CHALMERS UNIVERSITY OF TECHNOLOGY

MASTER THESIS

# Modeling of Bivariate Stock Returns with Copulas

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September 25, 2019

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Chalmers University of Technology, Gothenburg, Sweden 2019

# Abstract

The dependence between stock returns within a diversified portfolio is an important characteristic to identify the risk of the portfolio. Traditionally, the linear correlation can describe the dependence between stock returns with their joint elliptical distributions. However, when the dependence structure between the stock returns is not sure or the joint distribution of stock returns is not elliptical, the linear correlation cannot express the dependence between stock returns. Copula method combined with the bivariate Gaussian model is superior to the traditional bivariate Gaussian model in quantifying the dependence between stock returns, especially for the stock returns that have unknown dependence structure and non-elliptical joint distribution. This thesis is to describe, by giving examples, how to apply the Copula method to the traditional bivariate Gaussian model to get a deeper understanding of bivariate dependencies and the risk of portfolio on stock market. The advantages of the application of Copula method are shown by comparing the kernel density contour plots and surfaces of the simulation data from the Copula models and the bivariate traditional Gaussian models, and risk analysis of the portfolio.

**Key Words:** Gaussian Copula, Gaussian model, Portfolio, Risk

# Acknowledgements

I would like to express my gratitude to my supervisor, Patrik Albin, for his inspiration, help and advice. Meanwhile, I am also very grateful to my family for their understanding, support and help.

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# 1 Introduction

## 1.1 Background and problem statement

In the stock market, the diversified portfolio is one of the most important technologies for reducing investment risk, which means that the investment risk can be reduced by investing in different assets. If the stock prices do not rise or fall in full synchronism, the risk of a well-designed diversified portfolio will be smaller than that of investing only one asset. The risk of a portfolio is strongly associated to the dependence of the stock returns within the diversified portfolio. So dependence analysis of stock returns is an important approach to optimize a diversified portfolio, and provide basis for decision maker to make a low-risk portfolio plan (Guan 2011).

An effective approach to carry out dependence analysis is called Pearson correlation analysis, in which the dependence of two random variables X and Y is quantified using correlation coefficient $\rho = \frac{Cov(X,Y)}{\sqrt{Var(X)}\sqrt{Var(Y)}}$. Due to its simplicity and convenience, it has been widely used for interpretation of data dependence in science. However the Pearson correlation coefficient can only quantify the linear dependence of the random variables X and Y which meet their joint elliptical distribution (D' Avino 2013). If the dependence structure of X and Y are not determined or the joint distribution of X and Y is not elliptical, the Pearson correlation analysis will lose the precondition of its application, so that another approach should be used to depict the dependence.

In finance the Copula technique is suggested as a better way to describe the dependence of stock returns. A copula is a multivariate cumulative distribution function for which the marginal probability distribution of each marginal variable is a uniform distribution. Therefore, the dependence between random variables can be described as a multivariate distribution on the uniforms (Nelsen 2006).

The Copula technique can be used to combine with traditional bivariate Gaussian model to get a deeper understanding of bivariate dependencies and the risk of portfolios on the stock market. This thesis report will describe, together with examples, how the copula techniques can be applied to improve the bivariate Gaussian model, and to better understand the risk of stock portfolios.

## 1.2 Outline

In the thesis, the bivariate Gaussian Copula model and the bivariate Gaussian model will be firstly built for getting a better understanding of the dependence structure between stock returns, and then the kernel density contour plots and surfaces will be applied to compare the accuracies of the simulation of the bivariate Gaussian Copula models with the traditional bivariate Gaussian models. By repeating the simulations of the copula models and the Gaussian models, respectively, the portfolios of the random variables will be established, and for more clear comparison of the copula model and the Gaussian model, the risk of the portfolios will be then estimated using probability density of the 100 end points of the portfolios by 100 times simulations of the bivariate Gaussian copula models and the bivariate Gaussian models, respectively.

This thesis is organized as the following: Chapter 2 reviews some basic definitions and theories of copula. Chapter 3 illustrates the processes of building the bivariate Gaussian copula model and the traditional bivariate Gaussian model and the method of simulating and comparing of the copula model and the Gaussian model. Chapter 4 shows the characteristics and the preprocessing of the stock data and the results of the modeling, simulations and fitness of copula models. The risk analysis of the portfolios is covered in Chapter 5. Conclusions are drawn in Chapter 6.

# 2 Theory

In statistics, a copula is a multivariate cumulative distribution function with the uniform marginal probability distribution of each variable, and it can describe the dependence between random variables. In this Chapter, I introduced the basic concepts, properties and probabilistic interpretation relevant to copula function.

## 2.1 Bivariate Copula

In order to define copulas, the definitions of grounded and 2-increasing need first to be introduced.

Firstly, Let us consider two non-empty subsets $A_1$ and $A_2$ of $R$, and the function $f : A_1 \times A_2 \to R$ is a real function.

**Definition 2.1** *If $A_1$ and $A_2$ have a least element $a_1$ and $a_2$, respectively, then the function $f$ is said to be* ***grounded*** *if and only if*

$$f(a_1, v) = f(u, a_2) = 0, \quad \forall (u, v) \in A_1 \times A_2$$

**Definition 2.2** *The function $f : A_1 \times A_2 \to R$ is called* ***2-increasing*** *if and only if for $\forall (u_1, v_1) \times (u_2, v_2) \in A_1 \times A_2$ with $u_1 \leq u_2$, $v_1 \leq v_2$, we have*

$$f(u_1, v_1) - f(u_1, v_2) - f(u_2, v_1) + f(u_2, v_2) \geq 0$$

Based on the above two definitions, it is ready to define copulas.

**Definition 2.3** *A* ***bivariate copula*** *$C$ is a real function which is defined on the unit square $I^2 = [0, 1] \times [0, 1]$*

$$C : I^2 \to R,$$

*and $C$ is fulfilling the three following properties:*

***i.*** *$C$ is grounded, i.e. $C(u, 0) = C(0, v) = 0, \quad \forall u, v \in [0, 1]$;*

***ii.*** *$C$ is 2-increasing;*

***iii.*** *for $\forall u, v \in I$, $C(u, 1) = u$ and $C(1, v) = v$.*

## 2.2 The probability Density Function of Copulas

Since there is a virtual similarity between copula functions, it is difficult to visualize differences between the distribution functions, so it is convenient to study copulas density functions.

The density of a copula $C$ is defined by

$$c(u,v) = \frac{\partial^2}{\partial u \partial v} C(u,v), \quad \forall (u,v) \in I^2$$

## 2.3 Frechet-Hoeffding bounds

According to the definition of bivariate copula, we can observe that for $\forall u, v \in I$ the bivariate copula $0 \leq C(u,v) \leq 1$, i.e. the graph of the copula is a continious surface in the unit cube $I^3$. The following Theorem 2.1 states the bounds of the copula.

**Theorem 2.1 (Frechet-Hoeffding bounds inequality)** *For $\forall (u,v) \in I^2$, the copula $C(u,v)$ satisfies the following inequality*

$$W(u,v) = max(u+v-1,0) \leq C(u,v) \leq min(u,v) = M(u,v)$$

*i.e. the Frechet-Hoeffding lower and upper bounds are $W(u,v)$ and $M(u,v)$, respectively.*

Fig. 2.1 and Fig. 2.2 show the upper and lower Frechet-Hoeffding bounds with their levels sets, respectively.

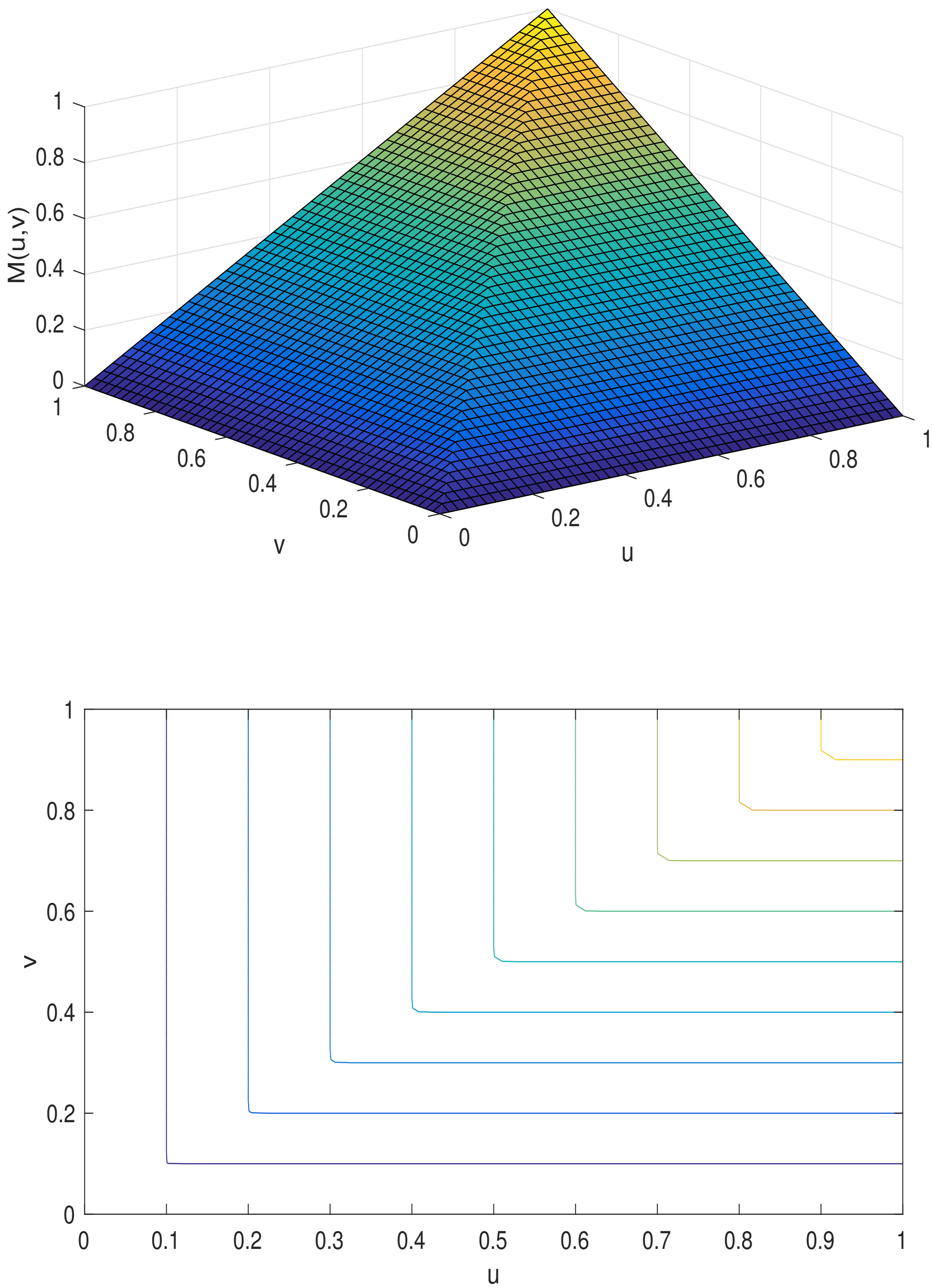


Figure 2.1: The upper Frechet-Hoeffding bound $M(u,v)$ and its level sets.

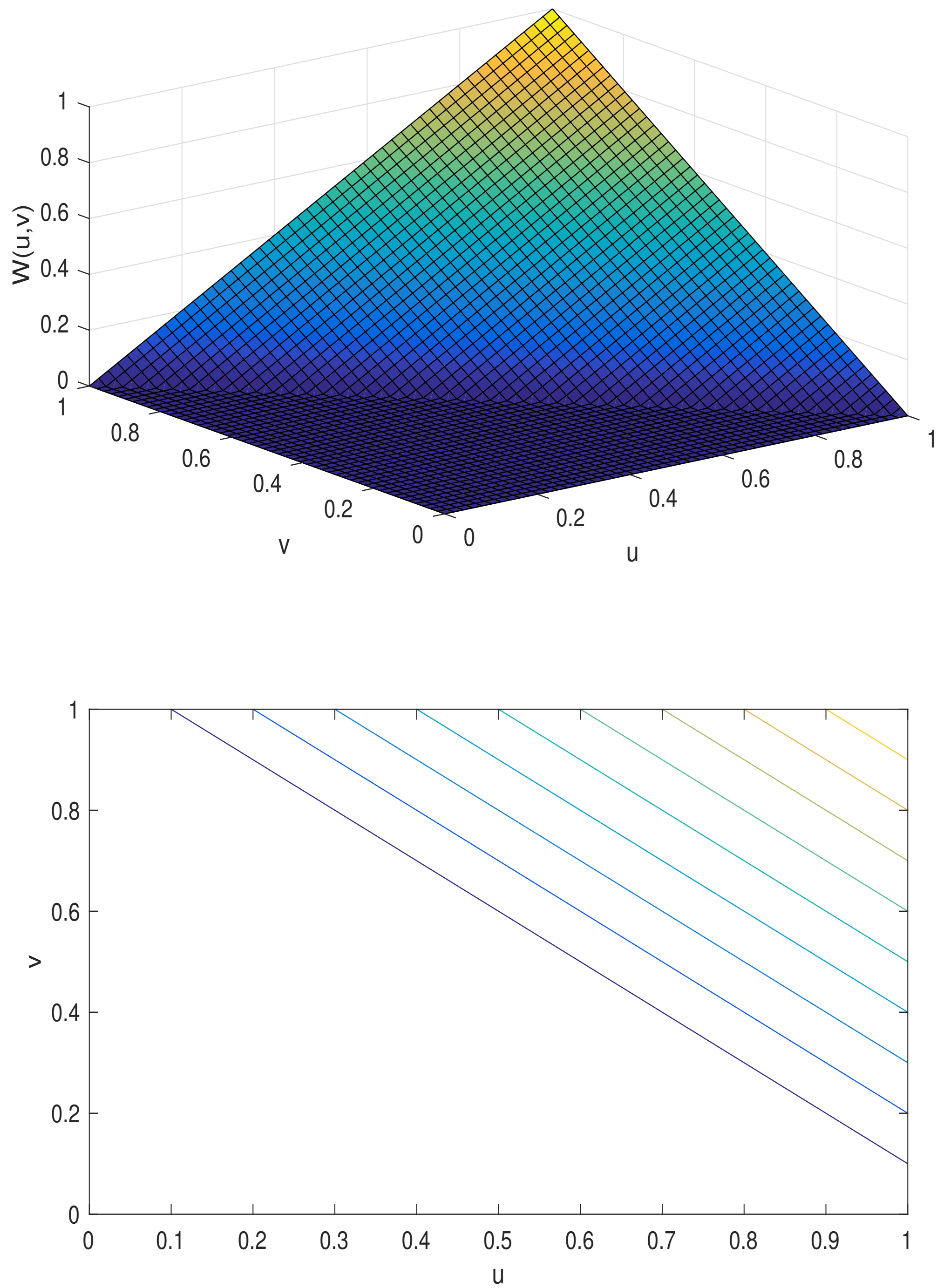


Figure 2.2: The lower Frechet-Hoeffding bound $W(u,v)$ and its level sets.

## 2.4 Product Copula

The other important definition of copula is the *product copula* which is defined as

$$\Pi(u,v) := uv, \quad \forall (u,v) \in I^2$$

whose plot is shown in Fig. 2.3.

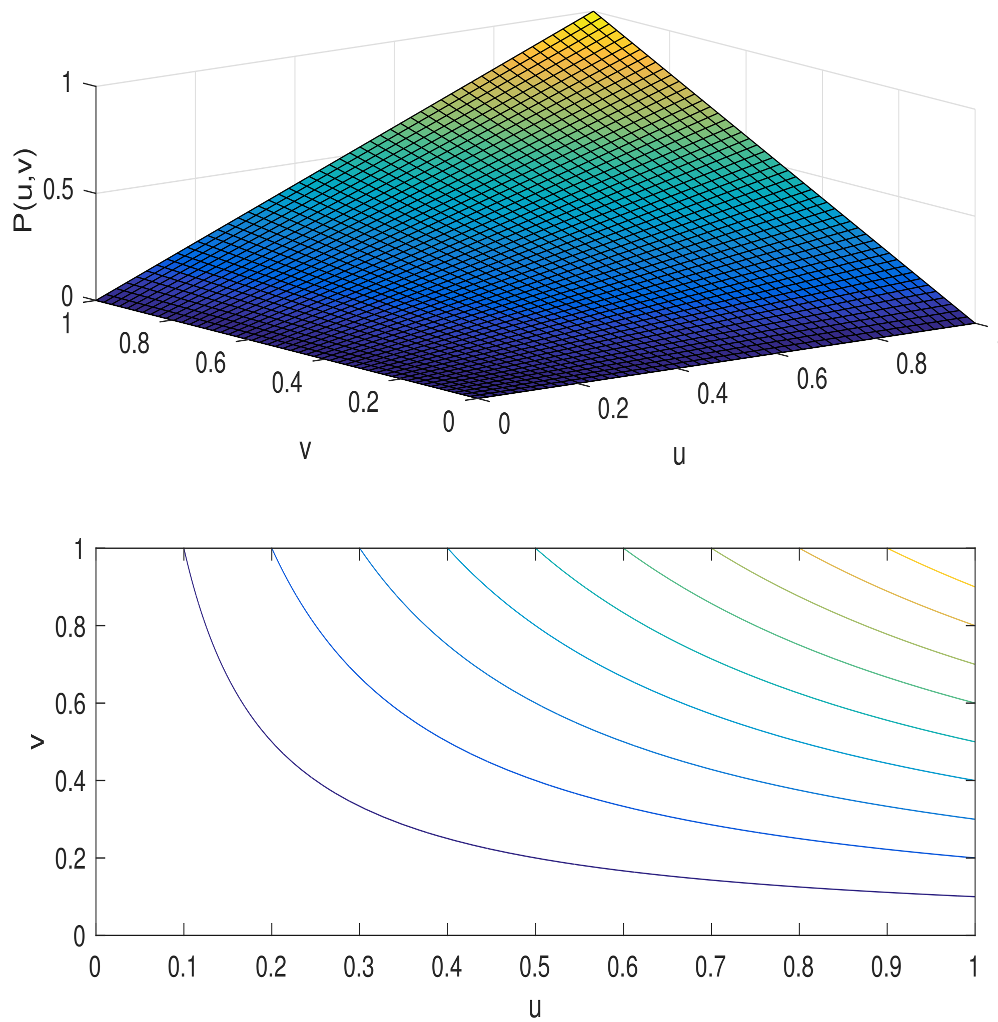


Figure 2.3: The product copula $\Pi(u,v)$.

