MASTER THESIS

Jump Modelling and Jump Risks of Exchange-Traded Certificates

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Mathematical Sciences
CHALMERS UNIVERSITY OF TECHNOLOGY
Gothenburg 2011

MASTER THESIS IN MATEMATHICAL STATISTICS
AT CHALMERS UNIVERSITY OF TECHNOLOGY

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Abstract

In this master thesis we study the jumps characteristics for prices in different asset classes and applying the models to exchange-traded certificates issued by SEB. The report presents a definition of jumps and looks at the behavior of them in terms of upward and downward jump sizes and the durations between them. The behavior differs between the various asset classes for both sizes and durations but distributions for sizes tend to be heavy-tailed compared to durations which tend to be light-tailed. From the results of fitting models to historical data a jump diffusion model was built where the jump part came from the model fitting which was added to the Black-Scholes model.

By simulations of possible future scenarios of the asset price one could use that to see how it affected the prices of certificates with different leverages. Each certificate has a certain stop loss level which means that the customer is protected from losing more than invested amount. This means that the issuer is exposed to jump risk by calculating the possibility of breaking the stop loss level and by how much, one can calculate the issuer's risk exposure.

The results show that the risk of breaking the stop loss level within 10 years is pretty big, and of course larger for more volatile asset classes like commodities compared to currencies which are less volatile. The tail events can unfortunately turn into big losses.

Acknowledgement

I would like to thank my supervisors Fredrik Fyring and Richard Tydén at SEB Merchant Bank for their guidance during my thesis work. I also like to thank my supervisor and examiner Patrik Albin at Chalmers for his comments and inputs.

Contents

1 Introduction

An always present subject is to find a model beyond the Black-Scholes that captures the behavior of financial asset prices in the best way. There are plenty of these kinds of models which can be divided into two parts, Stochastic-Volatility models and Jump Diffusion models. The first one extends the Black-Scholes model, which sees the volatility as constant, to a model that has a varying volatility over time. The Jump Diffusion model does instead add a jump process to the Black-Scholes, in form of discrete jumps which occur randomly and changes the prices. The jumps make it seem like the process captures the fluctuations of an asset price.
The jumps in a Jump-Diffusion model are often seen as the discontinuous changes in the sample path of a Brownian motion. The asset price is never continuous so usually the definition of a jump is that when the price changes over a certain time with a certain size, a jump has occurred. Almost all types of assets have this kind of behavior. It comes with a so called jump risk, i.e. the risk that the price makes a discrete jump through a stop loss, barrier or any other kind of level. Banks and other financial institutions sell products where they guarantee the buyer that they will not loose more than a certain amount of their money. Therefore the seller has a risk of loosing money if their margins are not big enough.
One product of that type is SEB:s Exchange-Traded Certificates, products that track prices of different assets. The certificates come with different levels of leverage which makes them more or less risky and the investor is guaranteed to not loose more than it has invested. SEB provides certificates in both currencies and commodities. Commodities are often very volatile and it's pretty common that they can drop in price with more then 10 percent in one day.
This master thesis explores Jump Diffusion models with different jump-distributions to match the wide range of underlying assets. When the models are found, simulations are made to find the probabilities and sizes of breaching stop losses for SEB:s exchange-traded certificates.

2 Theory

This section presents the theory behind all methods used in this master thesis.

2.1 Black-Scholes model

The Black-Scholes model gives a price of an European option for time $t \geq 0$ with a given starting capital $S(0)$ and interest μ. As everyone know, the future cannot with certainty be predicted. Therefore the model consists of an uncertainty parameter $B(t)$. $\{B(t)\}_{t\geq 0}$ belongs to a family of random processes, which together builds a stochastic process. The price of the option with these parameters is given by

$$S(t) = S(0)\,\mathrm{e}^{B(t)+\mu t} \quad \text{för } \; t \geq 0. \tag{1}$$

There are many stochastic processes but a proper choice is of great importance. The Lévy process is proper because its three abilities [1, s. 25]:

- $B(0) = 0$,
- $B(t)$ has independent increments:

  $$B(t_1) - B(t_0), \ldots, B(t_n) - B(t_{n-1}) \;\text{ are independent variables}$$

  for $0 \leq t_0 \leq t_1 \leq \ldots \leq t_n$,
- $B(t)$ has stationary increments:

  $$\text{Distribution for } \; B(t{+}h) - B(t) \; \text{ does not depend on } \; t, \; \text{ but only on } \; h \geq 0.$$

This means that the increments shall be independent of each other and the distribution of the increments shall be the same. The Lévy processes are a family of processes and in the Black-Scholes model the distribution for $B(t{+}h) - B(t)$ is normal $\mathrm{N}(0, \sigma^2 h)$ where σ^2 is the volatility.

2.1.1 Evaluation of the Black-Scholes Model

The Black-Scholes is widely used but it is at the same time known that it has its weaknesses. There are a couple of tests to evaluate how well the model imitates real world data. The test controls if the above criterias are fulfilled. One way is to take the logarithm of equation (1)

$$\{L(n{+}1) - L(n)\}_{n\in\mathbb{N}} = \{\ln(S(n{+}1)) - \ln(S(n))\}_{n\in\mathbb{N}} = \{B(n{+}1) - B(n) + \mu\}_{n\in\mathbb{N}}. \tag{2}$$

and evaluate the received, so called, log increments $\{\ln(S(n{+}1)) - \ln(S(n))\}_{n=0,\ldots,N}$, which also are assumed to be normal distributed, $\mathrm{N}(0, \sigma^2 h)$. The next step is to check if they are independent which means that they are also uncorrelated.

To visually check if the log increments are normal distributed a Q-Q plot can be used and to see how well the log increments fit a normal distribution, MLE is used. To get a numerical result of how well the model reflects real data the Kolmogorov-Smirnov method is a good choice. The tests are described under section (2.4).

2.2 Jump Diffusion Process

The assumption in the Black-Scholes model that the log increments are normal distributed is not correct, the log returns for real market data tends to be more heavy-tailed. This means that rare events are not as rare as the Black-Scholes models says and the Black-Scholes model has to be improved or replaced to get better results and that leads us to a Jump Diffusion model.

There are two basic blocks in a Jump Diffusion model, the Brownian motion part (the diffusion part) and a process capturing rare events (the jump part). For further discussion proper definitions of a jump, jump size and duration between jumps are given below.

2.2.1 A Jump

For a financial process X_t, a threshold α ($\alpha > 0$) is set on both positive and negative side of the devolatilized log increments of X_t,

$$|\frac{\ln X_t - \ln X_{t-1} - \hat{\mu}}{\hat{\sigma}}| \geq \alpha. \tag{3}$$

We say that a jump has occurred if a devolatilized log increment exceeds the threshold.

2.2.2 Jump Size

When a jump occurs at time t, the jump size A_t is the difference between the log prices, i.e. the value of the log increment at time t,

$$A_t = \ln X_t - \ln X_{t-1}.$$

If $A_t > 0$ then the jump is upward and if $A_t < 0$ then the jump is downward.

2.2.3 Jump Duration

When a jump occurs at time s and the next consecutive jump occurs at time t ($t > s$), the jump duration is $t - s$.

2.2.4 A First Jump Diffusion Model

For a basic standard jump diffusion model the jump part consists of a Poisson process. The Poisson process (2.2.6) has independent and stationary increments, which makes it a Lévy process.

When working with financial time series it is not very interesting to restrict oneself to having only one jump size, but rather a process. If the jump durations consider being exponential distributed and the jump sizes vary over time by a arbitrary distribution, the Compound Poisson process (2.2.6) can come in hand.

If a Brownian motion (B_t) with drift (μ) is combined with a Compound Poisson process we obtain a so called jump diffusion model:

$$X_t = \mu t + B_t + \sum_{i=1}^{N_t} Y_i. \quad (4)$$

A stock price can be expressed by $S_t = S_0^{X_t}$ with X_t as in equation (4). Taking jumps $\{Y_i\}$ with a Gaussian distribution one obtains the famous Merton Model [2, s. 125-144].

The process given by equation (4) is a Lévy process, but stock prices are often modeled as exponentials of Lévy processes to ensure that the price is positive and the log increments are independent and stationary. This gives the jump diffusion model

$$S_t = S_0^{\mu t + B_t + \sum_{i=1}^{N_t} Y_i}. \quad (5)$$

When comparing equation (5) with the Black-Scholes model, equation (1), it is easily seen that equation (5) is a generalization of equation (1) and gives the possibility to behave more like real financial time series.

2.2.5 A New Jump Diffusion Model

The Lévy processes are not limited to the form of equation (5). Therefore it gives an opportunity to elaborate with the jump process. In finance it is well known that downward jumps tend to be larger than upward jumps. A natural assumption is therefore that upward and downward jumps have different distributions for the jump size. Furthermore, the assumption in model (5) that the durations between jumps are exponential distributed is somehow restricted. To build a model that can have different distributions for upward and downward jump sizes and different distributions for the duration between upward and downward jumps are of great desire. In this master thesis four different distributions have been used with varying abilities. The distributions can be found under section (2.2.6). When taking different processes for both upward and downward jumps and also for the durations between them, equation (5) is changed to

$$S_t = S_0 e^{\mu t + \sigma B_t + X_t = \sum_{i=1}^{N_t} Y_i^1 \cdot 1_{U_i^1 \leq t} - \sum_{j=1}^{N_t} Y_j^2 \cdot 1_{U_j^2 \leq t}} \quad (6)$$

where μ is the drift, σ is the volatility, B_t is the Brownian motion, U_t^1 and U_t^2 are the duration processes and Y_t^1 and Y_t^2 are the processes for jump sizes.

