LARGE DEVIATIONS
HOMEWORK 4

Deadline for handing in solutions: March 28.

1. This problem is meant to illustrate Frank den Hollander's words "ANY LARGE DEVIATION IS DONE IN THE LEAST UNLIKELY OF ALL THE UNLIKELY WAYS". Suppose a fair 6-sided die is thrown 100 000 times, and let $X_1, \ldots, X_{100\,000}$ denote the successive outcomes. Let L_n denote the empirical measure after n tosses. Suggest typical approximate values of $L_{100\,000}$ conditional on the event A, in each of the following cases.

   (a) $A = \{X_i \geq 4$ for at least 60 000 of the 100 000 X_i's$\}$

   (b) $A = \{$For at least one $k \in \{1, \ldots, 6\}$ we have that k comes up at least 30 000 times in the 100 000 throws$\}$

   (c) $A = \{\sum_{i=1}^{100\,000} X_i \geq 400\,000\}$

   (d) $A = \{\sum_{i=1}^{100\,000} X_i \geq 300\,000\}$

   (e) $A = \{\sum_{i=1}^{100\,000} X_i^2 \geq 2\,000\,000\}$