## 2.5 Empirical Copula

The empirical copula is obtained through empirical cumulative density transform (rank transform) of the original data.

**Definition 2.4** *Let* $(x_i, y_i)_{i=1}^n$ *denote a sample with size n from a continuous bivariate distribution. And let the random variable* $X = \{x_i\}_{i=1}^n$ *and the random variable* $Y = \{y_i\}_{i=1}^n$*. The empirical copula* $C_{emp}(u, v)$ *is given by*

$$C_{emp}(u,v) = \frac{\sharp\{(x_i, y_i) : F_X(x_i) \leq u, F_Y(y_i) \leq v\}}{n}$$

*and the empirical copula density function* $c_{emp}$ *is given by*

$$c_{emp}(u,v) = \frac{1}{n}\sum_{i=1}^{n} \delta(u - F_X(x_i), v - F_Y(y_i))$$

where $\sharp$ is the number of elements of a set, the function $\delta$ can be approximated by normal-kernel smoothing, and $F_X$ and $F_Y$ are the marginal distributions of $X$ and $Y$, respectively.

## 2.6 Bivariate Gaussian Copula

The bivariate Gaussian copula is defined as following

$$C_\rho^{Ga}(u,v) = \Phi_\rho(\Phi^{-1}(u), \Phi^{-1}(v))$$

where $\Phi$ and $\Phi^{-1}$ are the joint distribution function and the inverse probability distribution function of a two dimensional standard normal distribution, respectively, and the parameter $\rho \in (-1, 1)$ is the correlation coefficient.

The culmulative distribution function of the bivariate Gaussian copula is (Cherubini et al. 2004)

$$C_\rho^{Ga}(u,v) = \int_{-\infty}^{\Phi^{-1}(u)} \int_{-\infty}^{\Phi^{-1}(v)} exp\left\{-\frac{u^2 - 2\rho uv + v^2}{2(1-\rho^2)}\right\} \frac{dudv}{2\pi\sqrt{1-\rho^2}}$$

Figure 2.4 shows the density surface and contour of the bivariate Gaussian copula.

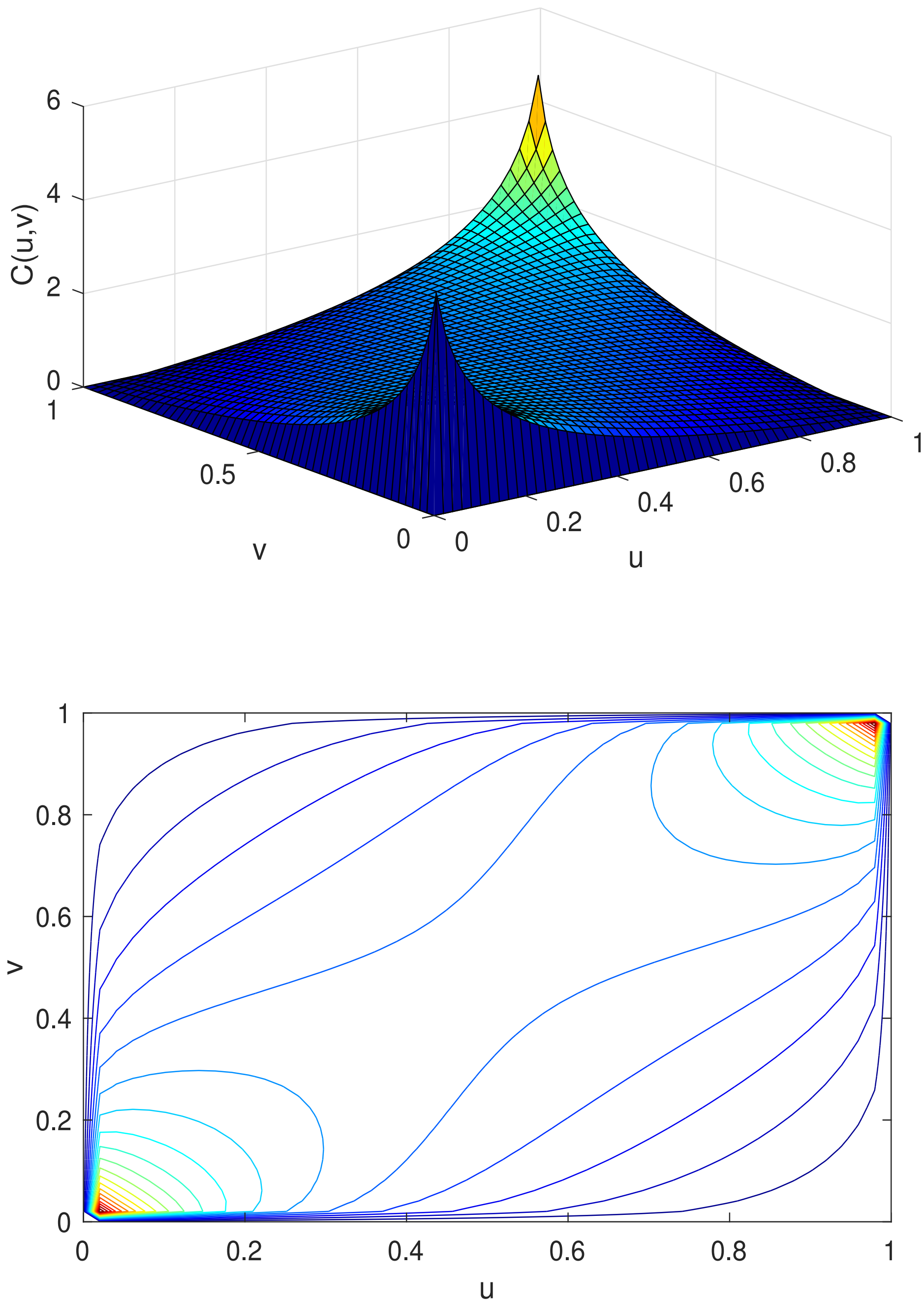


Figure 2.4: The bivariate Gaussian copula with $\rho = 0.5$.

## 2.7 Sklar's Theorem

In the application of copula, Sklar's theorem provides the theoretical foundation, and it states the role of copula which play the relation between multivariate distribution function and univariate margins. For the bivariate copula, Sklar's theorem is introduced as following

**Theorem 2.2 (Sklar's Theorem)** *Let $H(x)$ $(x = (x_1, x_2))$ be a two dimensional distribution function with marginal distribution $F_1(x_1)$ and $F_2(x_2)$. Then there exists a copula $C$ such that*

$$H(x) = C(F_1(x_1), F_2(x_2)), \quad \forall (x_1, x_2) \in R^2$$

*Conversely, for any univariate distributions functions $F_1(x_1)$ and $F_2(x_2)$ and any copula $C$, the function $H(x)$ is a two dimensional distribution function with marginals $F_1(x_1)$ and $F_2(x_2)$. Furthermore, if $F_1(x_1)$ and $F_2(x_2)$ are continuous, then the copula $C$ is unique.*

**Proposition 2.1** The Gaussian copula generates the standard joint Gaussian distribution function via Sklar's Theorem if and only if the margin distributions are standard Gaussian distribution (Jaworski et al. 2010).

# 3 Modeling

In the thesis, I used the bivariate Gaussian copula to improve the traditional Gaussian model, so the bivariate Gaussian copula model and the bivariate Gaussian model need to be built firstly. The processes of building the models are introduced in this section, and then the method and theory of the simulation and kernel density are introduced for comparing the copula model with the Gaussian model, respectively. The Kolmogorov-Smirnov distance is used for testing the fitness of the copula models.

## 3.1 Building the bivariate Gaussian Copula model

The bivariate Gaussian copula model will be built using two time series of the selected stock returns. The process of building the bivariate Gaussian copula has the following steps.

**Step 1. Calculating the daily returns of stock data**

The characteristics of normality is usually considered as a precondition of the data that can be used in a statistical analysis. However, stock return data usually do not follow normal distribution, so data transformation is needed to transfer the data to let them meet the condition of normal distribution. To do the data transformation, Log returns of the raw data (the daily stock return time series) are calculated as following:

Firstly, assume that there are two stock return time series $S_i(t)$, $i = 1,2$ and $t = 1,2,\ldots,n$. Then the log returns $X_i$ of the two stocks are

$$X_i(t) = log(S_i(t)) - log(S_i(t-1))$$

And $X_i(t)$ are two continuous random variables.

**Step 2. The empirical cumulative distribution function of the log returns $X_i s$**

The empirical cumulative distribution function (CDF) of $X_i$ is denoted by $U_i$, $i = 1,2$ and the formula is

$$U_i = \frac{1}{n}\sum_{j=1}^{n} \mathbf{1}(X_j \leq X_i)$$

In generally, the log returns $X_is$ are assumed independent identically distribution, so the empirical CDF $U_is$ are uniform distribution on interval $[0,1]$.

**Step 3. The standard normal inverse of the empirical CDF $U_is$**

In this step, the standard normal inverse of the empirical CDF $U_is$ is needed to be found, and is denoted by $Y_i$, $i = 1,2$, which are the following

$$Y_i(t) = \Phi^{-1}(U_i(t))$$

Since $U \sim Uniform[0,1]$, so $P(U \leq u) = u$. Then

$$P(Y \leq y) = P(\Phi^{-1}(U) \leq y) = P(U \leq \Phi(y)) = \Phi(y)$$

So $Y_i$ follow standard normal distribution.

**Step 4. The bivariate Gaussian copula**

According to Proposition 2.1, since the margin are standard normal distributed, so there exists the bivariate Gaussian copula $C^{Ga}$ with $Y_i$. In this step, the bivariate Gaussian copula $C^{Ga}$ is assumed to be the bivariate normal distribution with mean 0 and variance $V$, i.e. $C^{Ga} \sim \mathbf{N(0,V)}$. The mean is two dimensions and $V$ is the two dimensions covariance matrix with $Y_i$. So the bivariate Gaussian copula model is

$$\mathbf{C^{Ga}} \sim \mathbf{N}\left( \begin{pmatrix} 0 \\ 0 \end{pmatrix}^T, \begin{pmatrix} Var(Y_1) & Cov(Y_1,Y_2) \\ Cov(Y_2,Y_1) & Var(Y_2) \end{pmatrix} \right).$$

## 3.2 Building the bivariate Gaussian model

The bivariate Gaussian model $G_X$ of a random vector $X = [X_1, X_2]$ can be written as the following

$$G_X \sim N(\mu, \Sigma)$$

where $\mu$ and $\Sigma$ are the mean vector and the covariance of $X$, respectively, and the random vector $X = [X_1, X_2]$ is from Step1 of Section 3.1.

Then the bivariate Gaussian model is

$$\mathbf{G_X} \sim \mathbf{N}\left( \begin{pmatrix} mean(X_1) \\ mean(X_2) \end{pmatrix}^T, \begin{pmatrix} Var(X_1) & Cov(X_1, X_2) \\ Cov(X_2, X_1) & Var(X_2) \end{pmatrix} \right).$$

## 3.3 Simulation

The simulations using the bivariate Gaussian copula model and the bivariate Gaussian model are introduced in this section.

**1. Simulation of the bivariate Gaussian copula model.**

The bivariate Gaussian copula model $C^{Ga}$ is got from Section 3.1, so we can randomly choose the simulated data $\{y(t)\}_{t=1}^n$ from the model $C^{Ga}$, where $y(t)$ is a two dimensional vector which is denoted by $y(t) = [y_1(t), y_2(t)]$. And the data $y(t)$ is needed to return to the original values. Since

$$u(t) = (\Phi(y_1(t)), \Phi(y_2(t))),$$

so the original values are

$$x(t) = (F_1^{-1}(\Phi(y_1(t))), F_2^{-1}(\Phi(y_2(t)))) = (F_1^{-1}(u_1(t)), F_2^{-1}(u_2(t)))$$

Since $u_i(t)$, $i = 1, 2$ follow a uniform distribution on interval $[0, 1]$ which is a piecewise constant function, so the inverse $F_i^{-1}(u_i(t))$ can not be calculated. Under the situation, the concept of **Right inverse** will be introduced as the following:

**Definition 3.1 (Right inverse)** *Given a function $F : R \to [0,1]$, then we can write*

$$F^{-1}(u(t)) = inf\{x : F(x) \geq u(t)\} \quad \forall u(t) \in (0,1)$$

Then $F_i^{-1}(u_i(t))$, $i = 1,2$ is obtained using the right inverse. And from Step 1 of Section 3.1 we have

$$x_i(t) = log(s_i(t)) - log(s_i(t-1)), \quad i = 1,2,$$

then

$$\sum_{j=1}^{t} x_i(j) = log(s_i(t)) - log(s_i(1)),$$

so we get

$$\frac{s_i(t)}{s_i(1)} = exp\{\sum_{j=1}^{t} x_i(j)\},$$

and the original data of the simulated data $\{y(t)\}_{t=1}^{n}$ is

$$s_i(t) = s_i(1) \cdot exp\{\sum_{j=1}^{t} x_i(j)\}, \quad i = 1,2$$

**2. Simulation of the bivariate Gaussian model.**

The bivariate Gaussian model $G_X$ is obtained from Section 3.2, so we can randomly choose the simulated data $\{x^G(t)\}_{t=1}^{n}$ from the model $G_X$, where $x^G(t)$ is a two dimensional vector which is denoted by $x^G(t) = [x_1^G(t), x_2^G(t)]$. From the last equation, the original data of the simulated data $\{x^G(t)\}_{t=1}^{n}$ can be obtained which is

$$s_i^G(t) = s_i^G(1) \cdot exp(\sum_{j=1}^{t} x_i^G(j)), \quad i = 1,2$$

## 3.4 Kernel density

In this section, the raw data and the simulated datas will be compared using the contour plot and surface of the empirical copula density, respectively. But there is no giving detailed information for a possible empirical copula density model, so under this situation the Kernel density can be much more helpful estimating the density. In the thesis the bivariate Gaussian Kernel density is chosen to estimate the empirical copula density.

Let $x_1, x_2$ be a sample of 2-variate random vectors drawn from a common distribution described by density function $f$. The bivariate Gaussian kernel density is defined as following (Guan 2011)

$$\hat{f}(x,H) = \frac{1}{n}\sum_{i=1}^{n} K_H(x - x_i)$$

where $x = (x_1, x_2)^T$, $x_i = (x_{i1}, x_{i2})^T$, $i = 1, 2, \ldots, n$, $K_H(x) = |H|^{-\frac{1}{2}} K(H^{-\frac{1}{2}} x)$ is the kenel function, and here $K$ is chosen to be Gaussian, i.e.

$$K(x) = \frac{1}{2\pi} exp\{-\frac{1}{2} x^T x\}, \quad \forall x \in [0,1]^2$$

And $H$ is the bandwidth matrix with $H = \begin{pmatrix} h_1^2 & 0 \\ 0 & h_2^2 \end{pmatrix}$ and $h_1, h_2$ is the bandwidth which minimize

$$argmin_{h_1,h_2} E\Big[\int_0^1 \int_0^1 [\hat{f}(u,v) - f(u,v)] dudv\Big]$$

So the bivariate Gaussian Kernel density can be obtained as

$$\hat{f}(x,H) = \frac{1}{n}\sum_{i=1}^{n} \frac{1}{h_1 \cdot h_2} K(\frac{x_1 - x_{i1}}{h_1}, \frac{x_2 - x_{i2}}{h_2})$$

$$K(x) = \frac{1}{2\pi} exp\{-\frac{1}{2}(x_1^2 + x_2^2)\}$$

## 3.5 Test the goodness of fit for the Copula models

To compare the copula models, the Kolmogorov-Smirnov (KS) distance will be used to test the goodness of fitting of the copula models. The KS distance of the empirical copula can be obtain by the following formula

$$D_{KS}^{2D} = max_{u,v \in [0,1]} |C_{emp}(u,v) - C(u,v)|$$

A smaller KS distance implies the better fitting.

## 3.6 Portfolio

In this part, the portfolio of the two stocks $S_1(t)$ and $S_2(t)$ will be established using the bivariate Gaussian copula model and the bivariate Gaussian model.

**Step 1**. Establishing a portfolio of the two stocks: $P = S_1(t) + S_2(t)$;

**Step 2**. In Section 3.3, we have $s_i(t) = s_i(1) \cdot exp\{\sum_{j=1}^{t} x_i(j)\}$ for the bivariate Gaussian Copula model and $s_i^G(t) = s_i^G(1) \cdot exp\{\sum_{j=1}^{t} x_i^G(j)\}$ for the bivariate Gaussian model. Now we set $s_i(1) = M$ and $s_i^G(1) = M^G$ and then we do the simulations $T$ times for the copula model and the Gaussian model, respectively, after the process we will get $s_i(t)$ and $s_i^G(t)$, $i = 1,2$.

**Step 3**. Calculating the result $p = s_1(t) + s_2(t)$ and $p^G = s_1^G(t) + s_2^G(t)$ and observe the trends of the portfolio.

# 4 Data and Simulation

## 4.1 Data

The adjusted closing price (ACP) of four short-term stocks and two long-term stocks in the Swedish-A stock market were downloaded from the website of Yahoo Finance. The ACP was chosen for analysis, because it is a more accurate representation of the value of a stock compared to close price, since the ACP takes into account all corporate actions such as stock splits, dividends and new stock offerings.

### 4.1.1 Characteristics and transformation of the short-term data

The four short-term stocks include those from companies of Ericsson, SEB, Volvo and AstraZeneca. Daily data were derived from working days between 2018.01.01 and 2018.12.31 with a sample size of 252.

Figure 4.1 shows the ACP curves of the four short-term stocks. We can see that all the ACP fluctuate over the time with Ericsson and AstraZeneca showing obviously positive trends over the whole year, Volvo showing an obviously negative trend from late autumn to end of the year, SEB showing seasonal fluctuation without a trend.