2.2.6 Distributions and Processes

In this section six different types of processes and distributions are presented in terms of definition and abilities.

The Poisson Process

Take a sequence $\{\tau_i\}_{i\geq 1}$ of independent and exponential random variables with parameter λ with the cumulative distribution function as $P[\tau_i \geq y] = e^{\lambda y}$ and take $T_n = \sum_{i=1}^{n} \tau_i$. This gives the Poisson process:

$$N_t = \sum_{n\geq 1} 1_{t \geq T_n}.$$

The Poisson process is often used for waiting times were the intervals between events are exponentially distributed. A drawback is when applying this to jump diffusion models, the jump sizes are always one.

The Compound Poisson Process

Take a sequence $\{Y_i\}_{i\geq 1}$ of independent random variables with law f and let N be a Poisson process with parameter λ. Then the Compound Poisson process is given by

$$X_t = \sum_{i=1}^{N_t} Y_i.$$

The Compound Poisson Process is a development of the Poisson process. The waiting times are still exponentially distributed but the jump sizes can have an arbitrary distribution.

Exponential Distribution

The probability density function (pdf) for the Exponential distribution is given by

$$f(x;\lambda) = \begin{cases} \lambda e^{-\lambda x} & \text{for} \quad x \geq 0, \\ 0 & \text{for} \quad x < 0, \end{cases}$$

where λ can be seen as a inverse scale parameter. The cumulative distribution function (CDF) can be expressed as

$$F(x;\lambda) = \begin{cases} 1 - e^{-\lambda x} & \text{for} \quad x \geq 0, \\ 0 & \text{for} \quad x < 0, \end{cases}$$

Gamma distribution

The probability density function for the Gamma distribution is given by

$$f(x;k,\Theta) = x^{k-1}\frac{e^{-x/\Theta}}{\Theta^k \Gamma(k)} \; for\; x \geq 0 \; and\; k,\ \Theta > 0,$$

where k is the shape parameter, Θ is the scale parameter and $\Gamma(k)$ is the gamma function. Furthermore the cumulative distribution function can be expressed as

$$F(x;k,\theta) = \frac{\gamma(k, x/\Theta)}{\Gamma(k)}.$$

Generalized Pareto Distribution

The probability density function for the Generalized Pareto distribution is given by

$$f_{\xi,\mu,\sigma}(x) = \frac{1}{\sigma}(1 + \frac{\xi(x-\mu)}{\sigma})^{-\frac{1}{\xi}-1} \text{ for } x \geq \mu \text{ when } \xi \geq 0 \text{ and } x \leq \mu - \sigma/\xi \text{ when } \xi < 0,$$

where μ is the location parameter, σ is the scale parameter and ξ is the shape parameter. The cumulative distribution function can be expressed as

$$F_{(\xi,\mu,\sigma)}(x) = \begin{cases} 1 - (1 + \frac{\xi(x-\mu)}{\sigma})^{-1/\xi} & \text{for} \quad \xi \neq 0, \\ 1 - exp(-\frac{x-\mu}{\sigma}) & \text{for} \quad \xi = 0, \end{cases}$$

with the parameters as for the pdf.

The AR(1) Process

An autoregressive (AR) model is a type of linear prediction of an output depending on previous outputs. The AR(1) means that the next output only depends on the last one. The AR(1) model is given by

$$y_n = a_0 + a_1 y_{n-1} + \sigma \epsilon_n,$$

where a_0 and a_1 are constants and $\epsilon_n \approx N(0,1)$.

2.3 Exchange-Traded Certificates

An exchange-traded certificate is a debt security issued by a bank where the price is tracking the price of a reference asset. The American equivalent security is called exchange-traded notes, ETN. The market benchmarks can for example be commodities, currencies or stocks. The advantage for certificates is that they provide the opportunity to invest in markets that can be difficult to get access to. The certificates have issue dates, maturity dates, are exchange-traded and are as easy to buy as stocks. The certificates come in many forms where one can take a short or a long position with different leverages.

2.3.1 Bull and Bear Certificates

Certificates issued by SEB [3] are divided into three types with different kind of yield potential and risk:

- Long - The price goes up when the price of the reference asset rises and down when the reference price falls. It has no leverage.
- Bull - The price goes up when the price of the reference asset rises and down when the reference price falls. It comes with different sizes of leverage.
- Bear - The price goes up when the price pf the reference asset falls and down when the reference price rises. It comes with different sizes of leverage.

SEB:s Bull and Bear certificates gives leverage based on the total percent change between issue price and current price for the reference asset. Hence, the leverage changes over time when the price changes.

2.3.2 Prices of SEB:s Certificates

For exchange-traded certificates with currencies as underlying, the pricing looks like as follows (USD/SEK as underlying).

$$
\begin{aligned}
CV_t &= CV_{t-1} \cdot (1 - Fee \cdot InteresRatePeriod) + VF_t, \\
\text{where} & \\
CV_t &= \text{Value of the certificate at day t}, \\
CV_{t-1} &= \text{Value of the certificate at day t-1}, \\
CV_0 &= \text{Starting value}, \\
VF_t &= NumOfUSD_{t-1} \cdot [(1 + F_{t-1}^{USD}) \cdot Ref_t - Ref_{t-1}] + \\
&\quad + F_{t-1}^{SEK} \cdot NumOfSEK, \\
VF_t &= \text{Change in certificate value between day t-1 and day t}, \\
NumOfUSD_t &= NumOfUSD_{t-1} \cdot (1 + F_{t-1}^{USD}) + \\
&\quad + \frac{NumOfSEK \cdot F_{t-1}^{SEK} - Fee \cdot InterestRatePeriod \cdot CV_{t-1}}{Ref_t}, \\
F_{t-1}^{USD} &= (InterestRateBase_{t-1}^{USD} - Fee \cdot InterestRateMargin_{USD}) \cdot \\
&\quad \cdot InterestRatePeriod, \\
F_{t-1}^{SEK} &= (InterestRateBase_{t-1}^{SEK} - Fee \cdot InterestRateMargin_{SEK}) \cdot \\
&\quad \cdot InterestRatePeriod, \\
F_0^{USD} &= 0, \\
F_0^{SEK} &= 0, \\
F_{t-1}^{USD} &= \text{Finance USD at day t-1}, \\
F_{t-1}^{SEK} &= \text{Finance SEK at day t-1}, \\
Ref_t &= \text{Value of reference asset at day t}, \\
Ref_{t-1} &= \text{Value of reference asset at day t-1}, \\
Ref_0 &= \text{Value of reference asset at initial day}, \\
NumOfUSD_0 &= \frac{CV_0 \cdot InitialLeverage}{Ref_0}, \\
NumOfSEK &= CV_0 \cdot (1 - InitialLeverage).
\end{aligned}
\tag{7}
$$

The interest period refers to the period from the day before the valuation until the valuation day and applicable interest rate convention is actual number of days divided by 360.

The values for the Bear certificates are the same but with negative initial leverages.

The Bull and Bear exchange-traded certificates on commodities are calculated differently since the commodities are listed in USD. The investor is therefore exposed to both USD/SEK movements and the price of the commodity forward. The price of a Gold Bull certificate is calculated in the following way:

$$
\begin{aligned}
CV_t &= CV_{t-1} \cdot F_{t-1} \cdot \tfrac{ExchangeRate_t}{ExchangeRate_{t-1}} + NumOfRef_{t-1} \cdot \\
&\quad \cdot (Ref_t - Ref_{t-1} \cdot ExchangeRate_t, \\
\text{where} & \\
CV_t &= \text{Value of the certificate at day t}, \\
CV_{t-1} &= \text{Value of the certificate at day t-1}, \\
CV_0 &= \text{Starting value}, \\
F_{t-1} &= (1 + (InterestRateBase_{t-1} - InterestBaseMargin^{USD} \cdot \\
&\quad \cdot max[1, |NumOfRef_{t-2}| \cdot Ref_{t-1} \cdot \tfrac{ExchangeRate_{t-1}}{CV_{t-1}}]) \cdot \\
&\quad \cdot InterestRatePeriod) \cdot (1 - FeeInterestPeriod), \\
NumOfRef_t &= NumOfRef_{t-1} \cdot F_t, \\
NumOfRef_0 &= \tfrac{CV_0 \cdot InitialLeverage}{Ref_0 \cdot ExchangeRate_0}, \\
F_0 &= 1, \\
Ref_t &= \text{Value of reference asset at day t}, \\
Ref_{t-1} &= \text{Value of reference asset at day t-1}, \\
Ref_0 &= \text{Value of reference asset at initial day}, \\
ExchangeRate_t &= \text{Exchange rate at day t}, \\
ExchangeRate_{t-1} &= \text{Exchange rate at day t-1},
\end{aligned} \tag{8}
$$

The interest period refers to the period from the day before the valuation until the valuation day and applicable interest rate convention is actual number of days divided by 360.

The exchange rate refers to the exchange rate USD/SEK, expressed as number of SEK per USD.

