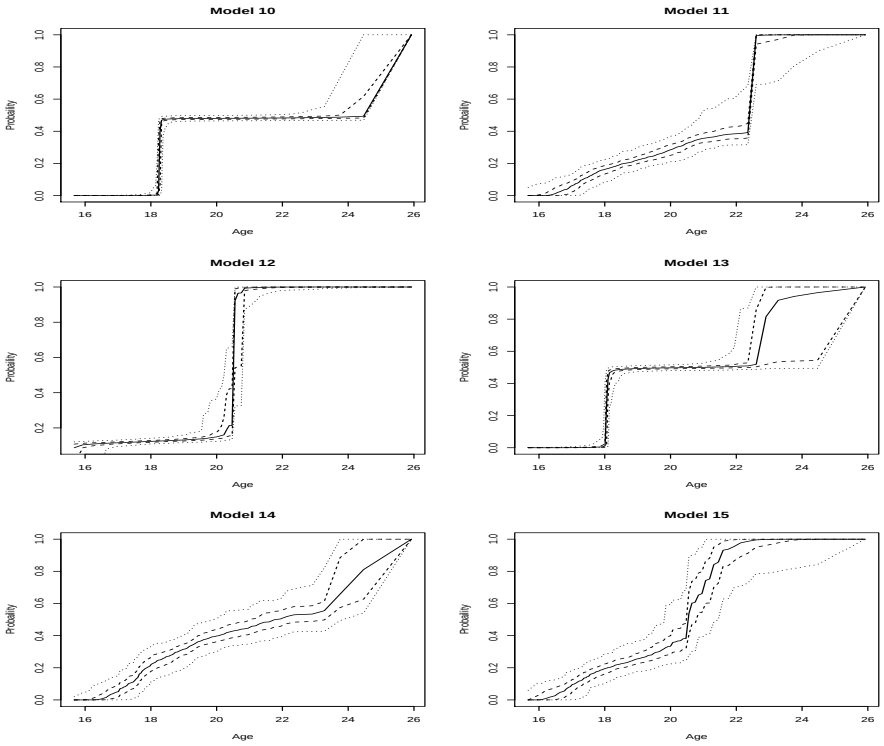

Figure 1: The posterior population profiles for models S10 through S15.

	Ottow prior	Wide knee prior	Adj. Ottow prior
Lucas	18.1 (18.0–18.1)	18.1 (18.0–18.1)	18. 1 (18.0–18.1)
	1.1 (1.0–1.3)	1.0 (0.9–1.2)	1.0 (1.0–1.2)
	17.0 (16.9–17.1)	16.9 (16.8–17.0)	16.9 (16.8–17.0)
	1.2 (1.1–1.4)	1.4 (1.1–1.6)	1.3 (1.1–1.5)
Mincer	18.1 (18.0–18.2)	18.1 (18.0–18.2)	18.1 (18.0–18.2)
	1.6 (1.4–1.7)	1.5 (1.4–1.7)	1.5 (1.4–1.7)
	17.0 (17.0–17.1)	17.0 (16.9–17.1)	17.0 (17.0–17.1)
	1.1 (1.1–1.2)	1.1 (1.1–1.2)	1.1 (1.1–1.2)
Wide	18.1 (18.0–18.1)	18.0 (18.0–18.1)	18.1 (18.0–18.1)
Prior	1.2 (1.0–1.4)	1.2 (1.0–1.4)	1.1 (1.0–1.3)
Teeth	17.0 (16.9–17.1)	16.9 (16.8–17.0)	16.9 (16.8–17.0)
	1.2 (1.1–1.3)	1.2 (1.1–1.5)	1.3 (1.1–1.5)

Table 2: Posterior expectations (and 95% credibility intervals) for model parameters θ_{11} θ_{12}, θ_{21}, and θ_{22}. The results are given for each of 9 different models, obtained by combining one of the three teeth priors (left margin) with one of the three knee priors (top). All results are obtained with the starting point age profile presented in Section 2.3 of our paper.