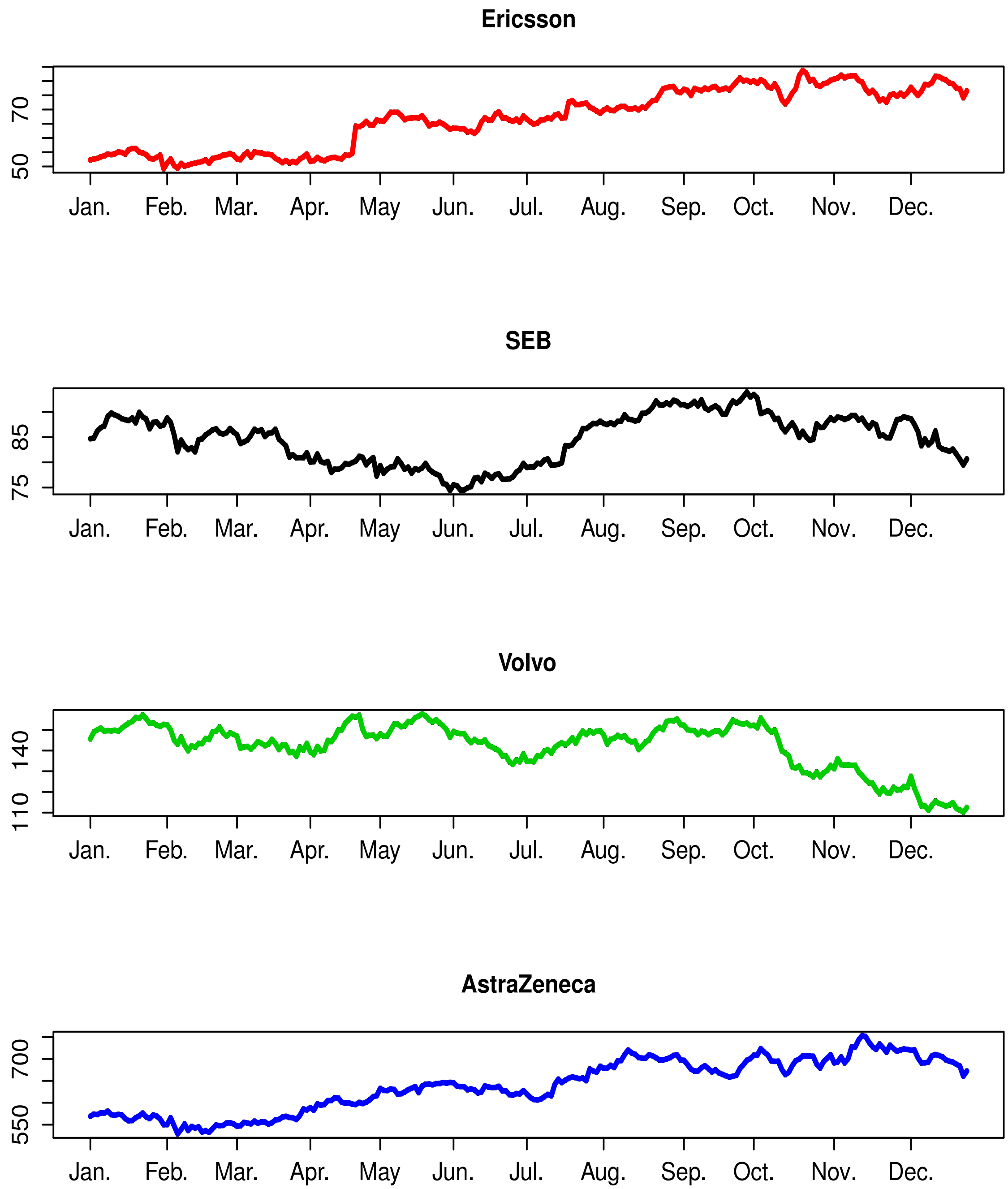


Figure 4.1: ACPs of the four short-term stocks in 2018.

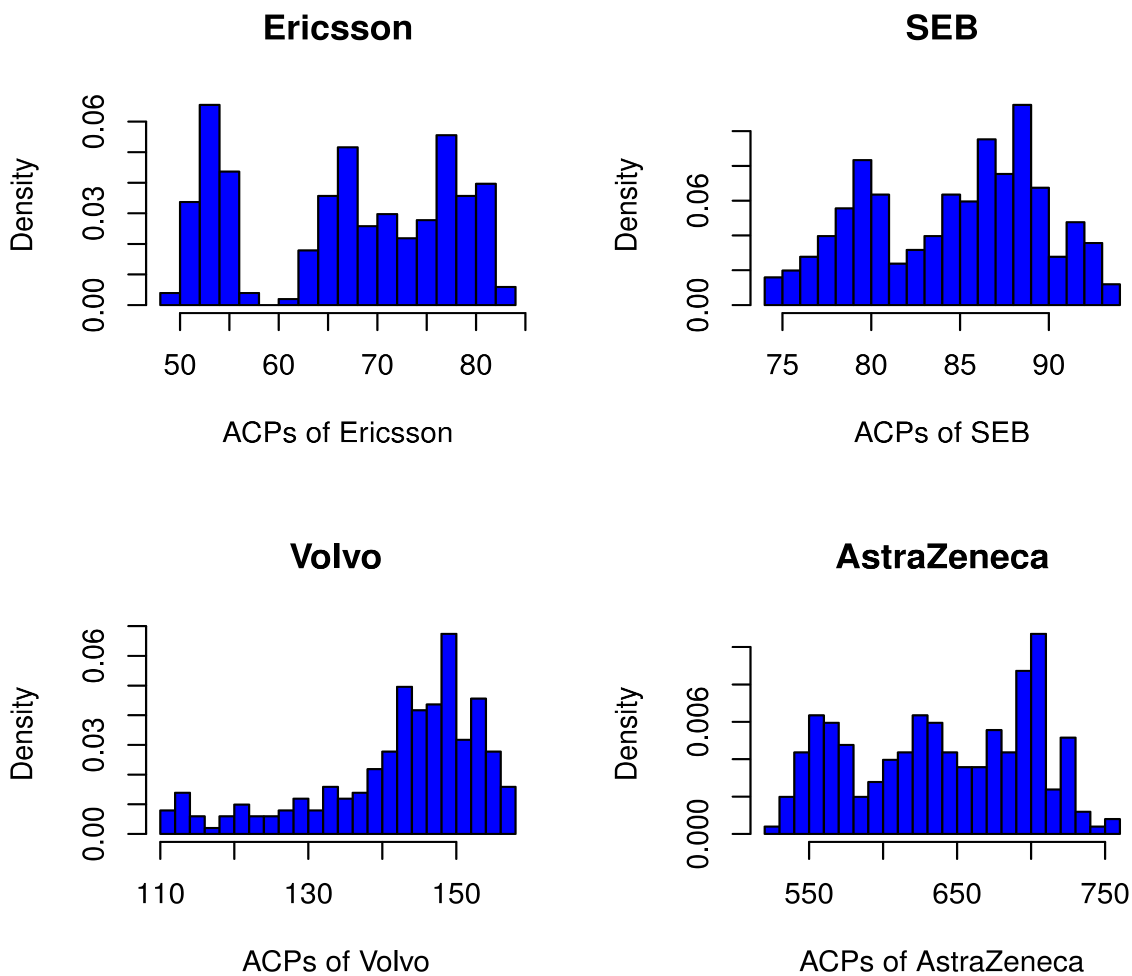


Figure 4.2: Histograms of the ACPs of the four short-term stocks.

Figure 4.2 shows that all the ACPs of the short-term stocks are not symmetrically distributed, and does not follow a normal distribution. The normality of the ACPs are also suggested by the Q-Q plots in Fig. 4.3.

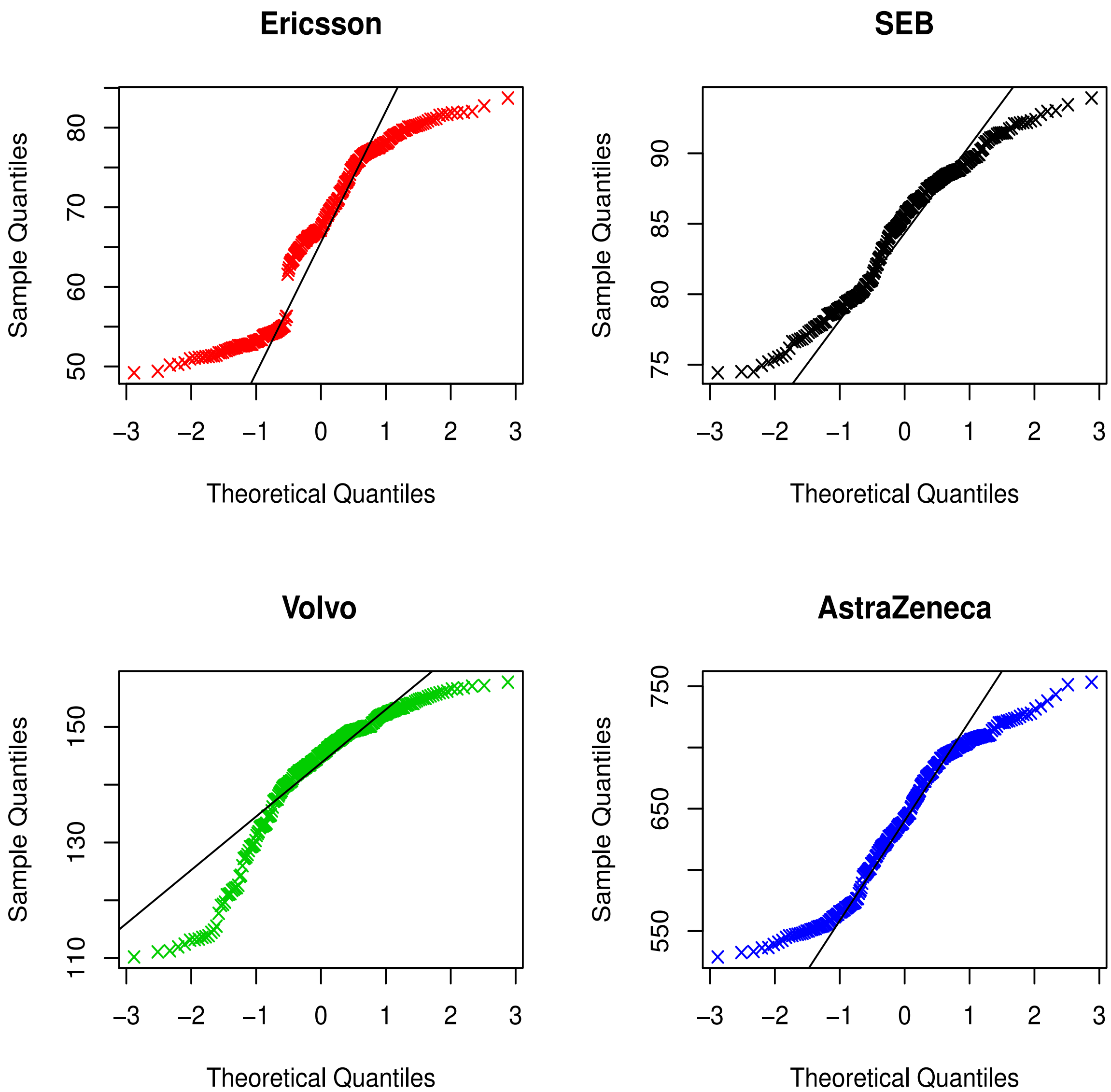


Figure 4.3: Normal Q-Q Plot of adjusted close prices in the four short term stocks.

The characteristics of normality is usually considered as a precondition of data from a stock market, which can be used in a statistical analysis, so I calculated the log returns of ACPs of every short-term stock, and check their normality characteristics in Fig. 4.4 and Fig. 4.5. Figure 4.4 shows that trends of the four ACPs were removed after log-return processing.

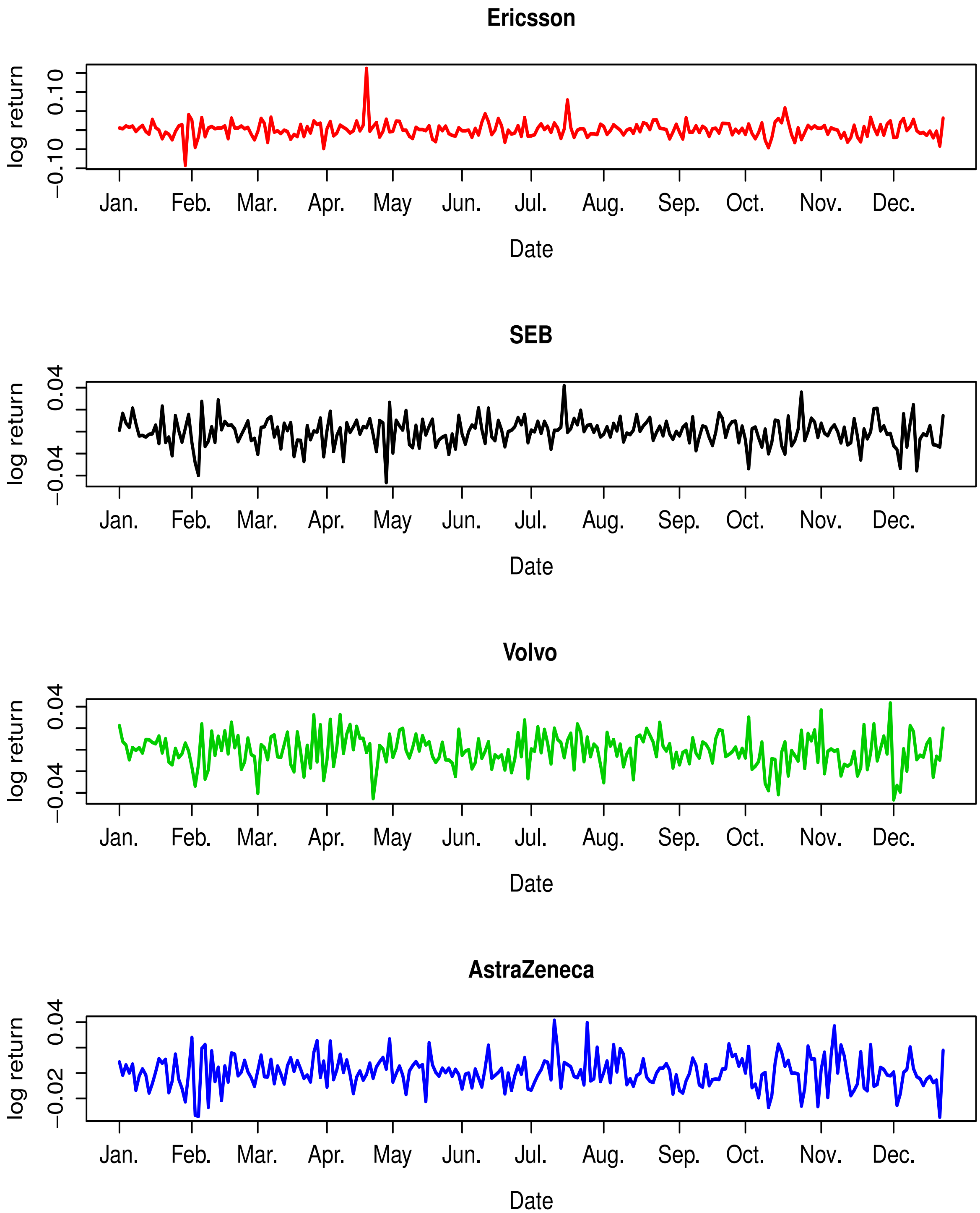


Figure 4.4: Log returns of the ACPs of the four short-term stocks in 2018.

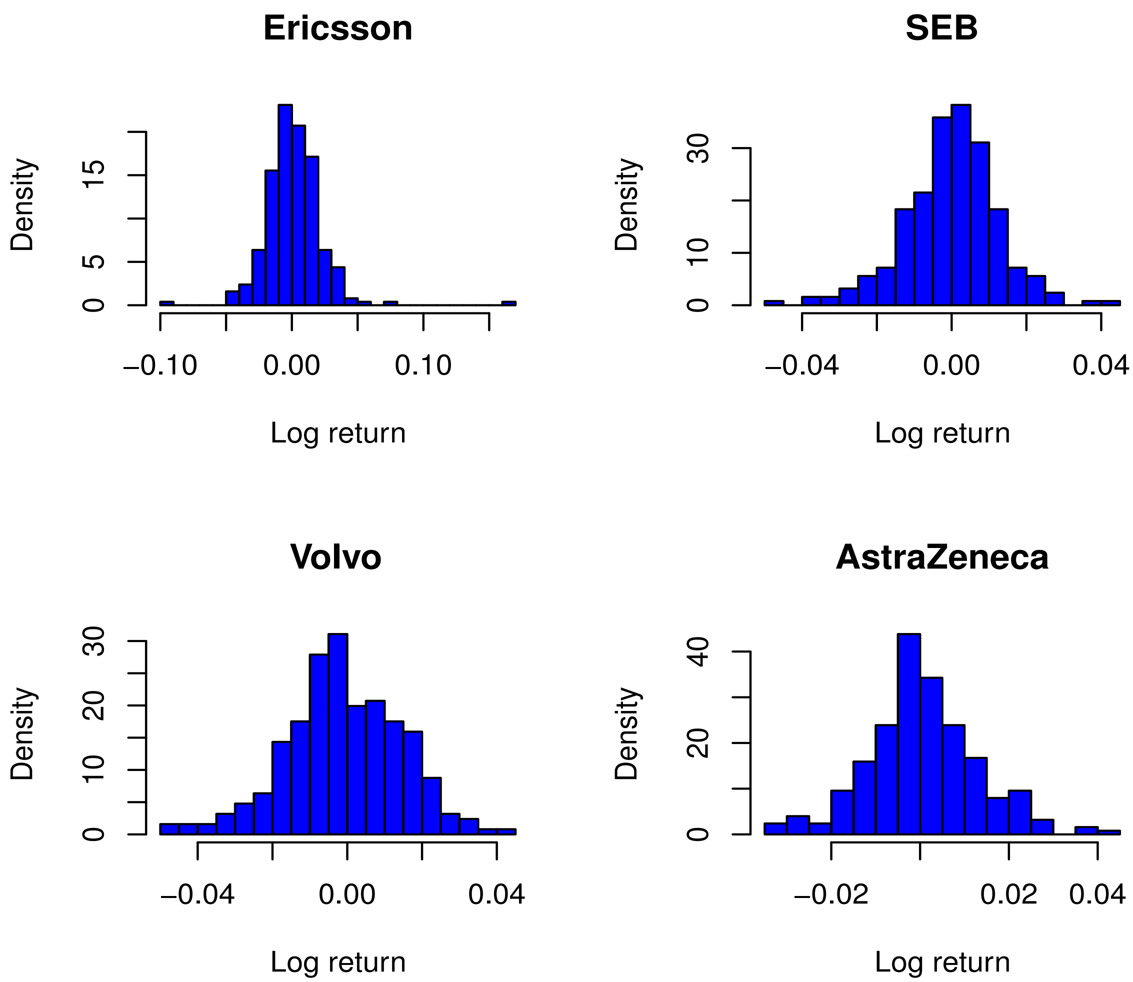


Figure 4.5: Histograms of the log returns of the four short-term stocks.