After rolling over to a new future contract, $NumOfRef_{t-1}$ is changed to

$$
\begin{aligned}
NumOfRef_{t-1}(after\,rolling) &= NumOfRef_{t-1}(before\,rolling) \cdot \\
&\quad \cdot \tfrac{Ref_{t-1}(before\,rolling)}{Ref_{t-1}(after\,rolling)},
\end{aligned}
$$

where $Ref_{t-1}(before\,rolling)$ refers to the price of the earlier contract and $Ref_{t-1}(after\,rolling)$ refers to the price of the new contract.

Valuations for the rest of the Exchange Trading Certificates can be found at [3].

2.3.3 Risks involved

Each certificate issued by SEB comes with a stop loss. The stop loss is set for the leverage which vary over time (7, 8). For example the leverage for commodities is calculated as

$$
Leverage_t = \frac{NumOfRef_t \cdot Ref_t \cdot ExchangeRate_t}{CV_t}.
$$

There is a margin between the stop loss level and the point where the issuer cannot unwind their positions in the reference asset without loosing money. Therefore, it is important to estimate the probability of breaching the stop loss and if breaching, estimate how much the leverage passes the stop loss level, i.e. the jump risk. This is important because the investor will not loose more than invested capital, i.e. the issuer is obligated to cover for any losses resulting from unwinding positions. If the stop loss is breached, the investor will get back the value based on the price where the issuer is able to unwind there positions at, with a minimum of zero.

2.4 Test Methods

Through the thesis many mathematically tests are used to confirm, strengthen or dismiss assumption that are given. The tests can be found under this subsection.

2.4.1 Q-Q Plot

A Q-Q plot gives a visual estimate on how similar a distribution is to another. The quantiles of two distributions are plotted against each other. The CDF of a distribution can be divided into q equally sized subsets. The point where two of those subsets meet is called a q-quantile. The value of the k:th q-quantile is the point x where the probability of a random variable is less then x where it has it's max $\frac{k}{q}$ and the probability that the random variable is larger or equal to x is at least $\frac{q-k}{q}$. These quantile values are the ones that build the Q-Q plot.

2.4.2 Maximum Likelihood Method

Maximum Likelihood Estimator, MLE, is a widely used method in statistics to estimate the parameters for a statistical model or distribution. When applying the method to a data set, it provides estimates for the model's parameters.

The method takes the parameters for the model that gives the greatest probability of fitting the distribution. This means the parameters that maximize the likelihood function.

If a sample of n independent and identically distributed random variables (iid) x_1, x_2,..., x_n is observed, which are coming from a certain distribution where the pdf $f_0(.)$ is unknown. However, the function $f_0(.)$ belongs to a certain family of distributions $\{f(|\theta), \theta \epsilon \Theta\}$, called the parametric model, so that $f_0 = f(.|\theta_0)$ where θ_0 is unknown.

The goal is therefore to find an estimator $\hat{\theta}$ which would be as close as possible to the true value θ_0.

The first step of the MLE is to build the joint density function for the iid data set

$$f(x_1, x_2, \ldots, x_n|\theta) = f(x_1|\theta) \cdot f(x_1|\theta) \cdots f(x_n|\theta).$$

If the parameters x_1, x_2,..., x_n are fixed and the function θ is variable, one comes to the distribution function called likelihood

$$L(\theta|x_1, \ldots, x_n) = f(x_1, x_2, \ldots, x_n|\theta) = \prod_{i=1}^{n} f(x_i|\theta).$$

It's often much easier to work with the logarithm of the likelihood function, log-likelihood.

$$\ln L(\theta|x_1, \ldots, x_n) = \sum_{i=1}^{n} \ln f(x_i|\theta), \ \hat{l} = \frac{1}{n} \ln L.$$

And when taking the maximum of the average of the log-likelihood function the maximum likelihood estimator (MLE) of θ_0 is received

$$\hat{\theta}_{mle} = argmax_{\theta \epsilon \Theta} \hat{l}(\theta|x_1, \ldots, x_n).$$

2.4.3 Kolmogorov-Smirnov Method

The Kolmogorov-Smirnov method is a nonparametric test for the equality of continuous, one-dimensional probability distributions that can be used to compare a sample with a probability distribution. This is called a one-sample K-S Test, or when comparing two samples it is called two-sample K-S test. The test gives the maximum distance between the empirical distribution function from the sample and the cumulative distribution function of the reference distribution. The null hypothesis is that the samples are drawn from the reference distribution. The test statistic is given by

$$KS = max|F_n(x) - F(x)|,$$

where $F_n(x)$ is the empirical distribution function:

$$F_n(x) = \frac{1}{n}\sum_{i=1}^{n} I_{X_i \leq x},$$

where $F(x)$ is the fitted distribution function and $I_{X_i \leq x}$ is the indicator function.

3 Implementation

This master thesis is in whole based on the Black-Scholes model which is widely used in the financial market. To show that the model has it's disadvantages and deficiencies a couple of tests were run on historical market data. The data series picked for the tests were the same as the underlying assets in the exchange-traded certificates. The number of daily closing prices that were used ranged between 10 and 20 years depending on the availability of data for the different assets. The prices that the commodity certificates are based on are forward contracts that are rolled monthly. Due to the difficulties to get historical prices for rolled forward contracts, daily spot prices were used for simplicity.

3.1 Tests on the Black-Scholes Model

For the Black-Scholes model the increments are assumed to be standard normal distributed and that the model fulfill the criteria of a Lévy process [1, s. 25]. Then by calculating devolatilized log increments, equation (2), from $N+1$ days of historical data S_n, it can be used in distribution tests.

3.1.1 Maximum Likelihood Estimation

The first test that was done to the log increments were the maximum likelihood estimation, section (2.4.2). The parameters that fit a normal distribution to the empirical data, the log increments, were calculated with the Matlab function mle. The parameters were calculated for some of the underlying assets to confirm if the assumption that the log increments are not standard normal distributed.

3.1.2 Q-Q Plot

The parameters received from the MLE were used to creat Q-Q plots, section (2.4.1), in Matlab, with the command qqplot. From the pictures one can visually see if the log increments fit a standard normal distribution.

3.1.3 Kolmogorov-Smitnov Test

To even get more information, in form of a numerical result, on how well the log increments fits to a normal distribution, the Kolmogorov-Smirnov goodness of fit test, section (2.4.3), was done on the increments. The largest distance between the empirical data and the normal distribution were calculated with the Matlab command kstest.

3.2 Building the Jump-Diffusion Process

To develop the Black-Scholes model the idea was to include a jump process to the Brownian motion. By the definitions of a jump the threshold was calculated as in equation (3) with $\ln X_t - \ln X_{t-1}$ as the size of a jump. This level was set depending on the volatility of the underlying asset. The minimum jump size for commodities was set to 2% and for currencies it was set to 1%. The thresholds were then calculated based on these assumptions.

3.2.1 Get Jumps

For each series of log increments, one series each based on historical data for the different underlying assets, upward jumps, downward jumps and the jump durations were calculated and sorted out based on the definitions (2.2.1 ,2.2.2 ,2.2.3) and certain thresholds α. Though the downward jumps are always negative, the absolute value of the sizes were taken for simplicity.

The hypothesis that upward jumps and downward jumps have different distributions was tested by looking at histograms over the jump sizes and durations. Numerical results were calculated by counting the number of upward and downward jumps and also looking at the maximum and minimum value of the jump sizes.

3.2.2 Distribution Tests

To find an approximation of the distribution for upward and downward jump sizes and the jump durations, a couple of tests were done on the jump data. Some of the tests gave visual results and to determine which distribution that gives the best approximation, the results from the Kolmogorov-Smirnov goodness of fit test (2.4.3) were used as a numerical measure.

Three different distributions were tested for both jump sizes and durations. Those were the Exponential distribution, the Gamma distribution and the Generalized Pareto distribution (2.2.6). These have different tail behavior and skew which gave a wide range of possible types of distributions. To fit the empirical data, i.e. the upward and downward jump sizes and durations, to the chosen distributions the MLE (2.4.2) were used. The parameters received from the MLE were then used to create CDFs, these were compared to CDFs of the empirical data with the Kolmogorov-Smirnov test, that gave the largest distance between the compared CDFs. The results could then be visually confirmed by plotting the two CDFs in the same graph.

It's well known in finance that increments are not independent of each other, i.e. big movements tend to come in cluster. By plotting the jump sizes over time one could visually see if that was true. Due to this conclusion another process was tested for upward and downward jump durations, the AR(1) model (2.2.6). By taking the logarithm of jump up and jump down durations, and estimating the parameters a_0 and a_1 by least square method and σ by the residual by MLE (2.4.2) one gets the parameters for the autoregressive model. By simulating durations from the AR(1) model and building CDFs the Kolmogorov-Smirnov test were used again to find the maximum distance between the CDFs. The simulation and the Kolmogorov-Smirnov test were run 1000 iterations to get good estimates of the results from the KS-test.

3.2.3 Choice of Distributions

From the results of KS-tests, distributions were chosen for upward jumps sizes, downward jump sizes, durations between upward jumps and durations between downward jumps. The distribution that had the lowest Kolmogorov-Smirnov distance was chosen as the proper distribution.

3.2.4 The Final Process

By combining the Black-Scholes model (1) with processes from the chosen distributions in (3.2.3) the final process becomes (6). This model has much more flexibility to replicate real market behavior.

