

LARGE DEVIATIONS
HOMEWORK 3

Deadline for handing in solutions: March 21.

1. **Regularity properties of the rate function.** Prove Lemma I.14 in den Hollander's book.¹
2. **Relative entropy.** For two probability distributions $\rho = (\rho_1, \dots, \rho_k)$ and $\nu = (\nu_1, \dots, \nu_k)$ on $\{1, \dots, k\}$, the relative entropy of ν given ρ is defined $H(\nu|\rho) = \sum_{i=1}^k \nu_i \log \left(\frac{\nu_i}{\rho_i} \right)$.
 - (i) Is it always the case that $H(\nu|\rho) = H(\rho|\nu)$? Give a proof or a counterexample.
 - (ii) Show that $H(\nu|\rho)$ is non-negative, and minimized when $\nu = \rho$.

¹The book contains a somewhat telegraphic solution to this problem. If you decide to follow the book's solution, then you are required to phrase it in your own words and to add some detail.