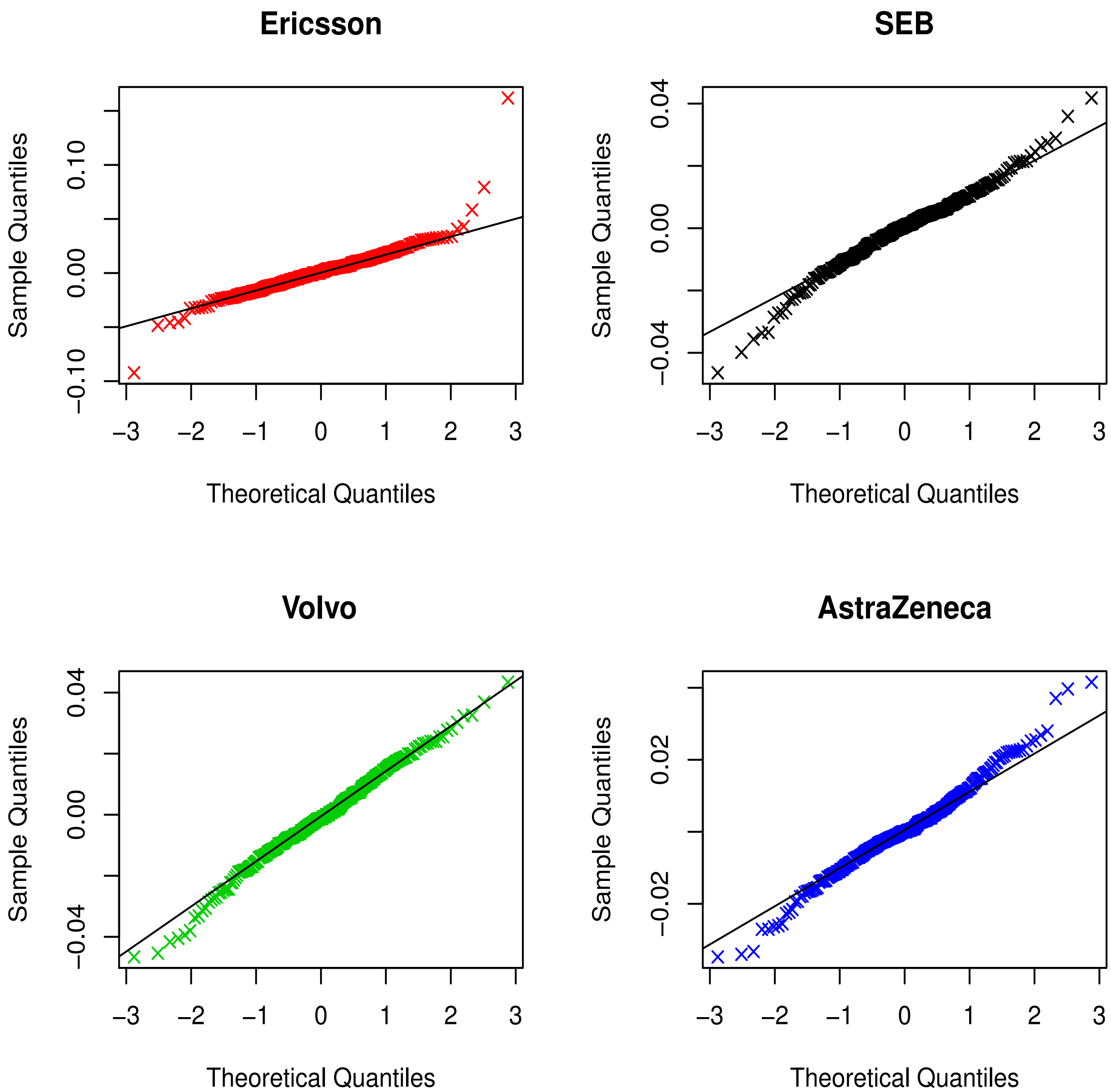


Figure 4.6: Normal Q-Q Plots of the log returns of the four short term stocks.

Figure 4.5 and 4.6 show that the ACPs approximately follow normal distributions after the log returns processing. However, the tails of the ACPs are heavy.

Generally, the log returns of ACPs should be assumed to be independent identically distributed, so the distribution of the log returns should be approximately distributed as a uniform on interval [0,1]. We can check the uniformity of the log returns on interval [0,1] by the empirical distributions plots. Figure 4.7 shows that the empirical distribution on interval [0,1] is approximately uniformly distributed.

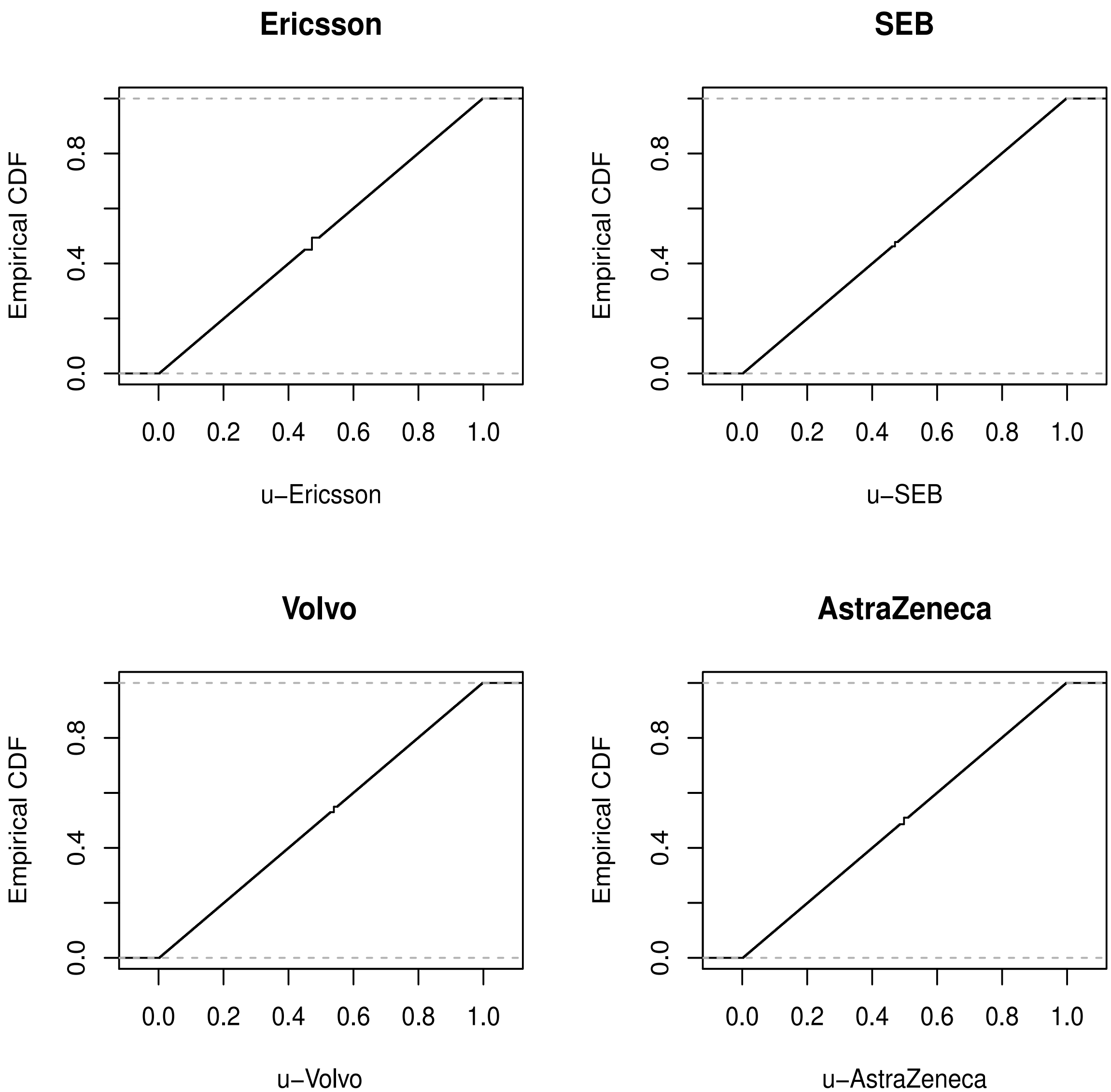


Figure 4.7: Empirical distributions of the ACPs of the four short-term stocks.

### 4.1.2 Characteristics and transformation of the long-term data

The two long-term stocks include those from companies of Volvo and AstraZeneca. Daily data were derived from working days between 2010.01.01 and 2018.12.31 with a sample size of 2264. Figure 4.8 shows the ACP variability of the two long-term stocks. We can see that both of them show obviously positive trends over the sampling period. However, Fig. 4.9 and 4.10 show that the ACPs of the two long-term stocks do not follow normal distributions over the sampling period, so that data transformation are needed before applying statistical analysis to the data.

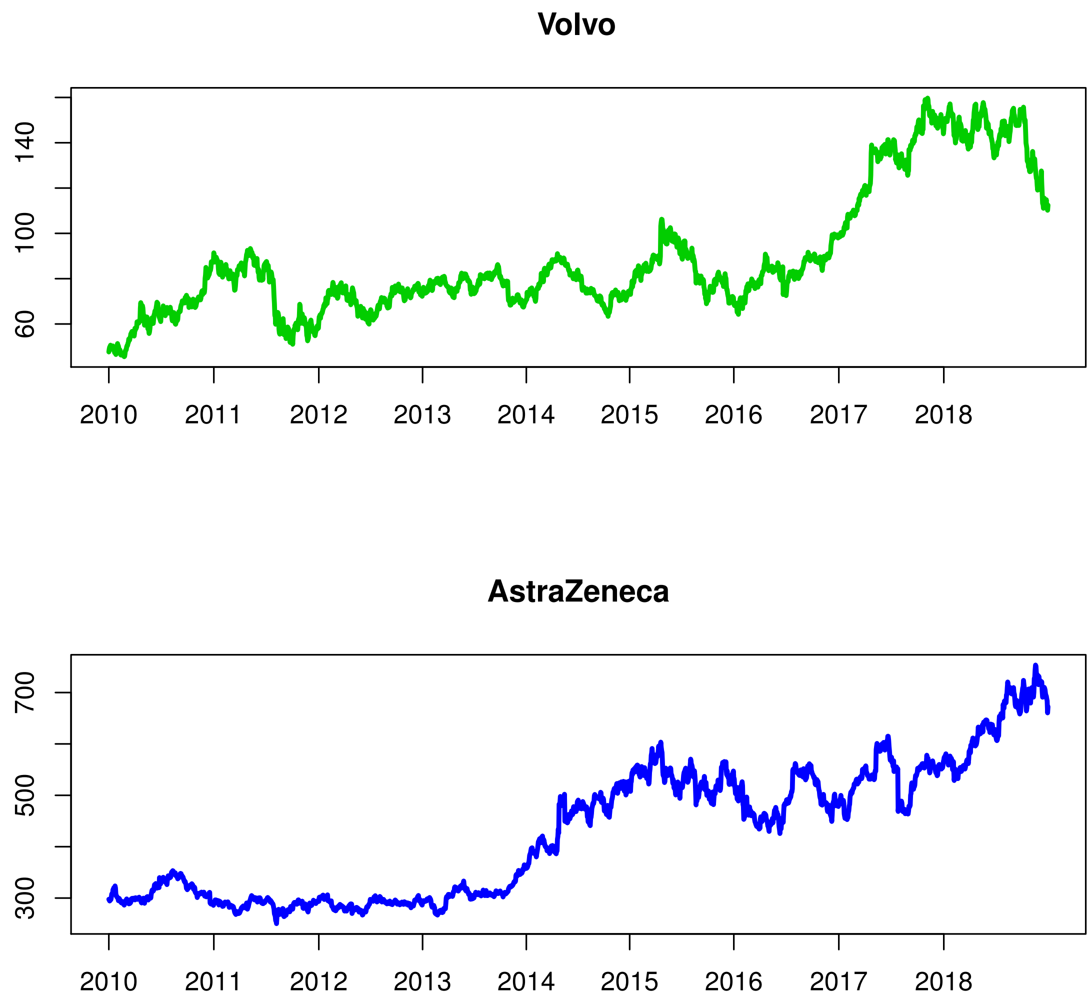


Figure 4.8: ACPs of the two long-term stocks.

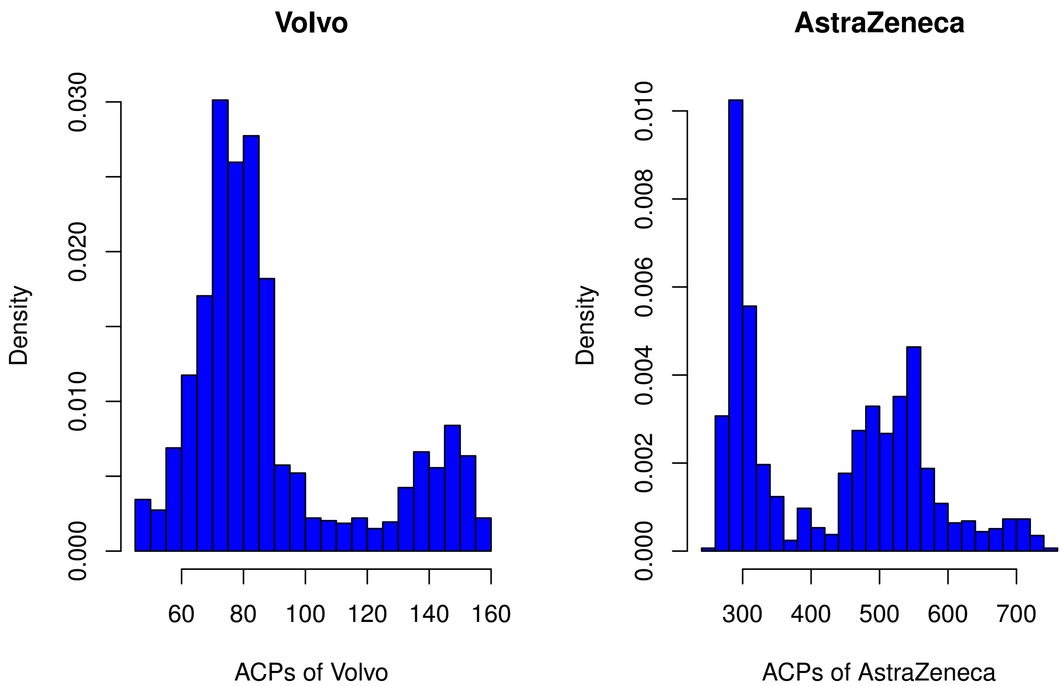


Figure 4.9: Histograms of the ACPs of the two long-term stocks.

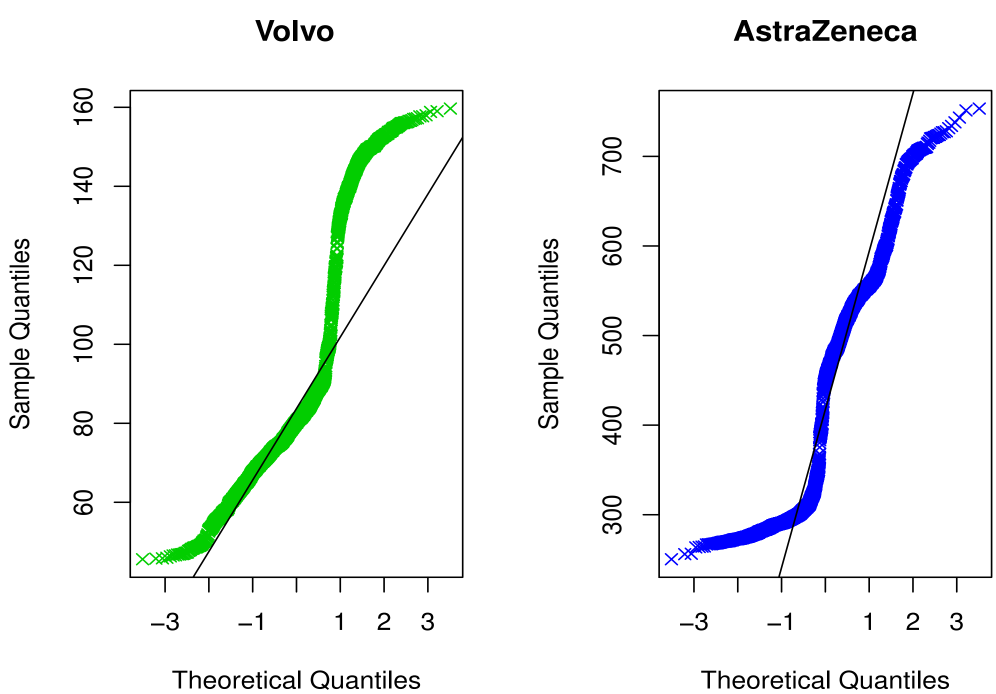


Figure 4.10: Normal Q-Q Plot of the ACPs of the two long-term stocks.

Figure 4.11 shows APC variability of the two long-term stocks after log-return transformations. We can see that the long-term trends were removed from both of the time series. Figure 4.12 and 4.13 show the histograms and Normal Q-Q plots of the two transformed ACP time series. We can see from the figures that both of the time series approximately follow normal distributions, but still the tails of them are very heavy.