3.3 Simulate Asset Prices With the Jump Diffusion Model

From the results of fitting jumps to different distributions the final jump diffusion model (6) were used to simulate future scenarios for the underlying asset. The simulation algorithm looks like

- Calculate the drift μ as the mean of historical data in form of log increments when jumps are excluded.
- Calculate the standard deviation from historical data with a moving window size of five days. Build a vector of the calculated values.
- Define a time vector $t_0, \dots, t_N$, where N is the number of days. (The time between issue date to maturity are for most of the Certificates 10 years, i.e. approximately 2500 days, hence N is set to 2500).
- Simulate N upward jump sizes from the chosen distribution with parameters received from the method under subsection (3.2), $\{Y_i^1\}_{i=1}^N$.
- Simulate N downward jump sizes from the chosen distribution with parameters received from the method under subsection (3.2), $\{Y_i^2\}_{i=1}^N$.
- Simulate upward jump durations from the chosen distribution with parameters received from the method under subsection (3.2) and take the cumulative sum to build a vector $\{U_i^1\}_{i=1}^N$.
- Simulate downward jump durations from the chosen distribution with parameters received from the method under subsection (3.2) and take the cumulative sum to build a vector $\{U_i^2\}_{i=1}^N$.
- The jump process is then given by $X_t = \sum_{i=1}^{N_T} Y_i^1 \cdot 1_{U_i^1 \leq t} - \sum_{i=1}^{N_T} Y_i^2 \cdot 1_{U_i^2 \leq t}$.
- Uniformly randomly draw of N values of σ from the vector of possible σ-values.
- Draw N randomly values from the standard normal distribution to create the process B_t.
- The Jump Diffusion process is then given by

$$S_t = S_0 e^{\mu t + \sigma_t B_t + X_t = \sum_{i=1}^{N_T} Y_i^1 \cdot 1_{U_i^1 \leq t} - \sum_{i=1}^{N_T} Y_i^2 \cdot 1_{U_i^2 \leq t}}. \tag{9}$$

3.4 Calculate the Prices for Exchange Traded Certificates

SEB is issuing a wide range of exchange traded certificates. Simulated prices were calculated for the certificates in table (1)

Table 1: The names of the 12 certificates tested in this master thesis.

EURSEK Bull	EURSEK Bear
USDSEK Bull	USDSEK Bear
Gold Bull	Gold Bear
Silver Bull	Silver Bear
Oil Bull	Oil Bear
Wheat Bull	Wheat Bear

The prices for certificates are calculated as in subsection (2.3.2), where the constant values can be found in table (2).

Table 2: Constants for calculations of certificate prices.

$InterestRateBase^{SEK}$ is $Stibor$	$=$	0.02
$InterestRateBase^{EUR}$ is $Eonia$	$=$	0.015
$InterestRateMargin^{SEK}$	$=$	0.002
$InterestRateMargin^{EUR}$	$=$	0.002

As explained in section (2.3) the calculations for the different kind of certificates are pretty similar to each other, but can basically be divided into two different classes, commodities and currencies. No more simulations were needed for the currency certificates though the price is calculated as in equation (7) with the simulated data from section (3.3) used as Ref_t. For the commodities one more simulation were done in the same manner as before because a simulation for the price of the commodity and for price of USDSEK were needed. Then the price of the commodity certificate are calculated as in equation (8).

3.5 Probability of Exceeding Stop Losses

The different exchange-traded certificates comes with different level of stop losses. There is a risk that the price of the certificate exceeds the level of the stop loss and a risk that the crossing is done by a discrete jump. The probability of crossing those levels can be approximated by repeated simulations of future time series of certificate prices. Therefore 1000 simulations for each certificate were simulated to receive a good estimate of the probability of crossing the stop loss level and if crossing, estimate the size of the jump through the level.

3.5.1 Valuation of the Results

To see how sensitive the simulation of the future time series is, depending on the parameters for the distributions, more simulations were done. By randomly draw the values of the parameters from a normal distribution with the earlier used value as mean μ and standard deviation $\sigma = 0.1\mu$. The simulation was run 1000 times and the results are presented as a probability distribution and numerical results as max, min, mean and standard deviation.

4 Results

All the numerical and visual results from the implementation in section (3) are presented below. The visual results are presented for two different underlying assets and certificates.

4.1 Valuation of the Black-Scholes Model

To se how well the Black-Scholes model fitted to historical data, daily close prices for gold and USD/SEK, where tested. The close prices and log increments of the prices are plotted in figures (1, 2).

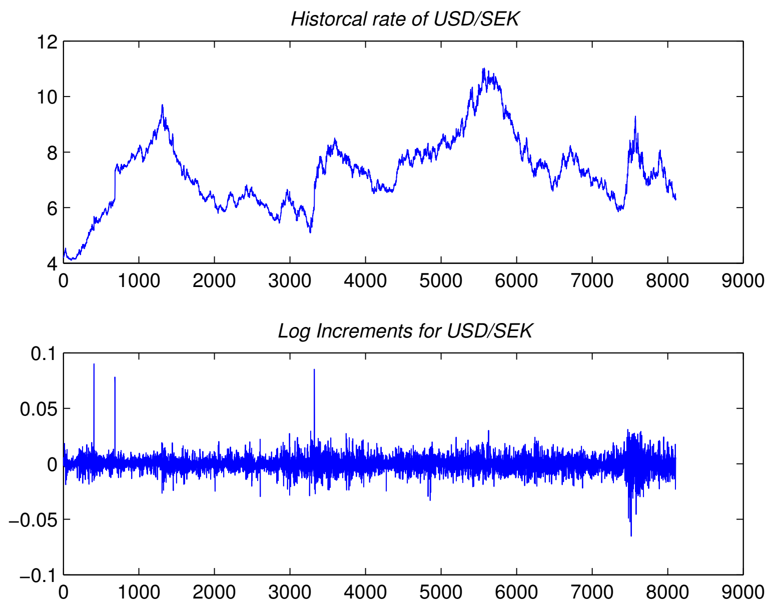

Figure 1: The top picture illustrates the daily rate of USD/SEK.
The bottom picture shows the corresponding log increments of USD/SEK.

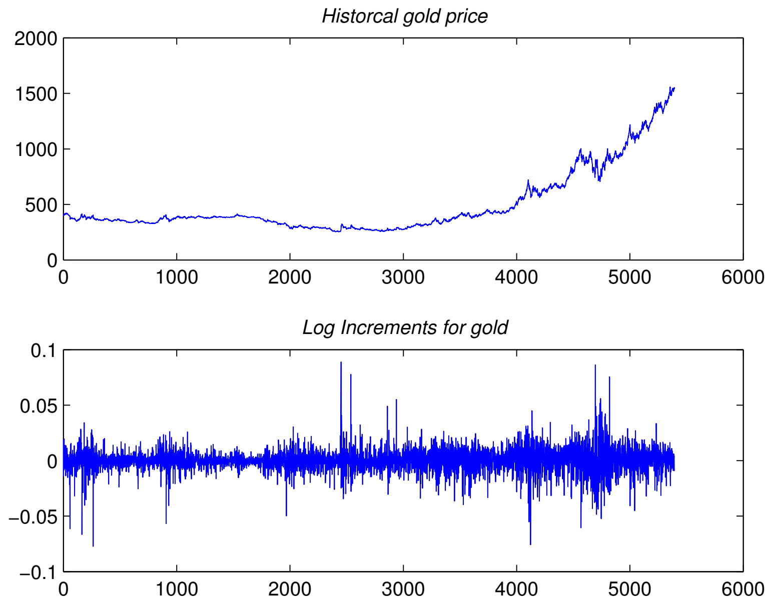

Figure 2: The top picture illustrates the daily price of gold in USD.
The bottom picture shows the corresponding log increments of gold.

The log increments of the historical data were fitted to a normal distribution with MLE (2.4.2), and the parameters received can be found in table (3).

Table 3: Parameters received from MLE with normal distribution.

Asset	Mean	Standard Deviation
USD/SEK	0.0001	0.007
Gold	0.0002	0.0101

To visually see the results, Q-Q plots were created and can be seen in figure (3). The conclusion that the log increments does not fit to a normal distribution can easily be drawn since the market data are much more heavy-tailed.

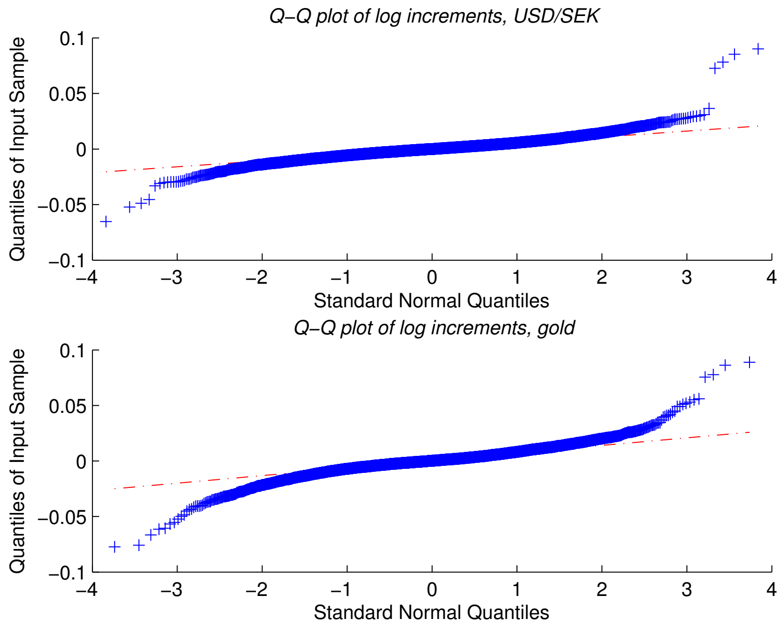

Figure 3: Q-Q plots over log increments for USD/SEK and gold.