Model S16 combines the narrow Lucas prior with the wide Ottow prior.

Model S17 combines the narrow Mincer prior with the wide Ottow prior.

Model S18 combines the wide teeth prior with the narrow Ottow prior.

Model S19 combines the wide teeth prior with the adjusted Ottow prior.

The resulting posterior population age profiles can be seen in Figure 2. The posteriors for the age indicator parameters are illustrated in Figure 3 and Table 3. Error rates are shown in Table 4.

Finally, we try out some models where we attempt to set the knee maturity parameter as larger than or equal to the tooth parameters.

Model S20 combines the narrow Lucas prior with a prior for knee parameters that have the same values.

Model S21 is the same as model S16 except that knees now mature one year later than teeth. In other words, while θ_{11} is centered around 18.6 and θ_{12} is centered around 0.7, θ_{21} will be centered around 19.6 and θ_{22} will be centered around 0.7.

Model S22 combines the narrow Mincer prior with a prior for knee parameters that have the same values.

Model S23 is the same as model S16 except that knees now mature one year later than teeth. In other words, while θ_{11} is centered around 20.0 and θ_{12}

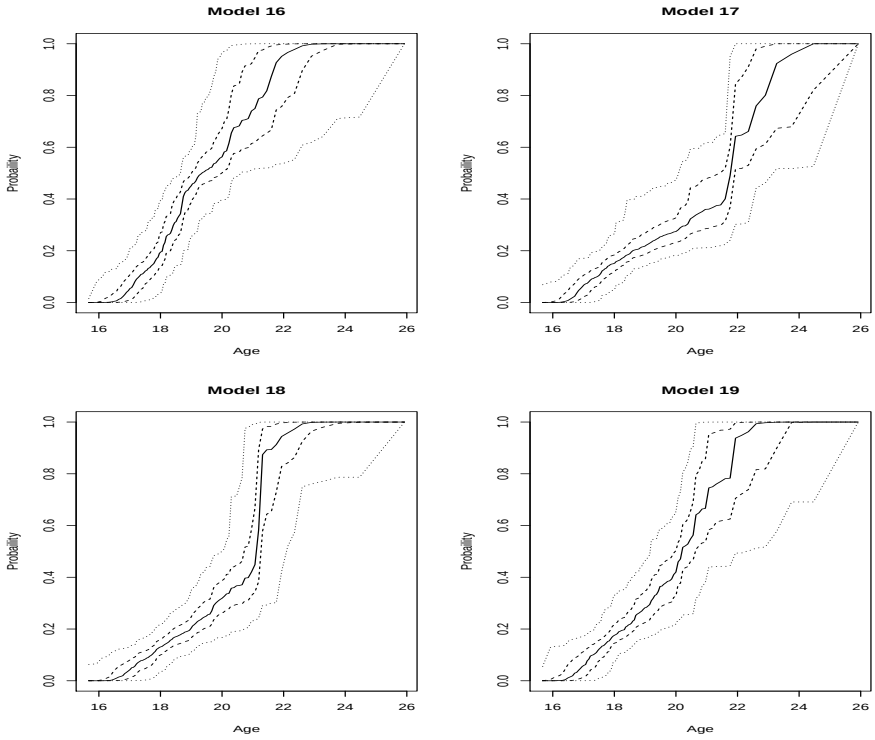

Figure 2: The posterior population profiles for models S16 through S19.