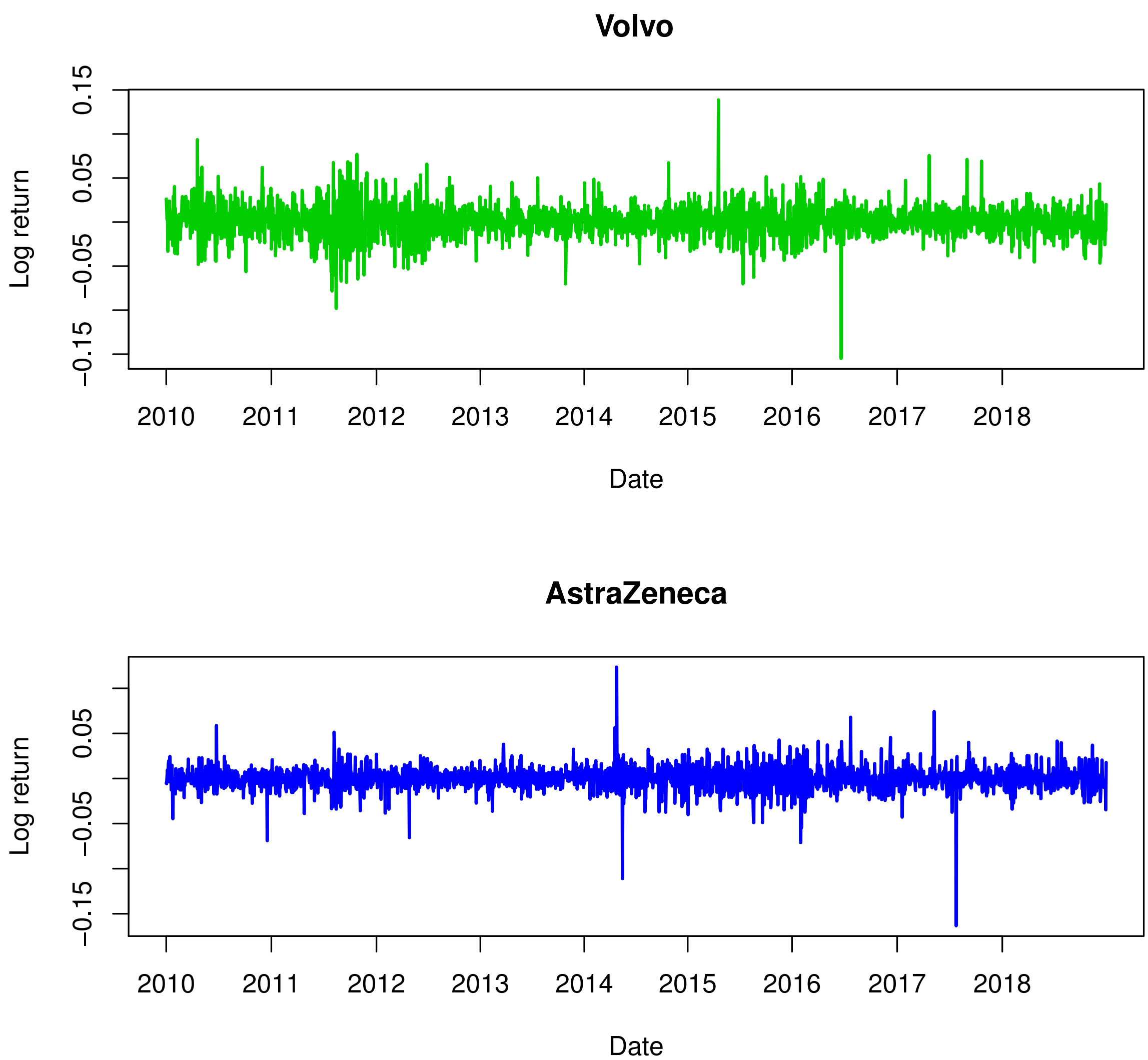


Figure 4.11: Log returns of the ACPs of the two long-term stocks.

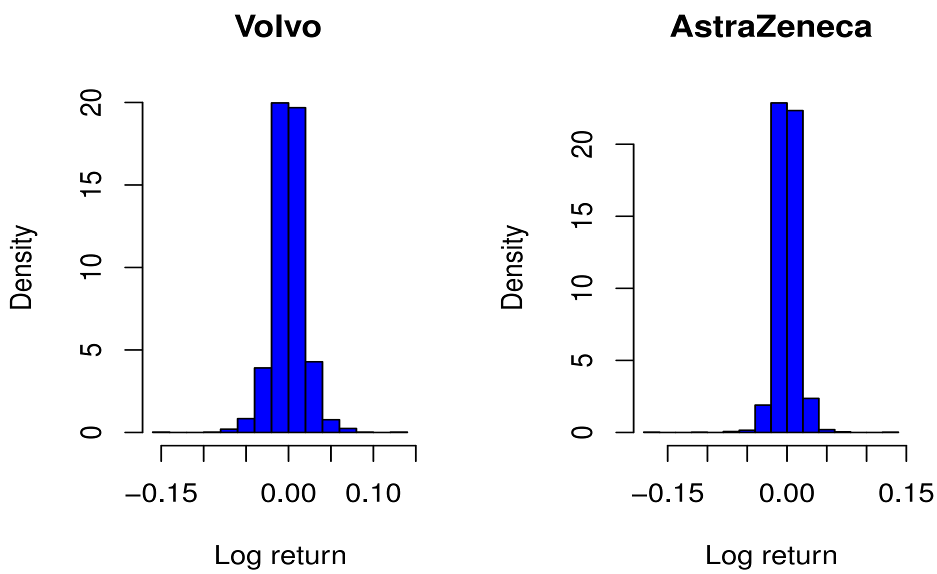


Figure 4.12: Histograms of the log returns of the two long-term stocks.

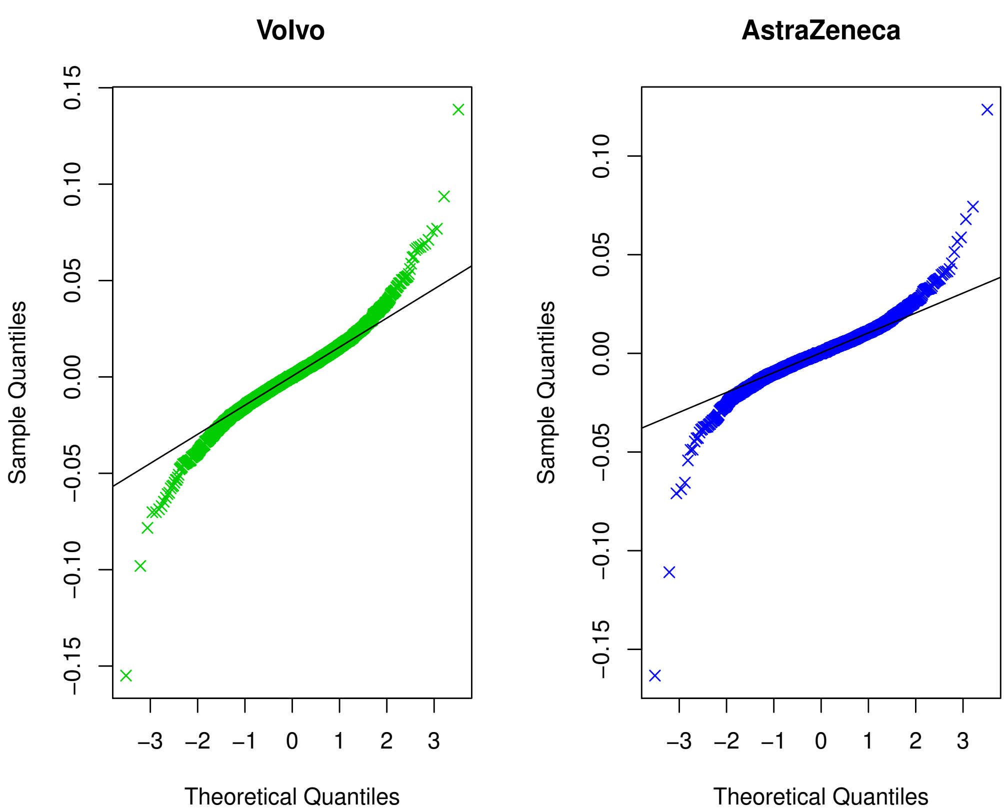


Figure 4.13: Normal Q-Q Plot of the log returns of the two long-term stocks.

The uniformities of the two transformed ACPs were checked using empirical distributions plots (Fig. 4.14). We can see that the two log returns were uniformly distributed on interval [0,1].

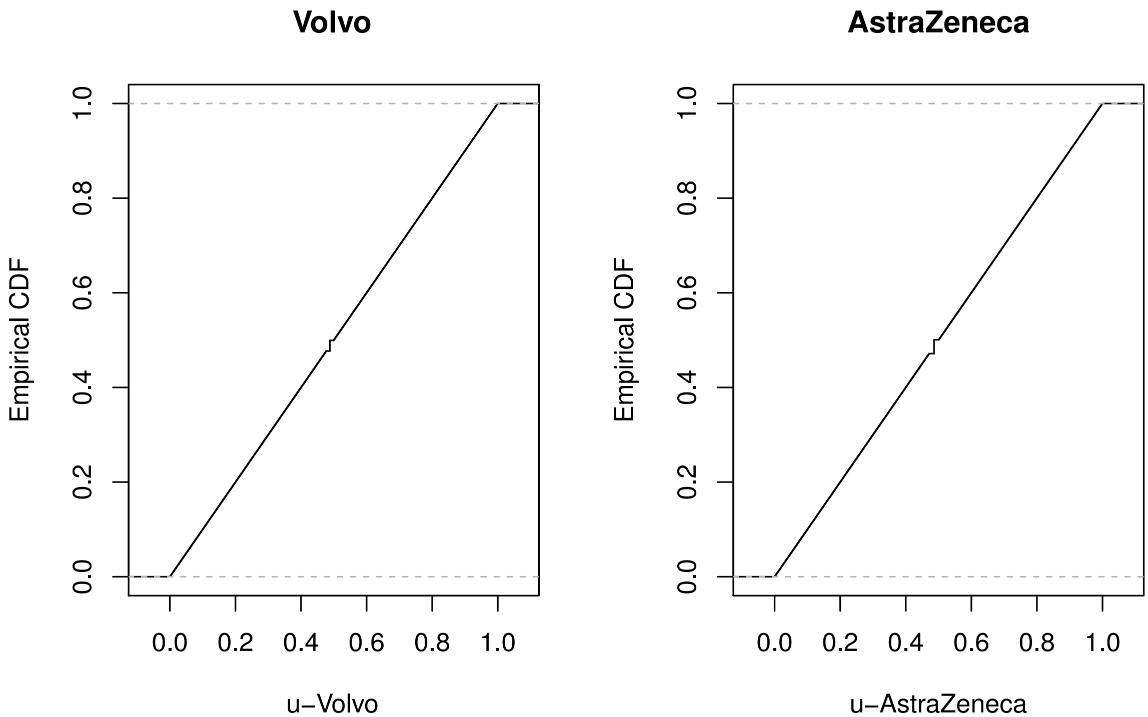


Figure 4.14: Empirical distributions of the two transformed APCs of the two long-term stocks.

## 4.2 Simulation

The bivariate Gaussian models and the bivariate Gaussian Copula models were built using the six different stock combinations of the four short-term stocks and the stock combination of the two long-term stocks respectively. Then $5000 \times 2$ dimensions simulated data matrices were randomly chosen from the Gaussian models and the Gaussian Copula models, respectively, and were used to compare with the raw data using the kernel density contour plots and surfaces plots.

**1**. The simulation of the Ericsson and SEB stock combination.

A bivariate Gaussian model of the combination of the stocks Ericsson and SEB were built. The Gaussian model of the Ericsson-SEB stock combination is denoted by $\mathbf{G_{eric-seb}}$ which is

$$\mathbf{G_{eric-seb}} \sim \mathbf{N}\left( \begin{pmatrix} 0.00151476 \\ -0.00019418 \end{pmatrix}^T, \begin{pmatrix} 0.00044926 & 0.00005207 \\ 0.00005207 & 0.00015601 \end{pmatrix} \right).$$

I randomly chose a $5000 \times 2$ data matrix from the model $G_{eric-seb}$.

Then I built a bivariate Gaussian Copula model using the Ericsson-SEB stock combination. The Gaussian Copula model of the Ericsson-SEB combination is denoted by $\mathbf{C_{eric-seb}}$ which is

$$\mathbf{C_{eric-seb}} \sim \mathbf{N}\left( \begin{pmatrix} 0 \\ 0 \end{pmatrix}^T, \begin{pmatrix} 0.9110 & 0.2264 \\ 0.2264 & 0.9219 \end{pmatrix} \right).$$

I randomly chose a $5000 \times 2$ dimensions data matrix from the model $C_{eric-seb}$.

Then I calculated the kernel densities of the Ericsson-SEB stocks combination from the raw data, simulated data based on Gaussian model and simulated data based on Gaussian Copula model, respectively. And the kernel density contour plots and surface plots of the raw data combination, the simulated data of $G_{eric-seb}$ and the simulated data of $C_{eric-seb}$ are shown in Fig. 4.15 to 4.20.

From Fig. 4.15 to Fig. 4.20, we see that the simulation with the Copula model $C_{eric-seb}$ is more similar with the raw data, so the simulation using the Copula model $C_{eric-seb}$ is better than the simulation using the Gaussian model $G_{eric-seb}$.

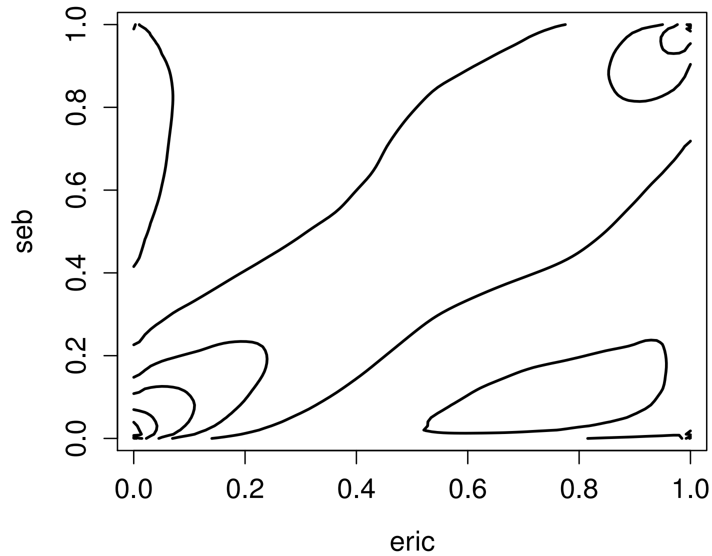


Figure 4.15: Empirical copula kernel density contour plot of the raw data of the Ericsson and SEB stock combination.

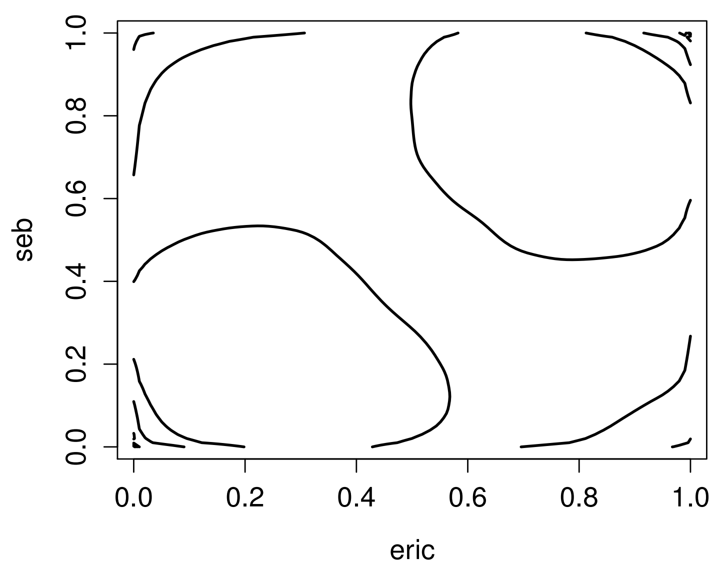


Figure 4.16: Empirical copula kernel density contour plot of the simulated data by the Gaussian model of the Ericsson and SEB stock combination.

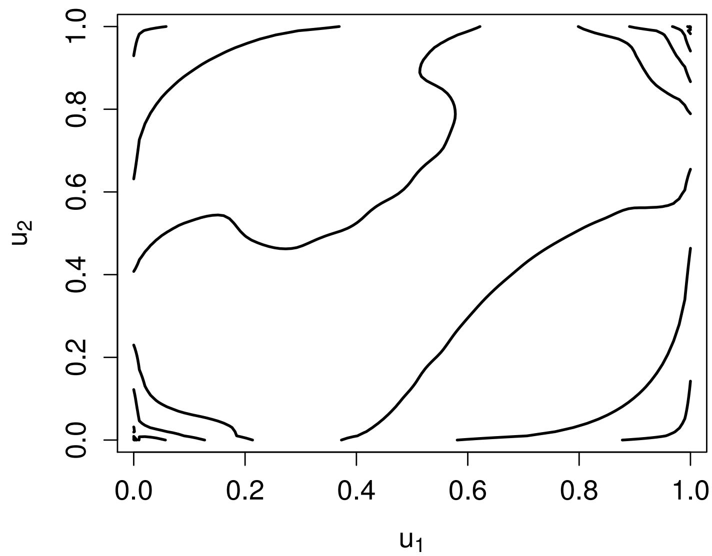


Figure 4.17: Empirical copula kernel density contour plot of the simulated data by the Copula model of the Ericsson and SEB stock combination.

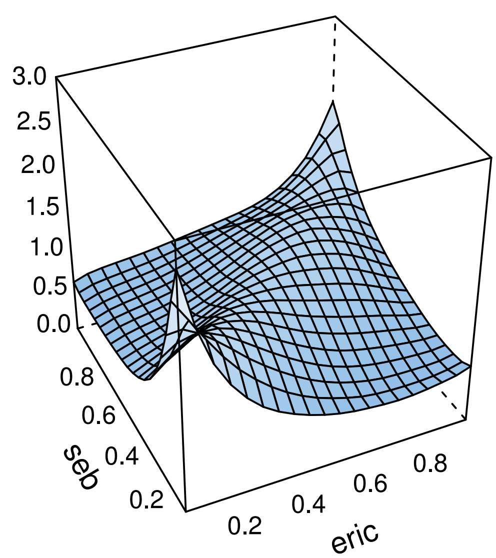


Figure 4.18: Empirical copula kernel density surface of the raw data of the Ericsson and SEB stock combination.