To further strengthen this conclusion a Kolmogorov-Smirnov test was run on the data. The results are displayed in table (4). It shows that the KS-distance are pretty big and the test rejects the null hypothesis that the data belongs to a normal distribution.

Table 4: Results from KS-test with normal distribution.

Asset	KS-Distance	Rejects null hypothesis	p-value
USD/SEK	0.4878	Yes	0
Gold	0.4828	Yes	0

4.2 Results From Building a Jump Diffusion Model

The tests on the Black-Scholes model showed that it had weaknesses and a jump-process might be a proper choice of a better model. The definition of jump sizes and durations were given. The chosen thresholds for the different assets can be found in table (5).

Table 5: Chosen thresholds for jumps.

Asset	Threshold
USD/SEK	1.4153
EUR/SEK	1.9834
Gold	1.9380
Silver	1.0853
Oil	0.8403
Wheat	1.5054

4.2.1 Jumps of Historical Data

By the definition of jumps (2.2.1, 2.2.2), upward and downward jumps and upward and downward durations were taken from historical data. The data are gathered in tables (6) and (7) and is visualized in figure (4) and (5). It shows that upward and downward jump sizes do not follow the same distribution. The results show that there tend to be more upward jumps but the downward jump sizes tend to be larger. When looking at the results from the duration, the durations between upward jumps are longer than for downward jumps.

Table 6: Upward and downward jump sizes.

Asset	Num. of upward jumps	Total Days	Max	Mean
USD/SEK	494	8107	0.0901	0.0151
EUR/SEK	146	5799	0.0938	0.0149
Gold	134	5392	0.0889	0.0283
Silver	534	5393	0.1220	0.0309
Oil	800	5458	0.1315	0.0343
Wheat	152	3195	0.1094	0.0317
Asset	Num. of downward jumps	Total Days	Max	Mean
USD/SEK	500	8107	0.0653	0.0144
EUR/SEK	137	5799	0.0315	0.0140
Gold	168	5392	0.0773	0.0285
Silver	519	5393	0.1479	0.0345
Oil	790	5458	0.4272	0.0356
Wheat	130	3195	0.2336	0.0336

Table 7: Upward and downward jump durations.

Asset	Num. of upward durations	Total Days	Max	Min	Mean
USD/SEK	493	8107	304	1	16.4037
EUR/SEK	145	5799	882	1	38.6759
Gold	133	5392	909	1	38.2857
Silver	533	5393	279	1	10.0938
Oil	799	5458	141	1	6.8273
Wheat	151	3195	710	1	21.0464
Asset	Num. of downward jumps	Total Days	Max	Min	Mean
USD/SEK	499	8107	163	1	16.1784
EUR/SEK	136	5799	976	1	41.2426
Gold	167	5392	855	1	31.7545
Silver	518	5393	206	1	10.2432
Oil	789	5458	101	1	6.9113
Wheat	129	3195	566	1	24.7442

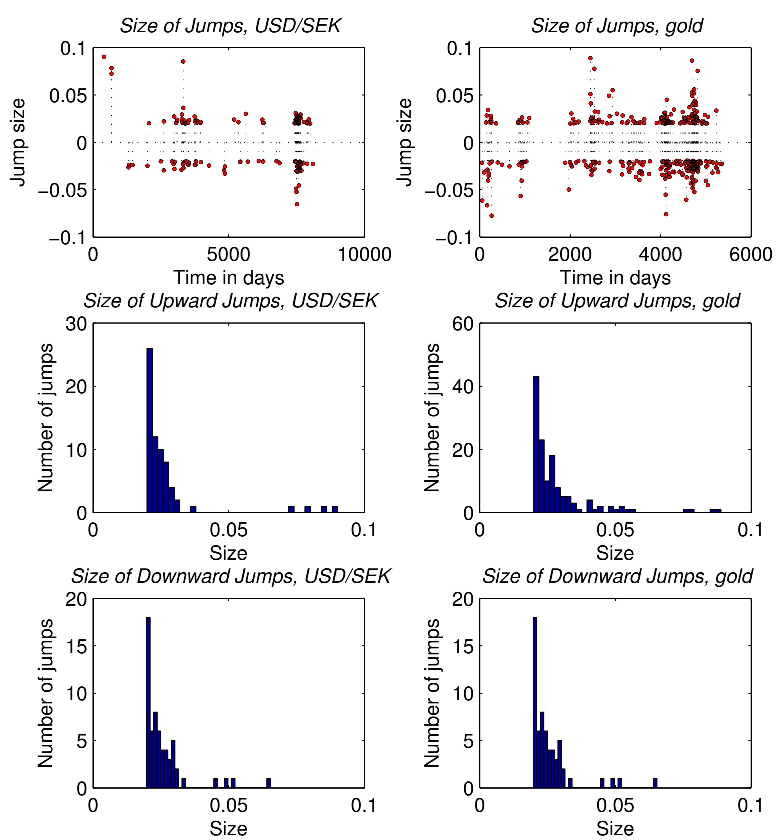

Figure 4: Jump sizes for USD/SEK and gold.

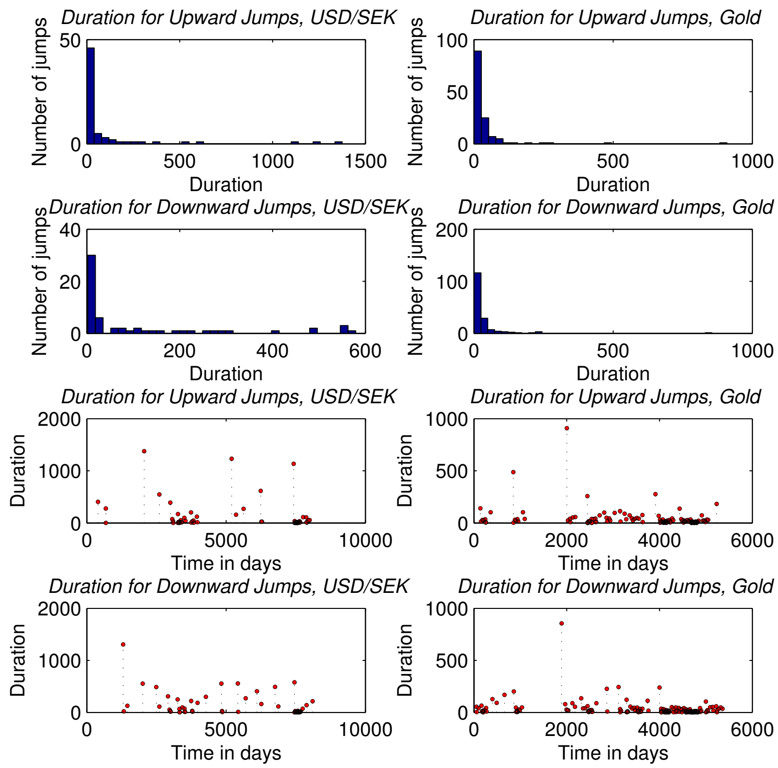

Figure 5: Jump durations for USD/SEK and gold.

4.2.2 Distribution of Jump Sizes and Jump Durations

Different distributions were used to test which distribution that fitted historical data the best. The parameters from the MLE can be found in tables (8, 9, 10, 11).

Table 8: Parameters for upward jump sizes received from MLE with different distributions.

Asset	Distribution	Parameters
USD/SEK	Exponential	$\lambda = 0.0151$
	Generalized Pareto	$\xi = -0.1588$, $\sigma = 0.0169$
	Gamma	$k = 8.5991$, $\Theta = 0.0018$
EUR/SEK	Exponential	$\lambda = 0.0149$
	Generalized Pareto	$\xi = -0.1319$, $\sigma = 0.0164$
	Gamma	$k = 8.3334$, $\Theta = 0.0018$
Gold	Exponential	$\lambda = 0.0283$
	Generalized Pareto	$\xi = -0.3883$, $\sigma = 0.0371$
	Gamma	$k = 8.1193$, $\Theta = 0.0035$
Silver	Exponential	$\lambda = 0.0309$
	Generalized Pareto	$\xi = -0.2981$, $\sigma = 0.0376$
	Gamma	$k = 7.9901$, $\Theta = 0.0039$
Oil	Exponential	$\lambda = 0.0343$
	Generalized Pareto	$\xi = -0.3044$, $\sigma = 0.0423$
	Gamma	$k = 6.6371$, $\Theta = 0.0052$
Wheat	Exponential	$\lambda = 0.0317$
	Generalized Pareto	$\xi = -0.3446$, $\sigma = 0.0398$
	Gamma	$k = 7.7282$, $\Theta = 0.0041$

Table 9: Parameters for downward jump sizes received from MLE with different distributions.