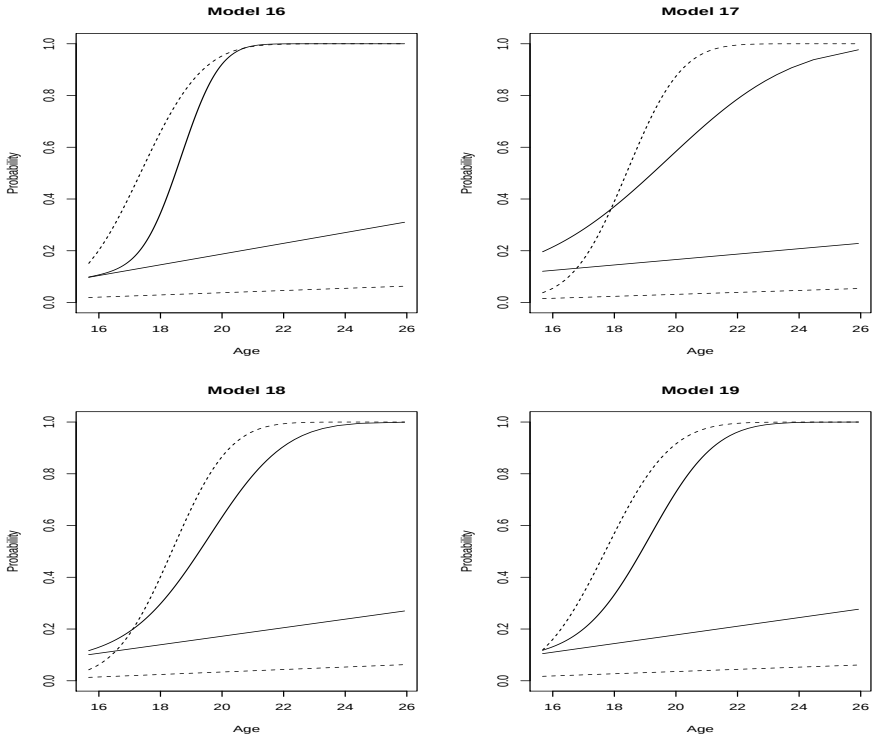

Figure 3: The posterior age indicator parameters for models S16 through S19.

		Prior	Posterior Model 16	Posterior Model 17	Posterior Model 18	Posterior Model 19
Lucas	θ_{11}	18.6 18.2 – 19.0	18.7 18.4 – 19.1			
	θ_{12}	0.7 0.3 – 1.1	1.0 0.9– 1.3			
Mincer	θ_{11}	20.0 19.6 – 20.4		20.0 19.6 – 20.4		
	θ_{12}	3.2 2.8 – 3.6		3.2 2.8 – 3.5		
Wide prior teeth	θ_{11}	19.3 17.7 – 20.9			19.7 19.3 – 20.2	19.3 18.8–19.8
	θ_{12}	2.0 0.4 – 3.6			1.9 0.8 – 3.1	1.6 0.9 – 3.0
Ottow	θ_{21}	18.5 18.1 – 18.9			18.4 18.1–18.8	
	θ_{22}	1.5 1.1 – 1.9			1.5 1.1–1.8	
Wide prior knees	θ_{21}	18.5 16.9 – 20.1	17.4 16.8–17.9	18.4 17.5–19.3		
	θ_{22}	1.5 0.0 – 3.1	1.6 1.0–2.4	1.4 0.9–2.3		
Ottow IIIc	θ_{21}	17.8 17.4 – 18.2				17.8 17.4–18.1
	θ_{22}	1.7 1.3 – 2.1				1.7 1.3–2.0

Table 3: Prior and posterior parameter distributions. The ranges indicate approximate 95% credibility intervals for each parameter.

RMV classification	N	Model 16	Model 17	Model 18	Model 19
Error rates when classifying as above 18					
K+, T+	4176	2 0–9	1 0–4	1 0-3	2 0–6
K+, T-	1735	37 4–84	11 0–37	11 2-27	26 6–60
K+, T0	1364	8 1–22	3 0–10	2 0–7	6 1–16
K-, T+	348	29 1–66	48 13–86	25 0–70	27 1–72
K0, T+	187	2 0–10	3 0–7	1 0–5	2 0–8
Error rates when classifying as below 18					
K-, T-	1087	29 5–84	28 4 – 73	35 8–82	26 5–70
K-, T0	237	48 16–90	38 8–79	51 19 –90	45 14–80
K0, T-	83	56 13–94	77 49–96	79 58–90	65 33–89

Table 4: Estimated error rates in percent. The ranges contain a 95% credibility interval.

is centered around 3.2, θ_{21} will be centered around 21.0 and θ_{22} will be centered around 3.2.