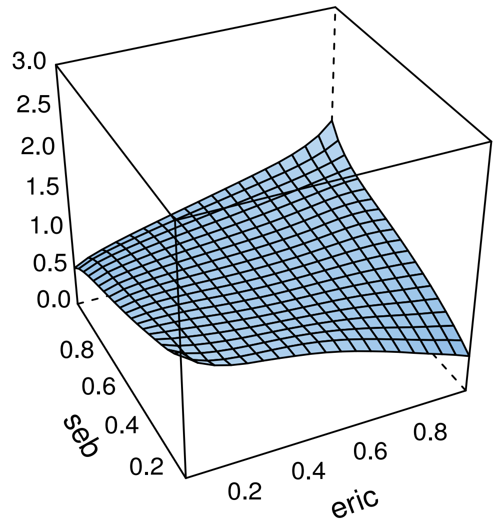


Figure 4.19: Empirical copula kernel density surface of the simulated data by the Gaussian model of the Ericsson and SEB stock combination.

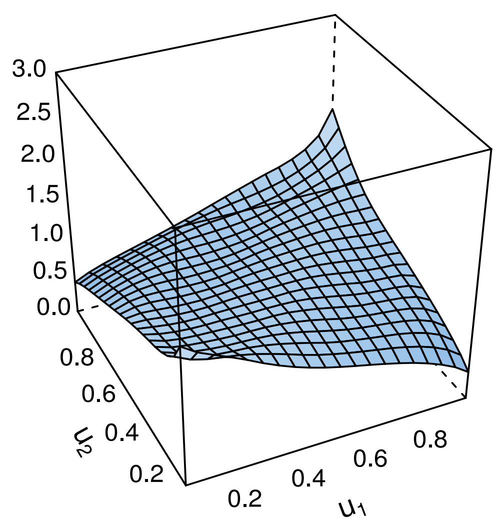


Figure 4.20: Empirical copula kernel density surface of the simulated data by the Copula model of the Ericsson and SEB stock combination.

**2**. The simulation of the Ericsson and Volvo stocks combination.

Using the same methods I built the bivariate Gaussian model and the bivariate Gaussian Copula model of the Ericsson and Volvo stock combination, which were denoted by $\mathbf{G_{eric-volvo}}$ and $\mathbf{C_{eric-volvo}}$ respectively. I randomly chose two $5000 \times 2$ dimensions data matrices from the models $G_{eric-volvo}$ and $C_{eric-volvo}$. The Gaussian model and Copula model are shown below

$$\mathbf{G_{eric-volvo}} \sim \mathbf{N}\left( \begin{pmatrix} 0.00151476 \\ -0.00103154 \end{pmatrix}^T, \begin{pmatrix} 0.00044926 & 0.00009798 \\ 0.00009798 & 0.00024042 \end{pmatrix} \right)$$

and

$$\mathbf{C_{eric-volvo}} \sim \mathbf{N}\left( \begin{pmatrix} 0 \\ 0 \end{pmatrix}^T, \begin{pmatrix} 0.9110 & 0.3267 \\ 0.3267 & 0.8970 \end{pmatrix} \right).$$

Figure 4.21 to Figure 4.26 show the kernel density contour plots and surfaces of the combination raw data, the simulated data of $G_{eric-volvo}$ and the simulated data of $C_{eric-volvo}$, respectively.

From Fig. 4.21 to 4.26, we can see that the simulation using the Gaussian model $G_{eric-volvo}$ is better than the simulation using the Gaussian Copula model $G_{eric-volvo}$, because the positive relations are in the similar place.

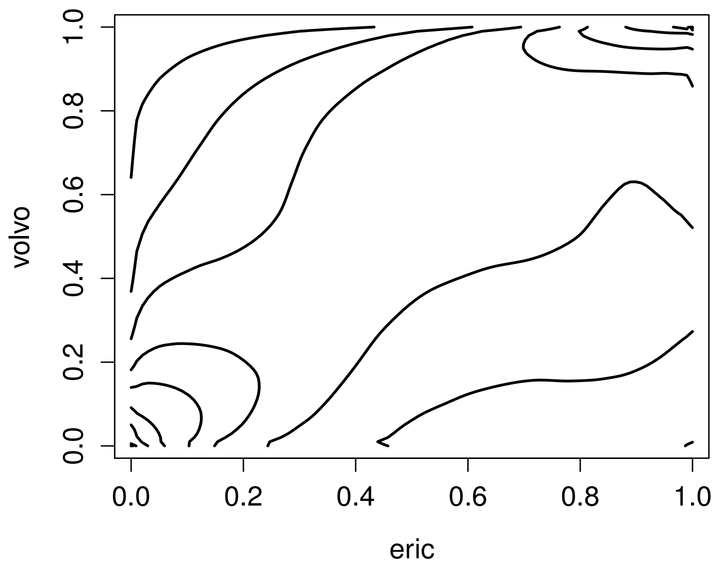


Figure 4.21: Empirical copula kernel density contour plot of the raw data of the Ericsson and Volvo stock combination.

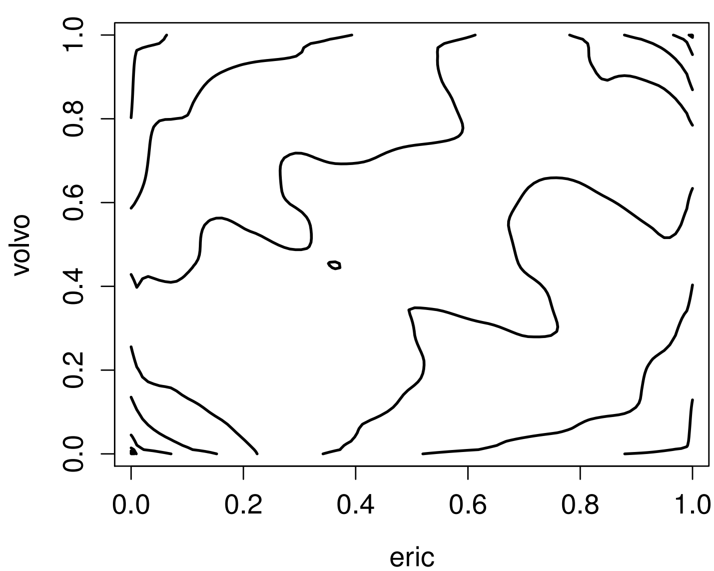


Figure 4.22: Empirical copula kernel density contour plot of the simulated data by the Gaussian model of the Ericsson and Volvo stock combination.

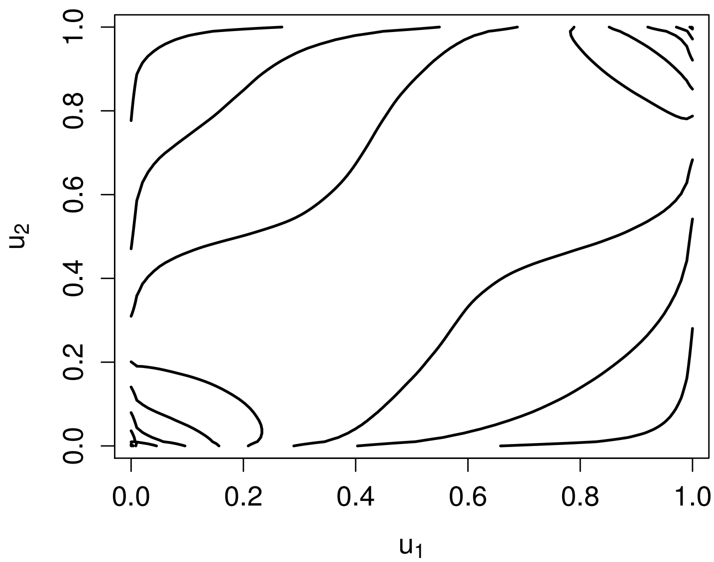


Figure 4.23: Empirical copula kernel density contour plot of the simulated data by the Copula model of the Ericsson and Volvo stock combination.

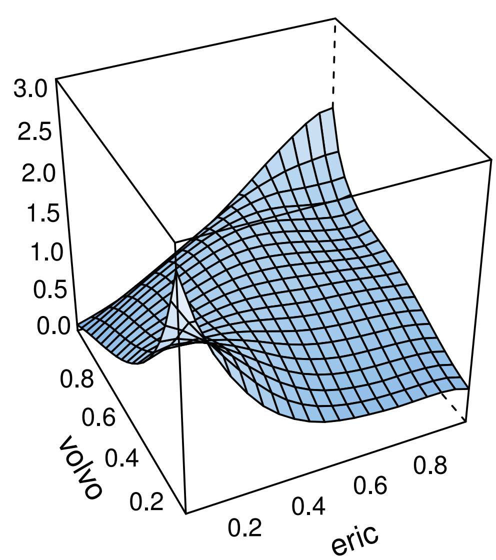


Figure 4.24: Empirical copula kernel density surface of the raw data of the Ericsson and Volvo stock combination.

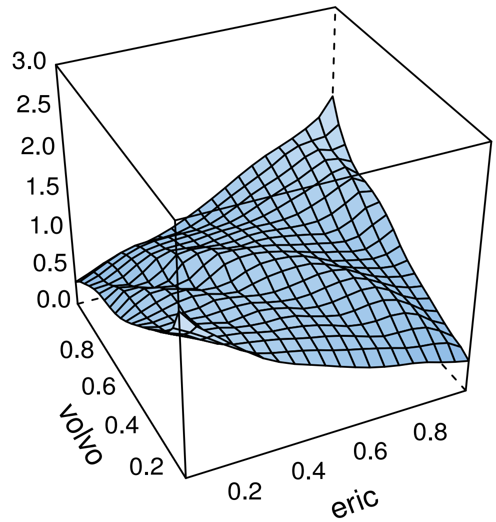


Figure 4.25: Empirical copula kernel density surface of the simulated data by the Gaussian model of the Ericsson and Volvo stock combination.

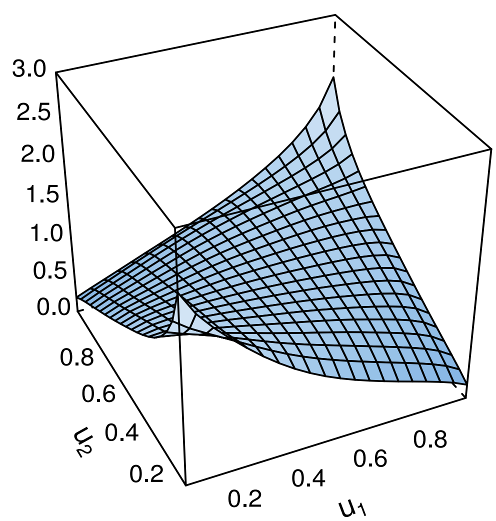


Figure 4.26: Empirical copula kernel density surface of the simulated data by the Copula model of the Ericsson and Volvo stock combination.

**3**. The simulation of the Ericsson and AstraZeneca stocks combination.

The bivariate Gaussian model and Gaussian Copula model of the Ericsson and AstraZeneca stock combination were denoted by $\mathbf{G_{eric-astra}}$ and $\mathbf{C_{eric-astra}}$, respectively. I randomly chose two $5000 \times 2$ dimensions data matrices from the models $G_{eric-astra}$ and $C_{eric-astra}$. The Gaussian model and Copula model are

$$\mathbf{G_{eric-astra}} \sim \mathbf{N}\left( \begin{pmatrix} 0.00151476 \\ 0.000665608 \end{pmatrix}^T, \begin{pmatrix} 0.00044926 & 0.00008093 \\ 0.00008093 & 0.00015445 \end{pmatrix} \right)$$

and

$$\mathbf{C_{eric-astra}} \sim \mathbf{N}\left( \begin{pmatrix} 0 \\ 0 \end{pmatrix}^T, \begin{pmatrix} 0.9110 & 0.3217 \\ 0.3217 & 0.9180 \end{pmatrix} \right)$$

The kernel density contour plots and surface plots of the raw data combination, the simulated data of $G_{eric-astra}$ and the simulated data of $C_{eric-astra}$ are shown in Fig. 4.27 to 4.32.

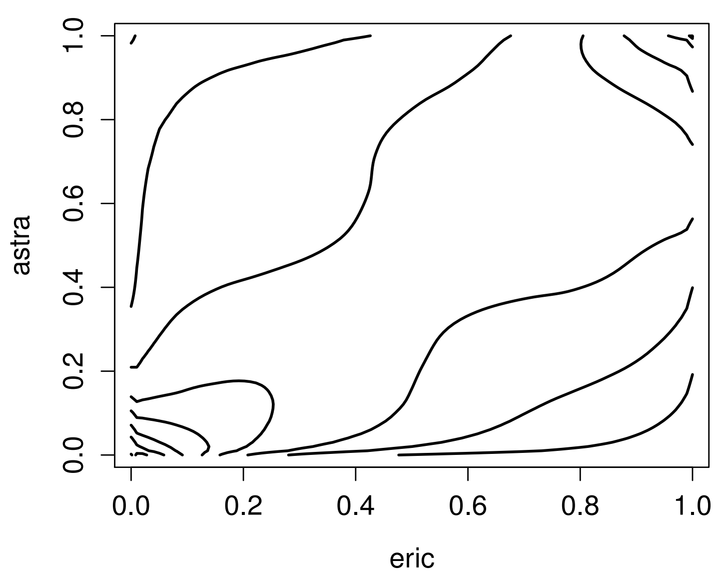


Figure 4.27: Empirical copula kernel density contour plot of the raw data of the Ericsson and AstraZeneca stock combination.

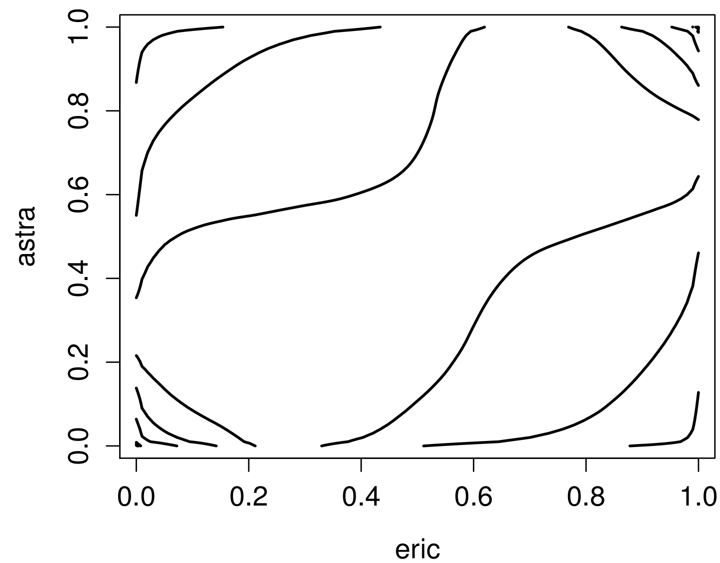


Figure 4.28: Empirical copula kernel density contour plot of the simulated data by the Gaussian model of the Ericsson and AstraZeneca stock combination.

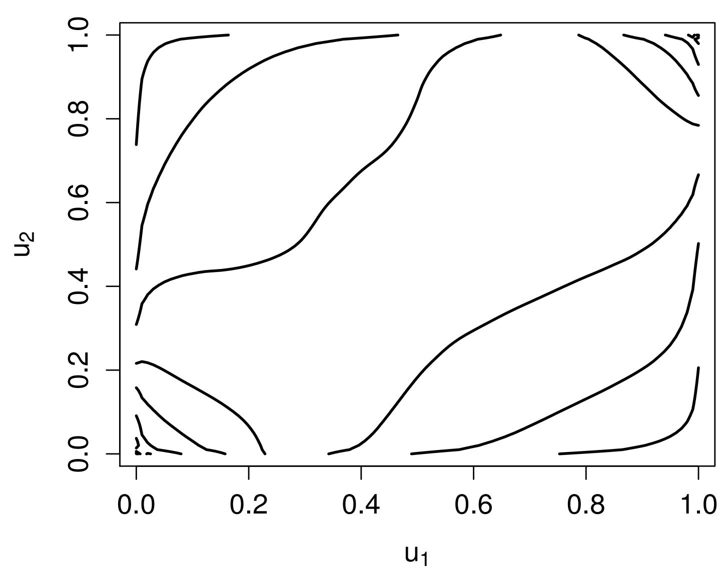


Figure 4.29: Empirical copula kernel density contour plot of the simulated data by the Copula model of the Ericsson and AstraZeneca stock combination.

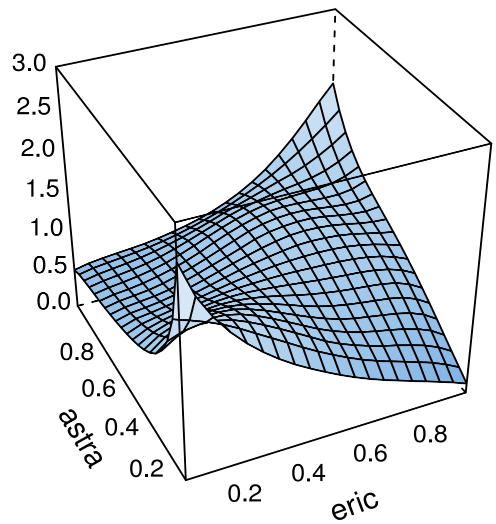


Figure 4.30: Empirical copula kernel density surface of the raw data of the Ericsson and AstraZeneca stock combination.

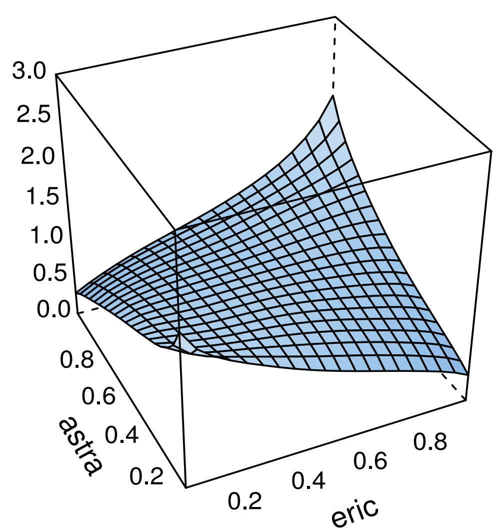


Figure 4.31: Empirical copula kernel density surface of the simulated data by the Gaussian model of the Ericsson and AstraZeneca stock combination.