Asset	Distribution	Parameters
USD/SEK	Exponential	$\lambda = 0.0144$
	Generalized Pareto	$\xi = -0.2466$, $\sigma = 0.0168$
	Gamma	$k = 10.2530$, $\Theta = 0.0014$
EUR/SEK	Exponential	$\lambda = 0.0140$
	Generalized Pareto	$\xi = -0.6386$, $\sigma = 0.0203$
	Gamma	$k = 14.6415$, $\Theta = 0.0010$
Gold	Exponential	$\lambda = 0.0285$
	Generalized Pareto	$\xi = -0.4877$, $\sigma = 0.0388$
	Gamma	$k = 10.4936$, $\Theta = 0.0027$
Silver	Exponential	$\lambda = 0.0345$
	Generalized Pareto	$\xi = -0.2630$, $\sigma = 0.0417$
	Gamma	$k = 5.7506$, $\Theta = 0.0060$
Oil	Exponential	$\lambda = 0.0356$
	Generalized Pareto	$\xi = -0.0637$, $\sigma = 0.0374$
	Gamma	$k = 5.0001$, $\Theta = 0.0071$
Wheat	Exponential	$\lambda = 0.0336$
	Generalized Pareto	$\xi = -0.0903$, $\sigma = 0.0363$
	Gamma	$k = 4.4301$, $\Theta = 0.0076$

Table 10: Parameters for upward jump durations received from MLE with different distributions.

Asset	Distribution	Parameters
USD/SEK	Exponential	$\lambda = 16.4037$
	Generalized Pareto	$\xi = 0.4323$, $\sigma = 9.6570$
	Gamma	$k = 0.7520$, $\Theta = 21.8140$
EUR/SEK	Exponential	$\lambda = 38.6759$
	Generalized Pareto	$\xi = 0.8479$, $\sigma = 11.6557$
	Gamma	$k = 0.5120$, $\Theta = 75.5460$
Gold	Exponential	$\lambda = 38.2857$
	Generalized Pareto	$\xi = 0.5587$, $\sigma = 16.8673$
	Gamma	$k = 0.6004$, $\Theta = 63.7656$
Silver	Exponential	$\lambda = 10.0938$
	Generalized Pareto	$\xi = 0.3880$, $\sigma = 6.0779$
	Gamma	$k = 0.8271$, $\Theta = 12.2039$
Oil	Exponential	$\lambda = 6.8273$
	Generalized Pareto	$\xi = 0.1353$, $\sigma = 5.8448$
	Gamma	$k = 1.2219$, $\Theta = 5.5875$
Wheat	Exponential	$\lambda = 21.0464$
	Generalized Pareto	$\xi = 0.7396$, $\sigma = 6.4800$
	Gamma	$k = 0.5158$, $\Theta = 40.8070$

Table 11: Parameters for downward jump durations received from MLE with different distributions.

Asset	Distribution	Parameters
USD/SEK	Exponential	$\lambda = 16.1784$
	Generalized Pareto	$\xi = 0.3234$, $\sigma = 11.0149$
	Gamma	$k = 0.8573$, $\Theta = 18.8715$
EUR/SEK	Exponential	$\lambda = 41.2426$
	Generalized Pareto	$\xi = 0.6520$, $\sigma = 16.6681$
	Gamma	$k = 0.5972$, $\Theta = 69.0614$
Gold	Exponential	$\lambda = 31.7545$
	Generalized Pareto	$\xi = 0.6154$, $\sigma = 13.4608$
	Gamma	$k = 0.5991$, $\Theta = 53.0062$
Silver	Exponential	$\lambda = 10.2432$
	Generalized Pareto	$\xi = 0.2065$, $\sigma = 8.0678$
	Gamma	$k = 0.9518$, $\Theta = 10.7621$
Oil	Exponential	$\lambda = 6.9113$
	Generalized Pareto	$\xi = 0.1584$, $\sigma = 5.7642$
	Gamma	$k = 1.1273$, $\Theta = 6.1306$
Wheat	Exponential	$\lambda = 24.7442$
	Generalized Pareto	$\xi = 0.8056$, $\sigma = 6.8927$
	Gamma	$k = 0.4871$, $\Theta = 50.7988$

The assumptions that durations are correlated were tested and the results can be seen in figure (5) of durations over time. Another process was therefore tested on the durations, in addition to the previous distributions, the AR(1) process. The fitted parameters for the AR(1) process are based on 1000 simulations and can be found in tables (12, 13).

Table 12: Parameters for upward jump durations with AR(1) processes.

Asset	a_0	a_1	σ
USD/SEK	1.7568	0.1207	1.2724
EUR/SEK	1.9660	0.1791	1.5531
Gold	2.2558	0.1315	1.3968
Silver	1.4927	0.0635	1.1082
Oil	1.3626	0.0672	0.9263
Wheat	1.2627	0.2970	1.4022

Table 13: Parameters for downward jump durations with AR(1) processes.

Asset	a_0	a_1	σ
USD/SEK	1.7568	0.1665	1.1816
EUR/SEK	2.3333	0.1322	1.3943
Gold	2.0516	0.1499	1.4094
Silver	1.4703	0.1394	1.1262
Oil	1.3226	0.0747	0.9804
Wheat	1.5958	0.1510	1.4822

CDFs were then created from the results of the MLE and the comparison between them and empirical CDFs are presented in figures (6, 7, 8, 9).

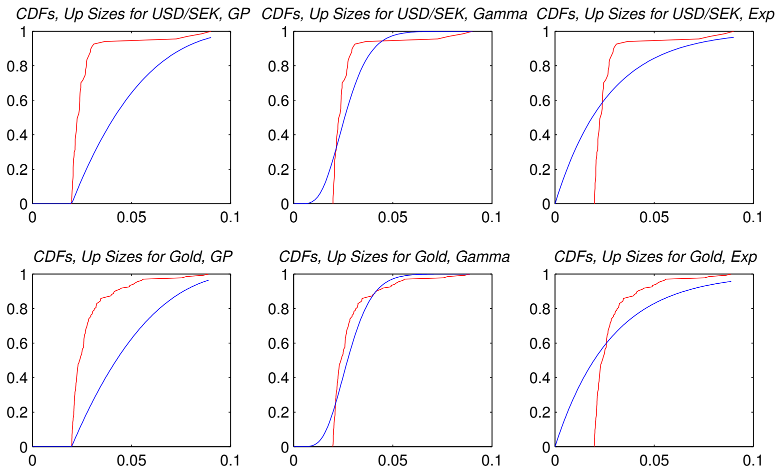

Figure 6: Fitted CDFs of upward jump sizes compared to empirical CDFs for USD/SEK and gold.

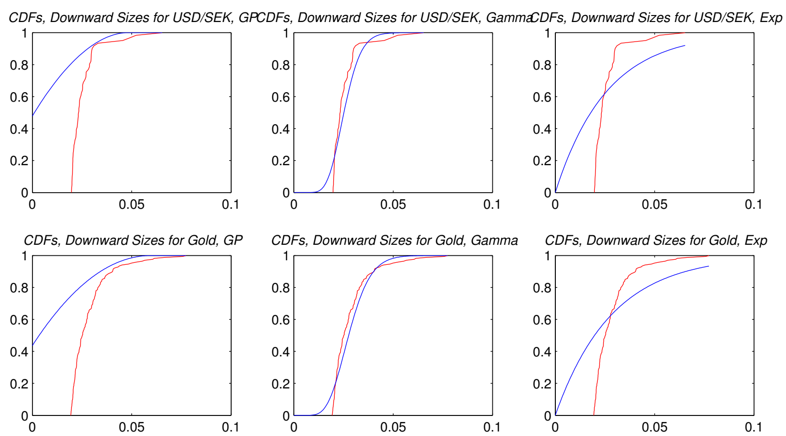

Figure 7: Fitted CDFs of downward jump sizes compared to empirical CDFs for USD/SEK and gold.

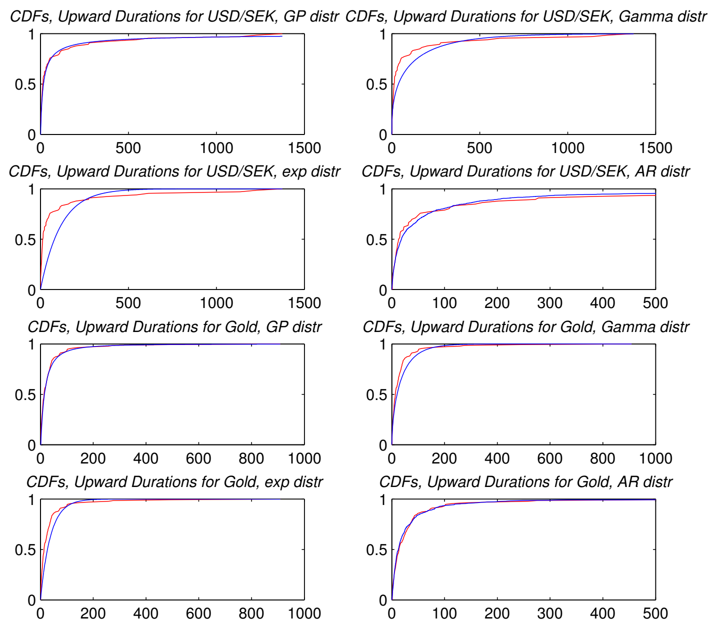

Figure 8: Fitted CDFs of upward jump durations compared to empirical CDFs for USD/SEK and gold.