For all the models S20-S23, the posterior distribution of the knee maturation parameter θ_{22} is centered well below the posterior distribution for the tooth maturation parameter θ_{11}. In other words, when the data of Table 1 of the main paper is taken into account, θ_{21} is forced to be smaller than θ_{11} even when it is larger in the prior. The most likely value for θ_{11} minus the most likely value for θ_{21} is 0.9, 0.8, 1.7, and 1.1 for models S20, S21, S22, and S23, respectively.

3 Influence of the age profile prior

As a final test, we recompute the results for models 1 through 5, those used in the main paper, using a hierarchical prior with a different starting point than that constructed in Section 2.3 of the main paper. Specifically, instead of a Gamma distribution, we use a normal distribution with expectation 22.5 years and standard deviation 4, truncated to the interval $[15, 30]$. The posterior distributions are shown in Table 5, and the posterior age distributions can be found in Figure 4. The posteriors for the age indicator model parameters is illustrated in Figure 5, while the error rates can be found in Table 6. We see that, indeed, numerical results are somewhat influenced by the change in the prior. However, the main conclusions listed in Section 3.2 of the main paper still hold.

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4 Female data

The RMV results for tested females during 2017 are given in Table 7. As the numbers are quite small compared to the counts for males, we have so far not carried out an analysis of these numbers.

		Prior	Posterior Model 1	Posterior Model 2	Posterior Model 3	Posterior Model 4	Posterior Model 5
Lucas	θ_{11}	18.6	19.3			19.0	
		18.2 – 19.0	19.0 – 19.5			18.7 – 19.2	
	θ_{12}	0.7	1.3			1.4	
		0.3 – 1.1	1.2 – 1.4			1.3 – 1.5	
Mincer	θ_{11}	20.0		20.1			20.0
		19.6 – 20.4		19.7 – 20.4			19.6 - 20.3
	θ_{12}	3.2		3.2			3.2
		2.8 – 3.6		2.8 – 3.6			2.8 – 3.6
Wide prior teeth	θ_{11}	19.3			20.0		
		17.7 – 20.9			19.1 – 21.0		
	θ_{12}	2.0			2.3		
		0.4 – 3.6			1.3 – 3.6		
Ottow	θ_{21}	18.5	17.9	18.5			
		18.1 – 18.9	17.6 – 18.2	18.1 – 18.8			
	θ_{22}	1.5	1.5	1.6			
		1.1 – 1.9	1.4 – 1.7	1.4 – 1.9			
Wide prior knees	θ_{21}	18.5			18.2		
		16.9 – 20.1			17.1 – 19.2		
	θ_{22}	1.5			2.0		
		0.0 – 3.1			1.4 – 2.9		
Ottow IIIc	θ_{21}	17.8				17.5	17.8
		17.4 – 18.2				17.2 – 17.8	17.5 – 18.2
	θ_{22}	1.7				1.7	1.8
		1.3 – 2.1				1.5 – 2.0	1.5 – 2.1

Table 5: Prior and posterior parameter distributions. The ranges indicate approximate 95% credibility intervals for each parameter. The difference to the corresponding table in the main paper is that we have here used the alternative starting point prior.