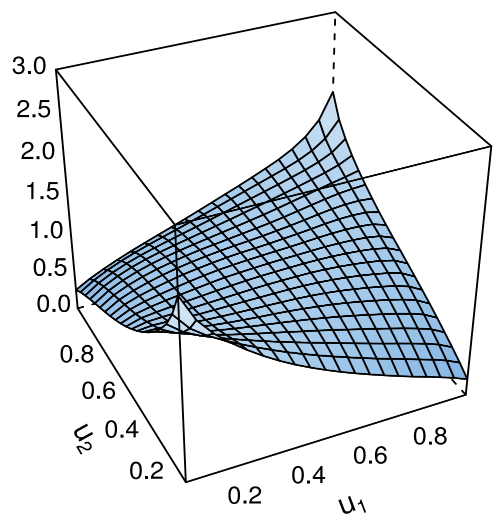


Figure 4.32: Empirical copula kernel density surface of the simulated data by the Copula model of the Ericsson and AstraZeneca stock combination.

From Fig. 4.27 to Fig. 4.32 we can see that the performance of the Gaussian Copula model $C_{eric-astra}$ is better than the simulation by the Gaussian model $G_{eric-astra}$, because the simulation by the Copula model $C_{eric-astra}$ is more similar with the raw data.

**4**. The simulation of the SEB and Volvo stock combination.

The bivariate Gaussian model and Gaussian Copula model of the SEB and Volvo stock combination are denoted by $\mathbf{G_{seb-volvo}}$ and $\mathbf{C_{seb-volvo}}$, respectively, and I randomly chose two $5000 \times 2$ dimensions data matrices from the models $G_{seb-volvo}$ and $C_{seb-volvo}$. The models $G_{seb-volvo}$ and $C_{seb-volvo}$ are built as

$$\mathbf{G_{seb-volvo}} \sim \mathbf{N}\left( \begin{pmatrix} -0.00019418 \\ -0.00103154 \end{pmatrix}^T, \begin{pmatrix} 0.00015601 & 0.00008869 \\ 0.00008869 & 0.00024042 \end{pmatrix} \right)$$

and

$$\mathbf{C_{seb-volvo}} \sim \mathbf{N}\left( \begin{pmatrix} 0 \\ 0 \end{pmatrix}^T, \begin{pmatrix} 0.9219 & 0.4283 \\ 0.4283 & 0.8970 \end{pmatrix} \right).$$

The kernel density contour plots and surface plots of the raw data, the simulated data of $G_{seb-volvo}$ and the simulated data of $C_{seb-volvo}$ are shown in Fig. 4.33 to 4.38.

From Fig. 4.33 to 4.38, we can see that the simulation of the Gaussian Copula model $C_{seb-volvo}$ is a little better than the simulation of the Gaussian model $G_{seb-volvo}$, because the positive relation and peak of the simulation of the Copula model $C_{seb-volvo}$ are in similar position with the raw data.

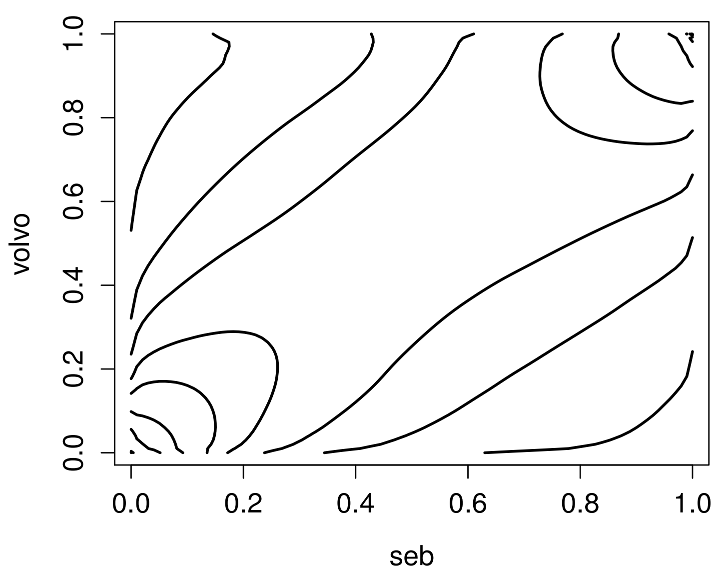


Figure 4.33: Empirical copula kernel density contour plot of the raw data of the SEB and Volvo stock combination.

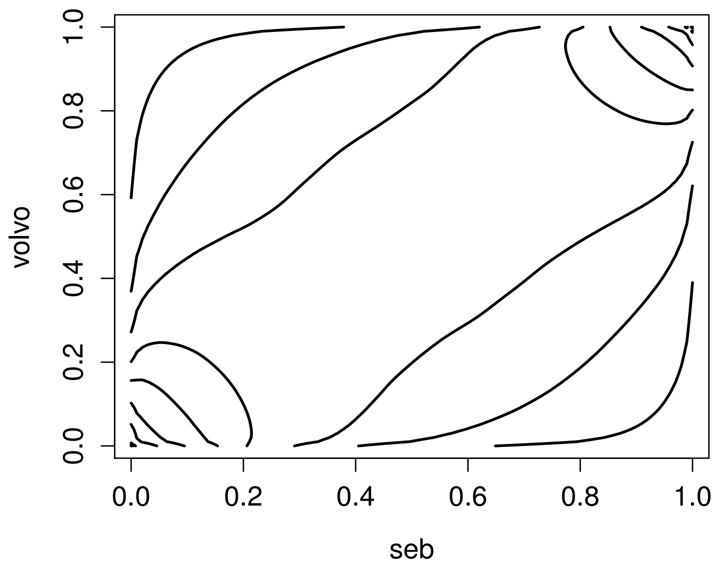


Figure 4.34: Empirical copula kernel density contour plot of the simulated data by the Gaussian model of the SEB and Volvo stock combination.

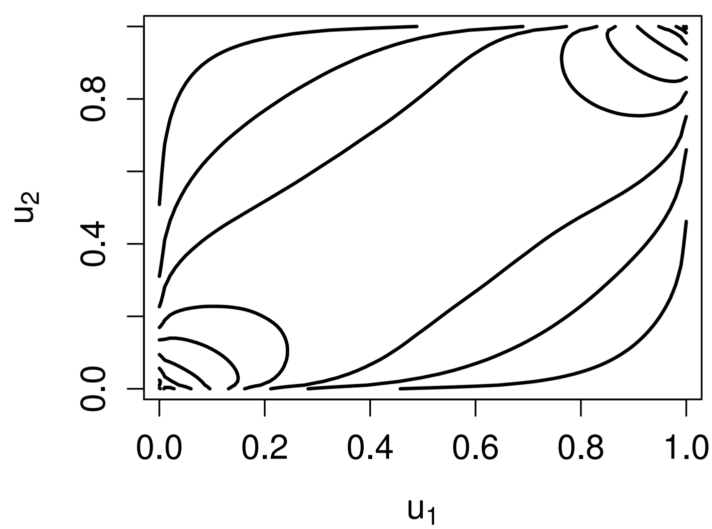


Figure 4.35: Empirical copula kernel density contour plot of the simulated data by the Copula model of the SEB and Volvo stock combination.

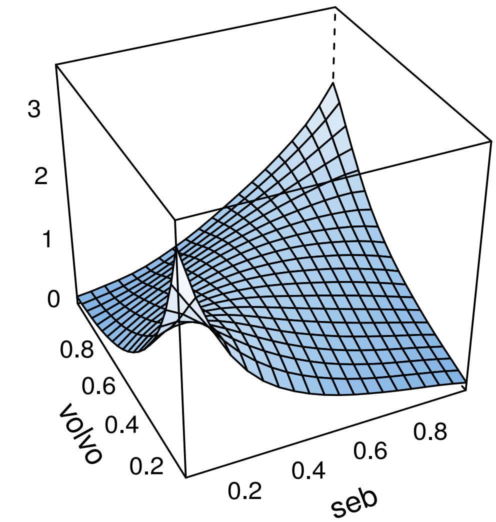


Figure 4.36: Empirical copula kernel density surface of the raw data of the SEB and Volvo stock combination.

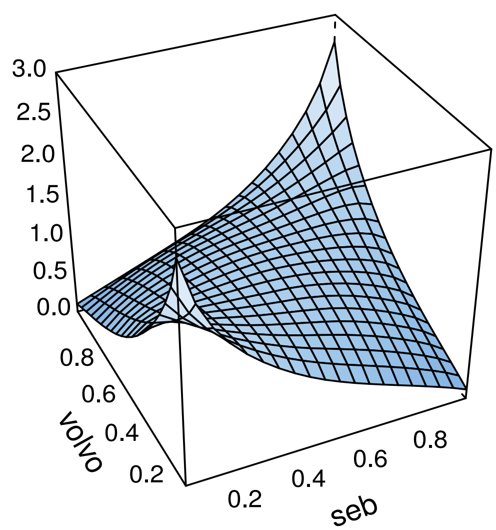


Figure 4.37: Empirical copula kernel density surface of the simulated data by the Gaussian model of the SEB and Volvo stock combination.

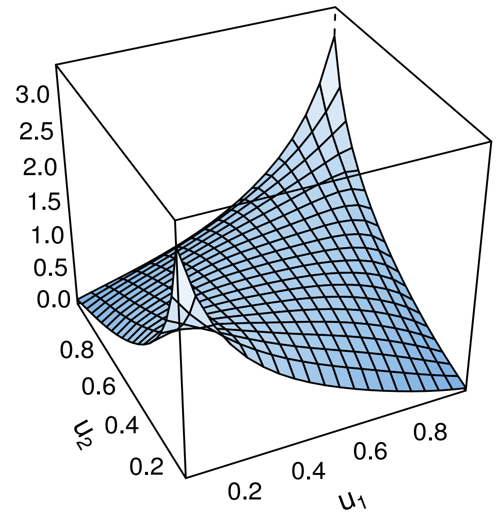


Figure 4.38: Empirical copula kernel density surface of the simulated data by the Copula model of the SEB and Volvo stock combination.

**5**. The simulation of the SEB and AstraZeneca stocks combination.

The bivariate Gaussian model and Gaussian Copula model of the SEB and AstraZeneca stock combination were denoted by $\mathbf{G_{seb-astra}}$ and $\mathbf{C_{seb-astra}}$, respectively, and I randomly chose two $5000 \times 2$ dimensions data matrices from the models $G_{seb-astra}$ and $C_{seb-astra}$. The models $G_{seb-astra}$ and $C_{seb-astra}$ are

$$\mathbf{G_{seb-astra}} \sim \mathbf{N}\left( \begin{pmatrix} -0.00019418 \\ 0.00066561 \end{pmatrix}^T, \begin{pmatrix} 0.00015601 & 0.00003631 \\ 0.00003631 & 0.00015445 \end{pmatrix} \right)$$

and

$$\mathbf{C_{seb-astra}} \sim \mathbf{N}\left( \begin{pmatrix} 0 \\ 0 \end{pmatrix}^T, \begin{pmatrix} 0.9219 & 0.2251 \\ 0.2251 & 0.9180 \end{pmatrix} \right)$$

The kernel density contour plots and surfaces of the raw data, the simulated data of $G_{seb-astra}$ and the simulated data of $C_{seb-astra}$ are shown in Fig. 4.39 to 4.44. We can see that the $C_{seb-astra}$ simulation is better in simulating the raw data than the $G_{seb-astra}$ simulation.

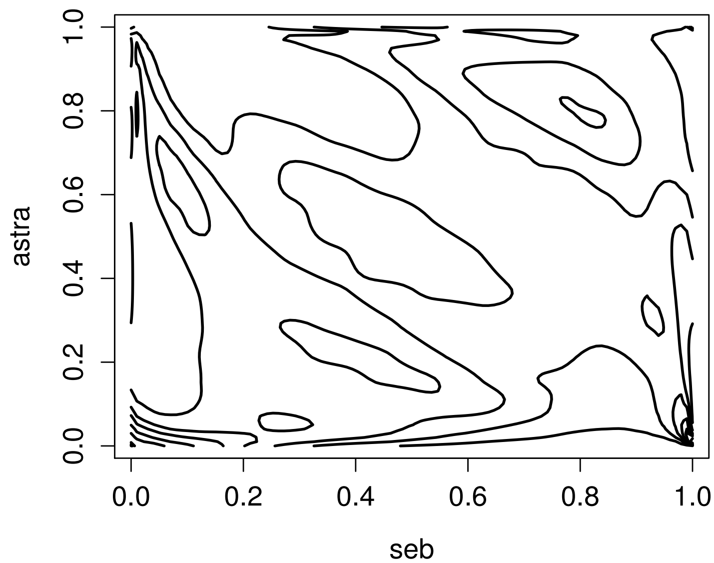


Figure 4.39: Empirical copula kernel density contour plot of the raw data of the SEB and AstraZeneca stock combination.

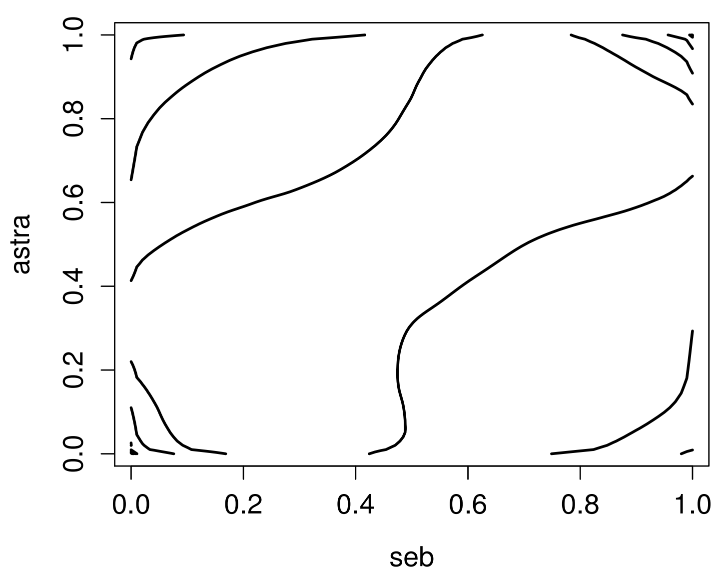


Figure 4.40: Empirical copula kernel density contour plot of the simulated data by the Gaussian model of the SEB and AstraZeneca stock combination.

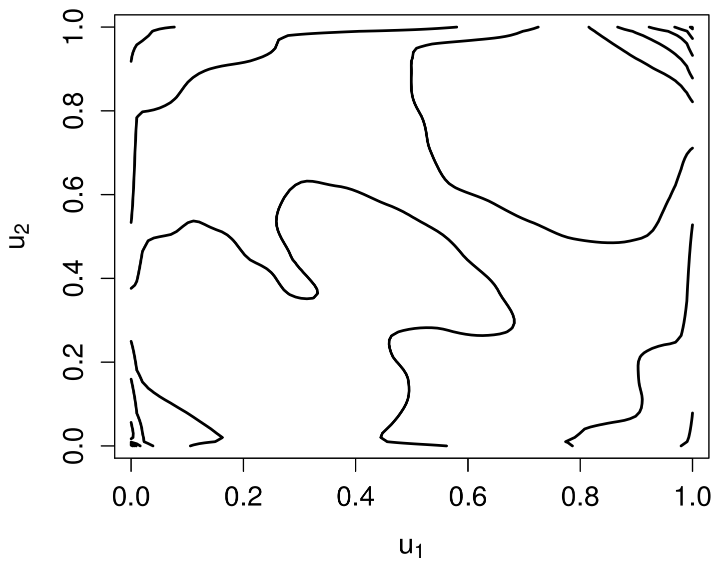


Figure 4.41: Empirical copula kernel density contour plot of the simulated data by the Copula model of the SEB and AstraZeneca stock combination.

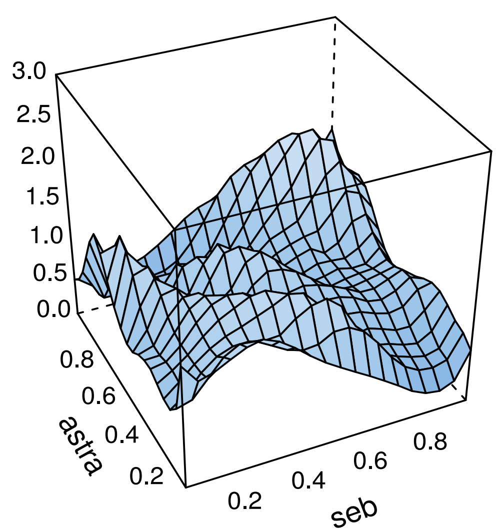


Figure 4.42: Empirical copula kernel density surface of the raw data of the SEB and AstraZeneca stock combination.

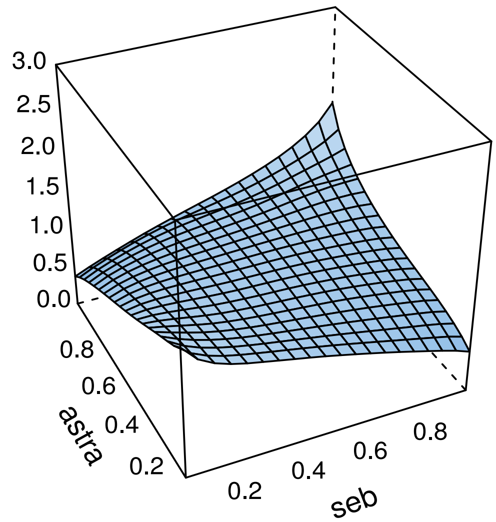


Figure 4.43: Empirical copula kernel density surface of the simulated data by the Gaussian model of the SEB and AstraZeneca stock combination.