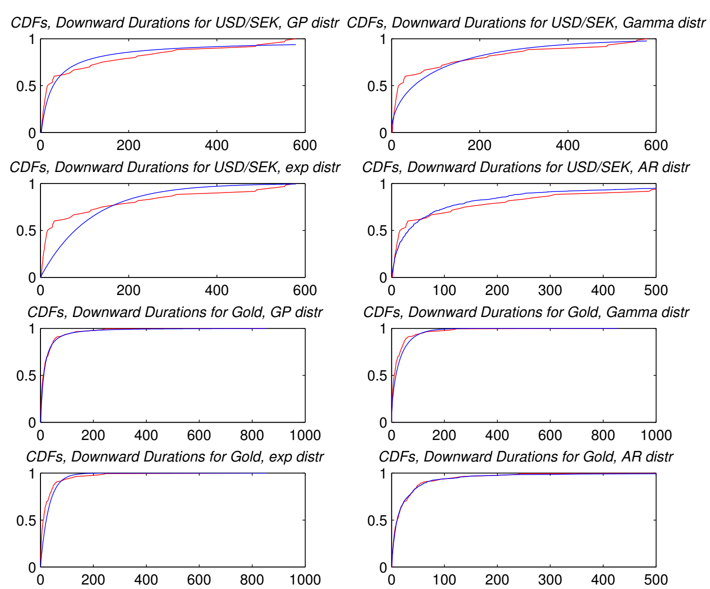

Figure 9: Fitted CDFs of downward jump durations compared to empirical CDFs for USD/SEK and gold.

The largest distance between the CDFs that are given from the Kolmogorov-Smirnov goodness of fit test are presented in tables (14, 15, 16, 17).

Table 14: Results from KS-tests on upward jump sizes for the different distributions.

Asset	Distribution	KS-Distance	Rejects null hyp.	p-value
USD/SEK	Exponential	0.4830	Yes	0
USD/SEK	Generalized Pareto	0.4354	Yes	0
USD/SEK	Gamma	0.1532	Yes	0
EUR/SEK	Exponential	0.4900	Yes	0
EUR/SEK	Generalized Pareto	0.4466	Yes	0
EUR/SEK	Gamma	0.1712	Yes	0
Gold	Exponential	0.5029	Yes	0
Gold	Generalized Pareto	0.5232	Yes	0
Gold	Gamma	0.1997	Yes	0
Silver	Exponential	0.4733	Yes	0
Silver	Generalized Pareto	0.4134	Yes	0
Silver	Gamma	0.1475	Yes	0
Oil	Exponential	0.4391	Yes	0
Oil	Generalized Pareto	0.3708	Yes	0
Oil	Gamma	0.1233	Yes	0
Wheat	Exponential	0.4663	Yes	0
Wheat	Generalized Pareto	0.4260	Yes	0
Wheat	Gamma	0.1487	Yes	0.0021

Table 15: Results from KS-tests on downward jump sizes for the different distributions.

Asset	Distribution	KS-Distance	Rejects null hyp.	p-value
USD/SEK	Exponential	0.4959	Yes	0
USD/SEK	Generalized Pareto	0.7489	Yes	0
USD/SEK	Gamma	0.1509	Yes	0.0229
EUR/SEK	Exponential	0.5061	Yes	0
EUR/SEK	Generalized Pareto	0.7833	Yes	0
EUR/SEK	Gamma	0.1288	Yes	0.0193
Gold	Exponential	0.4933	Yes	0
Gold	Generalized Pareto	0.7457	Yes	0
Gold	Gamma	0.1427	Yes	0.0019
Silver	Exponential	0.4248	Yes	0
Silver	Generalized Pareto	0.6494	Yes	0
Silver	Gamma	0.1261	Yes	0
Oil	Exponential	0.4173	Yes	0
Oil	Generalized Pareto	0.6543	Yes	0
Oil	Gamma	0.1372	Yes	0
Wheat	Exponential	0.4397	Yes	0
Wheat	Generalized Pareto	0.6766	Yes	0
Wheat	Gamma	0.1877	Yes	0

Table 16: Results from KS-tests on upward jump durations for the different distributions.

Asset	Distribution	KS-Distance	Rejects null hyp.	p-value
USD/SEK	Exponential	0.1739	Yes	0
USD/SEK	Generalized Pareto	0.1327	Yes	0
USD/SEK	Gamma	0.1203	Yes	0
USD/SEK	AR(1)	0.7744	Yes	0
EUR/SEK	Exponential	0.2785	Yes	0
EUR/SEK	Generalized Pareto	0.1368	Yes	0.0079
EUR/SEK	Gamma	0.1413	Yes	0.0055
EUR/SEK	AR(1)	0.7975	Yes	0
Gold	Exponential	0.2097	Yes	0
Gold	Generalized Pareto	0.0690	No	0.5291
Gold	Gamma	0.1357	Yes	0.0135
Gold	AR(1)	0.8352	Yes	0
Silver	Exponential	0.1797	Yes	0
Silver	Generalized Pareto	0.1440	Yes	0
Silver	Gamma	0.1512	Yes	0
Silver	AR(1)	0.8248	Yes	0
Oil	Exponential	0.1362	Yes	0
Oil	Generalized Pareto	0.1433	Yes	0
Oil	Gamma	0.1243	Yes	0
Oil	AR(1)	0.8068	Yes	0
Wheat	Exponential	0.3215	Yes	0
Wheat	Generalized Pareto	0.1368	Yes	0.0063
Wheat	Gamma	0.1810	Yes	0
Wheat	AR(1)	0.8607	Yes	0

Table 17: Results from KS-tests on downward jump durations for the different distributions.

Asset	Distribution	KS-Distance	Rejects null hyp.	p-value
USD/SEK	Exponential	0.1321	Yes	0
USD/SEK	Generalized Pareto	0.0925	Yes	0
USD/SEK	Gamma	0.1081	Yes	0
USD/SEK	AR(1)	0.6818	Yes	0
EUR/SEK	Exponential	0.2713	Yes	0
EUR/SEK	Generalized Pareto	0.1002	No	0.1216
EUR/SEK	Gamma	0.1717	Yes	0
EUR/SEK	AR(1)	0.8191	Yes	0
Gold	Exponential	0.2213	Yes	0
Gold	Generalized Pareto	0.1058	Yes	0.0441
Gold	Gamma	0.1239	Yes	0.0108
Gold	AR(1)	0.8342	Yes	0
Silver	Exponential	0.1110	Yes	0
Silver	Generalized Pareto	0.1539	Yes	0
Silver	Gamma	0.1013	Yes	0
Silver	AR(1)	0.7994	Yes	0
Oil	Exponential	0.1347	Yes	0
Oil	Generalized Pareto	0.1753	Yes	0
Oil	Gamma	0.1143	Yes	0
Oil	AR(1)	0.7653	Yes	0
Wheat	Exponential	0.3118	Yes	0
Wheat	Generalized Pareto	0.1480	Yes	0.0062
Wheat	Gamma	0.1579	Yes	0.0028
Wheat	AR(1)	0.8284	Yes	0

The Kolmogorov-Smirnov test rejects the null hypothesis that the data belongs to the tested distribution, for all distributions. But the distributions that had the smallest Kolmogorov-Smirnov distances were chosen as proper distributions. The chosen distributions for each asset can be seen in table (18).

Table 18: Chosen distributions for upward and downward jump sizes and durations.

Asset	Up Sizes	Down Sizes	Up Durations	Down Durations
USD/SEK	Gamma	Gamma	Gamma	Generalized Pareto
EUR/SEK	Gamma	Gamma	Generalized Pareto	Generalized Pareto
Gold	Gamma	Gamma	Generalized Pareto	Generalized Pareto
Silver	Gamma	Gamma	Generalized Pareto	Gamma
Oil	Gamma	Gamma	Gamma	Gamma
Wheat	Gamma	Gamma	Generalized Pareto	Generalized Pareto

4.3 Simulations of Future Asset Prices

The models received from section (3) were used to simulate possible future scenarios for asset prices. The parameters in tables (8, 9, 10, 11) were used in the model and by the algorithm (3.3) when the time series were simulated. Two examples of series are displayed in figures (10, 11), one for USD/SEK and one for gold.

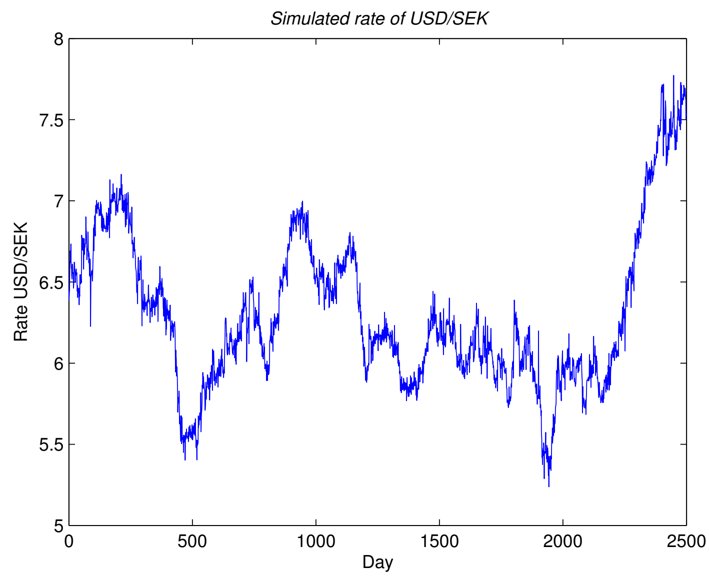

Figure 10: Example of simulated rate of USD/SEK ten years ahead.