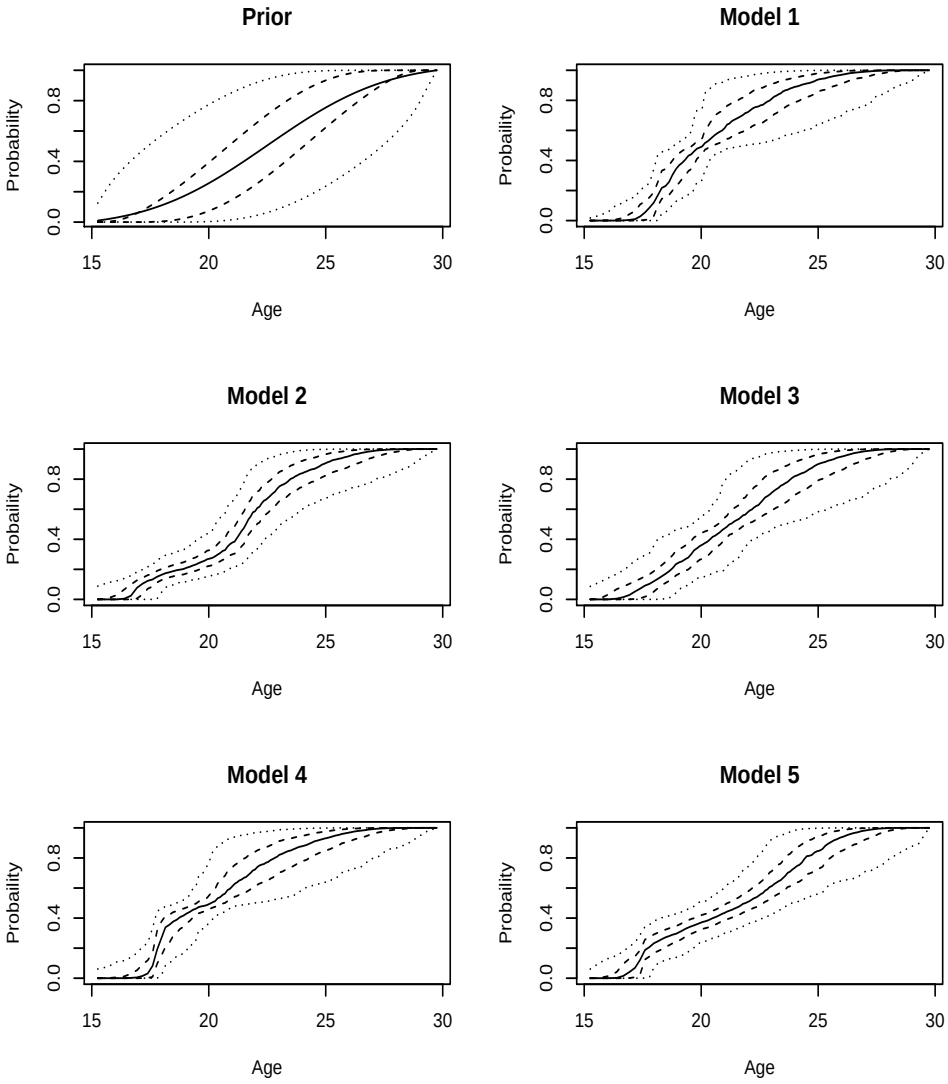

Figure 4: Prior and posterior population profiles. The middle line in each plot indicates the most likely profiles. The other lines delineate the 2.5%, 25%, 75%, and 97.5% quantiles, respectively. Thus, vertical intervals between the two dotted lines represent 95% credibility intervals. The difference to the corresponding table in the main paper is that we have here used the alternative starting point prior.