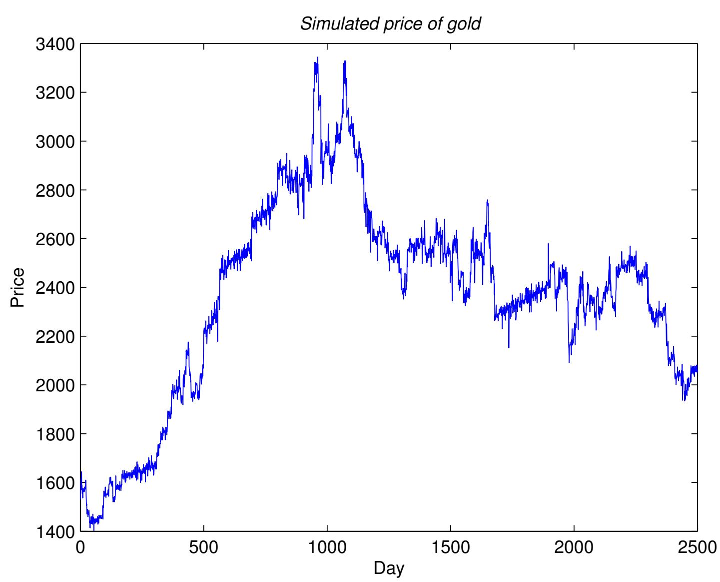

Figure 11: Example of simulated price of gold ten years ahead.

4.4 Calculations of Certificate Prices

The prices of different certificates (Bulls and Bears on currencies and commodities) were explicitly determined from the simulations of the asset prices by equations under section (2.3.2). The corresponding certificate prices of a Gold Bull and USDSEK Bear are found in figures (12, 13).

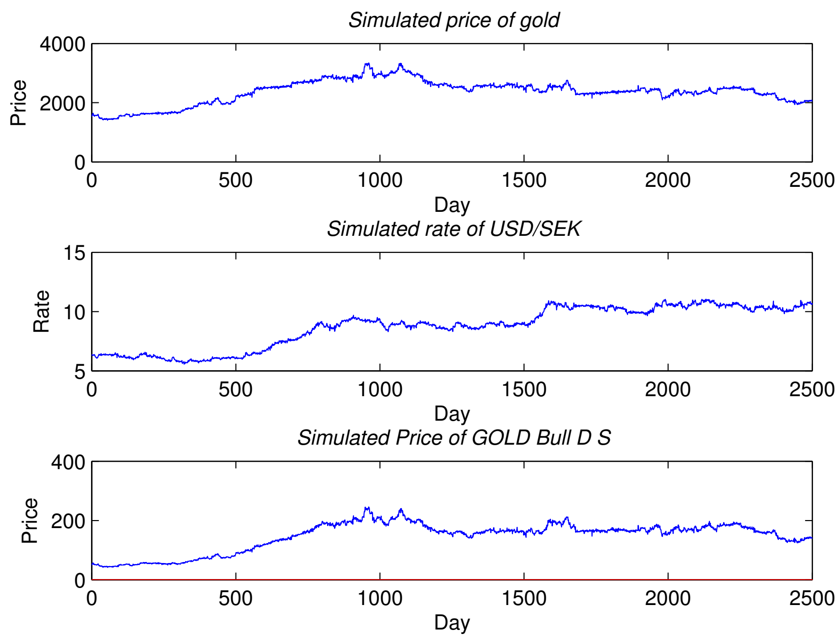

Figure 12: Example of simulated price of gold, rate of USD/SEK and corresponding price of Bull D S ten years ahead.

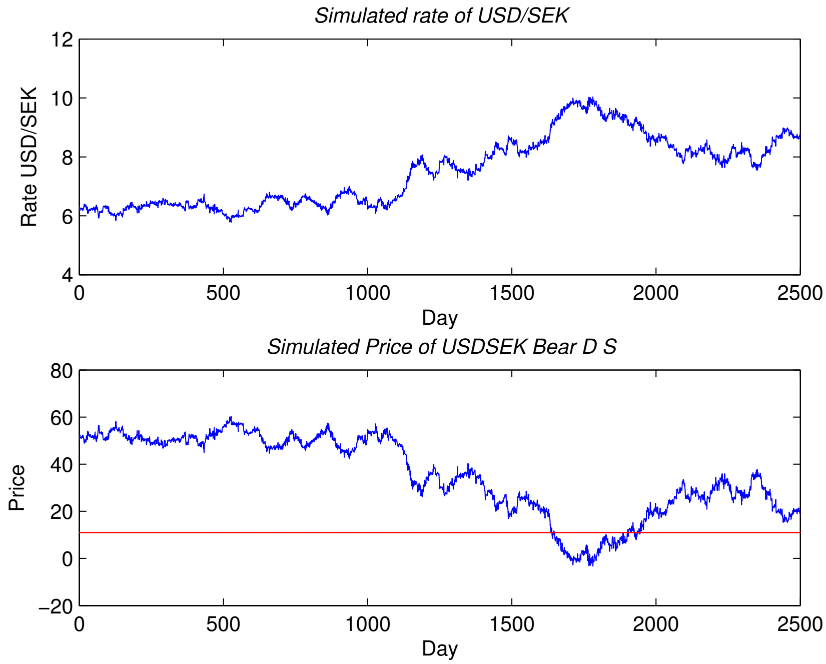

Figure 13: Example of simulated price of USDSEK Bear D S ten years ahead. The line indicates the level of the stop loss.

4.5 Probabilities of Exceeding Stop Losses

By the 1000 simulations for each certificate the probabilities of exceeding the stop losses for the leverages could be estimated. The estimated probabilities of exceeding stop losses within 10 years are put together in table (19). The same table shows the median-, max- and min-values on how much the leverage exceeded the maximum leverage (the stop loss), it is calculated as

$$Size = (\frac{Leverage\ after\ breaking}{Stop\ Loss} - 1) \cdot 100$$

Table 19: The table presents the probability of exceeding stop losses and sizes of the breaking. The median, max and min values are jump size through stop losses calculated as the percentage of the maximum leverage (the stop loss).

Certificate	Prob. of crossing stop loss	Median	Max	Min
USDSEK Bull	5%	5.3142	16.0782 %	1.1063 %
USDSEK Bear	57%	7.3460 %	62.0978 %	0.4313 %
EURSEK Bull	0 %	0 %	0 %	0 %
EURSEK Bear	56 %	5.6096 %	31.3385 %	0.0381 %
Gold Bull	20 %	6.1360 %	71.1140 %	0.2814 %
Gold Bear	74 %	6.1889 %	167.3845 %	0.2049 %
Silver Bull	38 %	7.4643 %	78.1238 %	0.0970 %
Silver Bear	90 %	9.4294 %	143.4668 %	0.1762 %
Oil Bull	44 %	8.8249 %	211.7520 %	0.4047 %
Oil Bear	78 %	10.3002 %	433.1139 %	0.3664 %
Wheat Bull	53 %	5.5904 %	49.9065 %	0.0573 %
Wheat Bear	85 %	4.1883 %	163.7760 %	0.0377 %

4.5.1 Valuation of the Results

By letting the input parameters for the distributions vary by a normal distribution one could se how sensitive the results are. The simulation results can be seen in table (20). It shows that the probability of exceeding a stop loss is similar to the results in table (19). The max values vary a lot as expected due to more extreme input values to the distributions.

Table 20: The table presents the probability of exceeding stop losses and sizes of the breaking. The median, max and min values are jump size through stop losses calculated as the percentage of the maximum leverage (the stop loss). The input parameters vary by a normal distribution.

Certificate	Prob. of crossing stop loss	Median	Max	Min
USDSEK Bull	7%	6.2875	19.1272 %	0.1483 %
USDSEK Bear	55%	5.2150 %	81.1722 %	0.2424 %
EURSEK Bull	0 %	0 %	0 %	0 %
EURSEK Bear	47 %	1.9130 %	27.4839 %	0.1413 %
Gold Bull	28 %	7.5450%	39.6160%	0.1308%
Gold Bear	72 %	6.3173 %	50.1201 %	0.1674 %
Silver Bull	49 %	8.4663 %	108.7231 %	0.4566 %
Silver Bear	77 %	11.7882 %	129.2536 %	0.1277 %
Oil Bull	46 %	7.7372 %	384.5597 %	0.3324 %
Oil Bear	61 %	11.4165 %	917.2793 %	0.0146 %
Wheat Bull	61 %	6.2458 %	96.8745 %	0.1503 %
Wheat Bear	90 %	5.2746 %	98.5555 %	0.0792 %

5 Conclusions

This master thesis is based on the Black-Scholes model and it is shown through tests that the model has its weaknesses. The tests shows that the Black-Scholes model underestimates the tail events. Therefore by adding a jump diffusion process to the Black-Scholes model, a new model is received that can capture the behavior of market data in a better way.

Definitions are given of upward and downward jumps and the durations between them. Different distributions are then fitted to the jumps and durations to find the distributions that imitates market data the best. For most of the assets the Gamma distribution is a proper choice for upward and downward jump sizes and the Generalized Pareto distribution is often the best choice for durations between upward and downward jumps.

Prices for different commodities and currencies are then simulated based on the new jump diffusion model. The simulated data are then used to calculate the price of different certificates issued by SEB. The simulated prices of the certificates are analyzed and the probability of exceeding the stop losses for the leverages of the certificate is estimated. It gives an approximation for the chance of crossing a stop loss and an estimate of how big the jumps through the stop losses can be.

Finally, the parameters are varied for the different distributions to test the goodness of the results. It is shown that the probability of exceeding a stop loss did not differ much from the previous results, but the maximum size of the jumps through the stop losses is even more extreme.

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