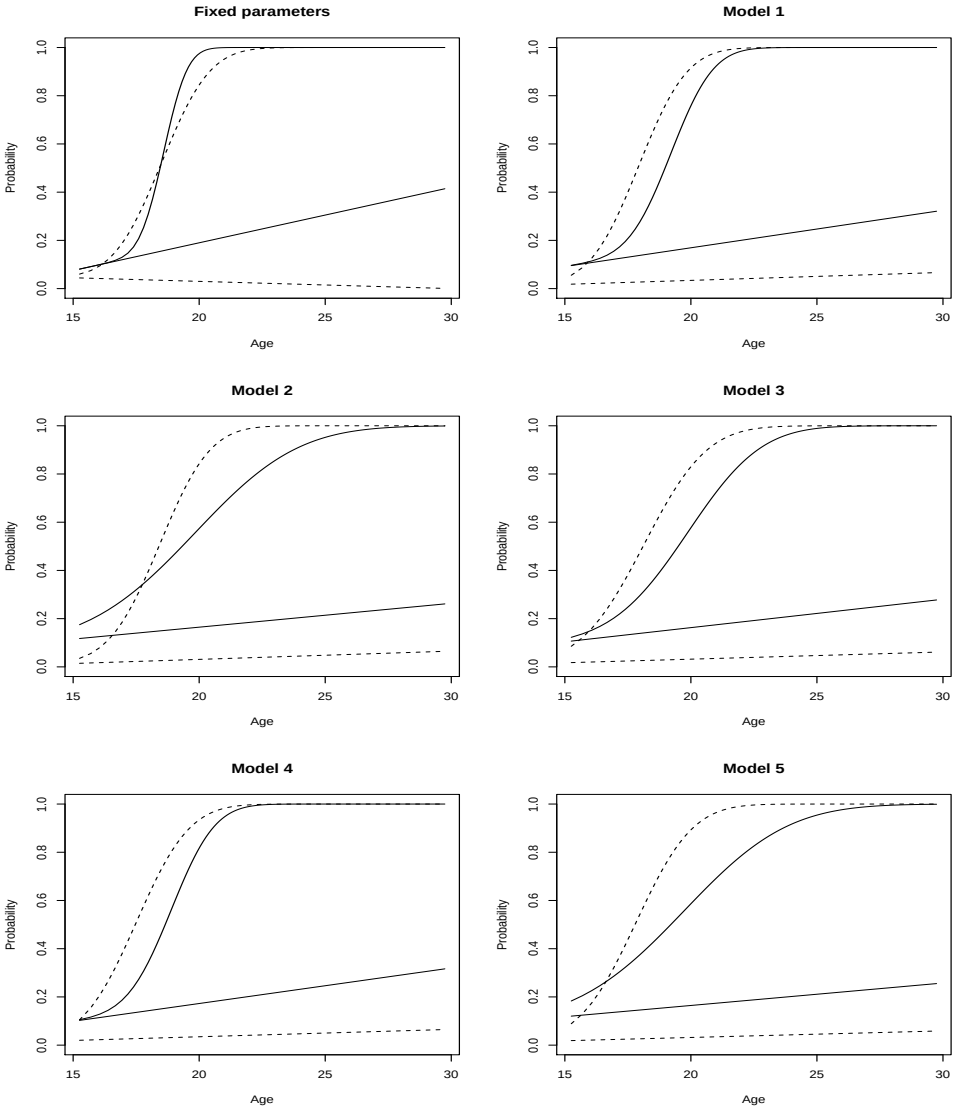

Figure 5: Illustration of the posterior expected model parameters, under various models. The straight lines indicate the posterior rates for missing data. The dotted lines correspond to the knee model; the continuous lines to the teeth model. The difference to the corresponding table in the main paper is that we have here used the alternative starting point prior.

RMV classification	N	Model 1	Model 2	Model 3	Model 4	Model 5
Error rates when classifying as above 18						
K+, T+	4176	1 0–7	1 0–5	1 0–8	5 0–13	4 0–9
K+, T-	1735	19 0–68	13 1–37	17 0–68	49 2–96	31 4–67
K+, T0	1364	4 0-17	3 0–10	4 0–18	12 0–27	8 1–19
K-, T+	348	15 0–65	49 8–89	23 0–85	43 0–95	62 13–97
K0, T+	187	1 0–8	3 0–8	2 0–10	5 0–16	5 0–12
Error rates when classifying as below 18						
K-, T-	1087	49 5–100	22 2 – 75	45 3–100	24 0–92	18 1–65
K-, T0	237	63 16–100	33 5–81	58 7 –100	36 2–95	26 1–73
K0, T-	83	73 25–100	74 48–96	75 25–100	44 2–96	58 25–90

Table 6: Estimated error rates in percent when classifying as over 18. The ranges contain a 95% credibility interval. The difference to the corresponding table in the main paper is that we have here used the alternative starting point prior.

	Knees mature	Knees immature	No data knees	SUM
Teeth mature	190	1	6	197
Teeth immature	79	12	5	96
No data teeth	40	1	3	44
SUM	309	14	14	337

Table 7: The data for females