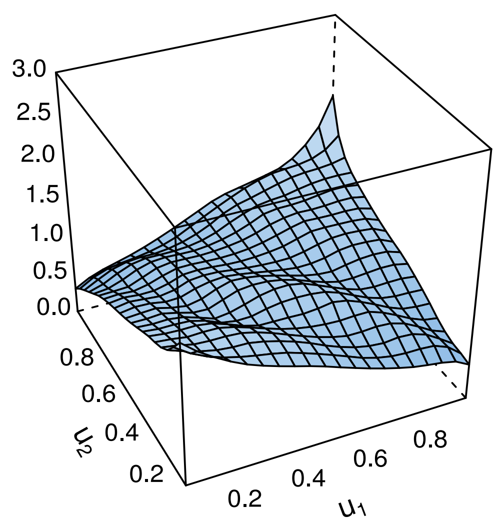


Figure 4.44: Empirical copula kernel density surface of the simulated data by the Copula model of the SEB and AstraZeneca stock combination.

**6**. The simulation of the Volvo and AstraZeneca stocks combination.

The bivariate Gaussian model and Gaussian Copula model of the Volvo and AstraZeneca stock combination were denoted by $\mathbf{G_{volvo-astra}}$ and $\mathbf{C_{volvo-astra}}$, respectively, and I randomly chose two $5000 \times 2$ dimensions data matrices from the models $G_{volvo-astra}$ and $C_{volvo-astra}$. The models $G_{volvo-astra}$ and $C_{volvo-astra}$ are

$$\mathbf{G_{volvo-astra}} \sim \mathbf{N}\left( \begin{pmatrix} -0.00103154 \\ 0.00066561 \end{pmatrix}^T, \begin{pmatrix} 0.00024042 & 0.00005532 \\ 0.00005532 & 0.00015445 \end{pmatrix} \right)$$

and

$$\mathbf{C_{volvo-astra}} \sim \mathbf{N}\left( \begin{pmatrix} 0 \\ 0 \end{pmatrix}^T, \begin{pmatrix} 0.8970 & 0.2663 \\ 0.2663 & 0.9180 \end{pmatrix} \right)$$

The kernel density contour plots and surface plots of the raw data, the simulated data of $G_{volvo-astra}$ and the simulated data of $C_{volvo-astra}$ are shown in Fig. 4.45 to 4.50. We can see that the $C_{volvo-astra}$ simulation is more similar with the raw data, so the $C_{volvo-astra}$ simulation is better than the $G_{volvo-astra}$ simulation.

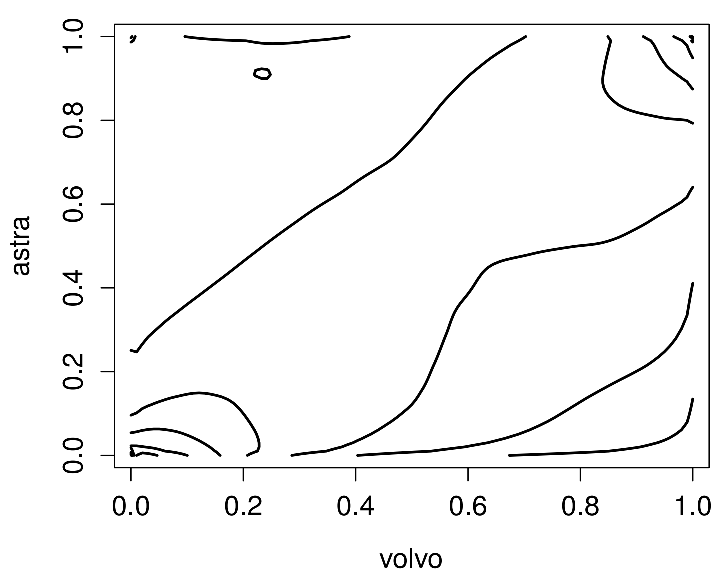


Figure 4.45: Empirical copula kernel density contour plot of the raw data of the Volvo and AstraZeneca stock combination.

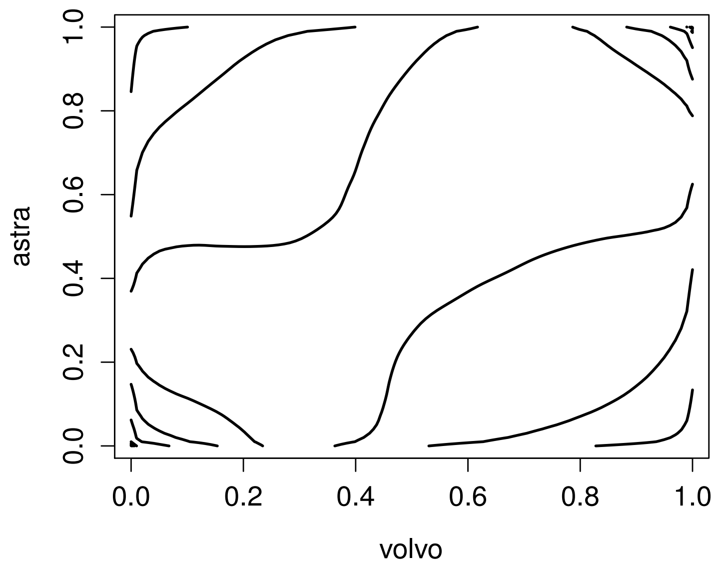


Figure 4.46: Empirical copula kernel density contour plot of the simulated data by the Gaussian model of the Volvo and AstraZeneca stock combination.

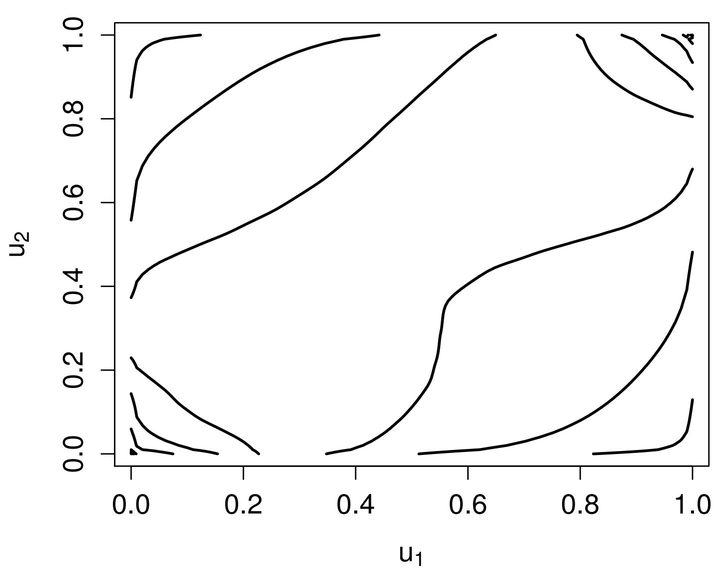


Figure 4.47: Empirical copula kernel density contour plot of the simulated data by the Copula model of the Volvo and AstraZeneca stock combination.

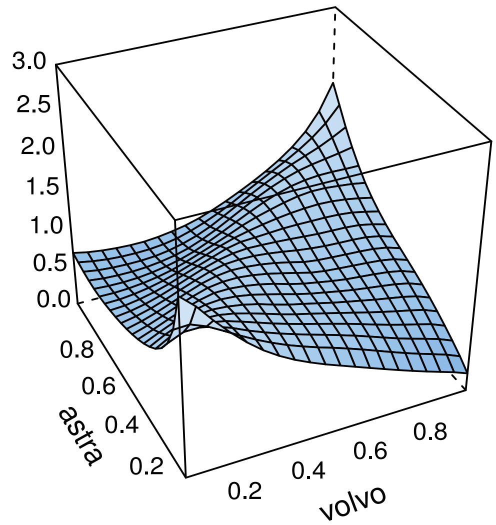


Figure 4.48: Empirical copula kernel density surface of the raw data of the Volvo and AstraZeneca stock combination.

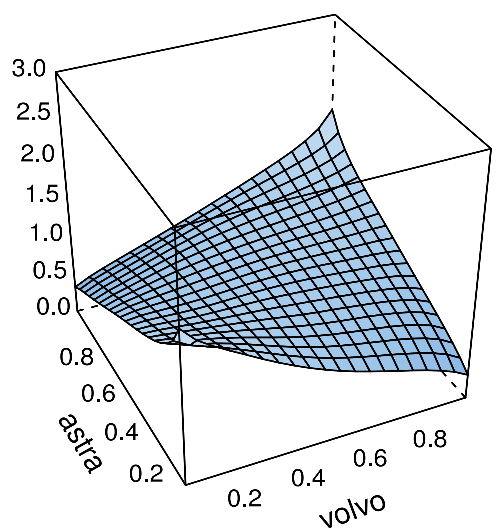


Figure 4.49: Empirical copula kernel density surface of the simulated data by the Gaussian model of the Volvo and AstraZeneca stock combination.

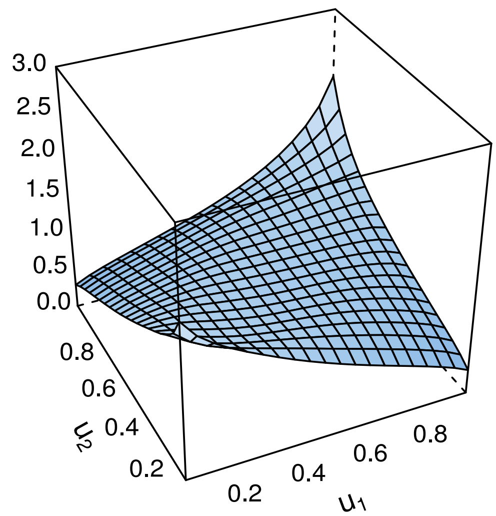


Figure 4.50: Empirical copula kernel density surface of the simulated data by the Copula model of the Volvo and AstraZeneca stock combination.

**7**. The simulation of the combination of the long-term stocks Volvo and AstraZeneca.

The bivariate Gaussian model and Gaussian Copula model of the long-term Volvo and AstraZeneca stock combination were denoted by $\mathbf{G_{l-volvo-astra}}$ and $\mathbf{C_{l-volvo-astra}}$, respectively, and I randomly chose two $5000 \times 2$ dimensions data matrices from the models $G_{l-volvo-astra}$ and $C_{l-volvo-astra}$. The models $G_{l-volvo-astra}$ and $C_{l-volvo-astra}$ are

$$\mathbf{G_{l-volvo-astra}} \sim \mathbf{N}\left( \begin{pmatrix} 0.00037907 \\ 0.00036023 \end{pmatrix}^T, \begin{pmatrix} 0.00015445 & 0.00006817 \\ 0.00006817 & 0.00017824 \end{pmatrix} \right)$$

and

$$\mathbf{C_{l-volvo-astra}} \sim \mathbf{N}\left( \begin{pmatrix} 0 \\ 0 \end{pmatrix}^T, \begin{pmatrix} 0.9681 & 0.2887 \\ 0.2887 & 0.9694 \end{pmatrix} \right).$$

The kernel density contour plots and surface plots of the raw data, the simulated data of $G_{l-volvo-astra}$ and the simulated data of $C_{l-volvo-astra}$ are shown in Fig. 4.51 to 4.56. We can see that the $C_{l-volvo-astra}$ simulation is more similar with the raw data, so the $C_{l-volvo-astra}$ simulation is better than the $G_{l-volvo-astra}$ simulation.

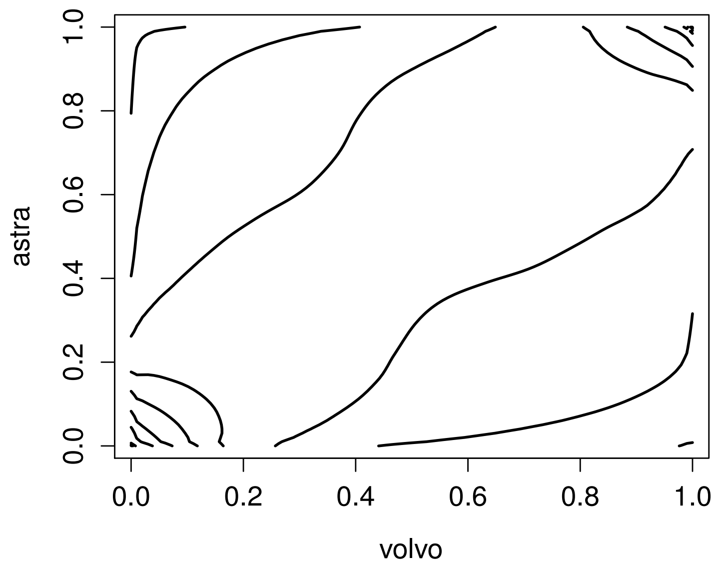


Figure 4.51: Empirical copula kernel density contour plot of the raw data of the long-term Volvo and AstraZeneca stock combination.

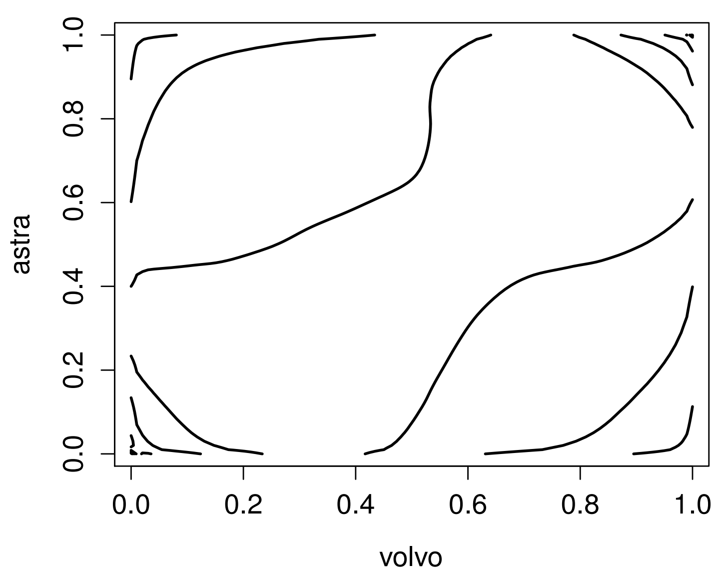


Figure 4.52: Empirical copula kernel density contour plot of the simulated data by the Gaussian model of the long-term Volvo and AstraZeneca stock combination.

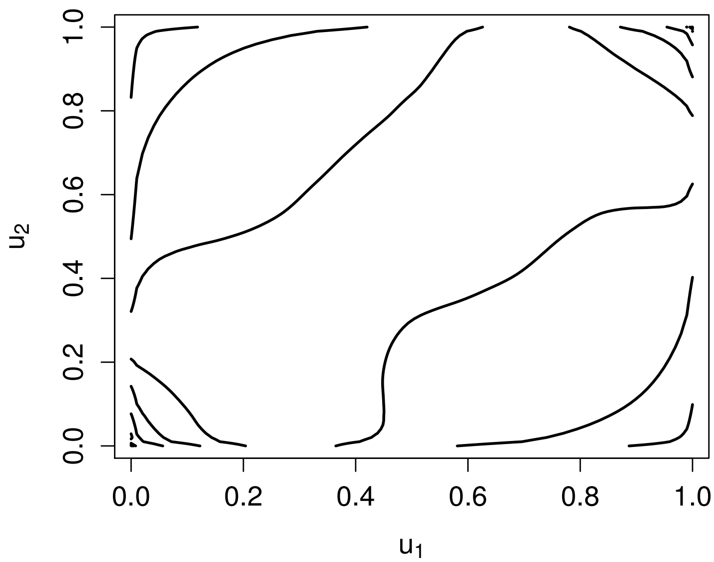


Figure 4.53: Empirical copula kernel density contour plot of the simulated data by the Copula model of the long-term Volvo and AstraZeneca stock combination.

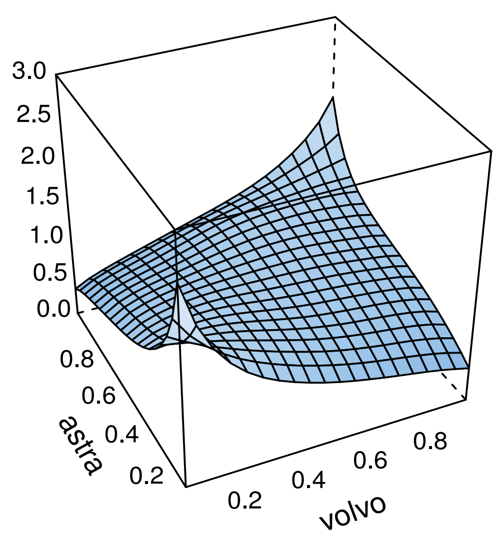


Figure 4.54: Empirical copula kernel density surface of the raw data of the long-term Volvo and AstraZeneca stock combination.

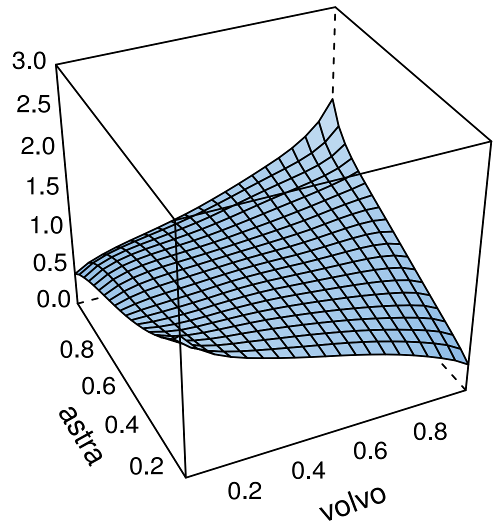


Figure 4.55: Empirical copula kernel density surface of the simulated data by the Gaussian model of the long-term Volvo and AstraZeneca stock combination.

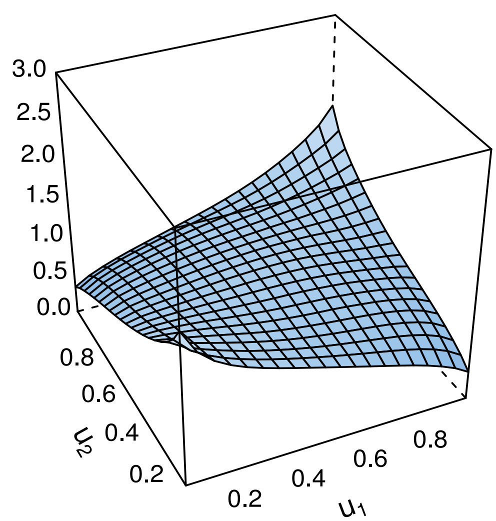


Figure 4.56: Empirical copula kernel density surface of the simulated data by the Copula model of the long-term Volvo and AstraZeneca stock combination.

## 4.3 The goodness of fitting of the copula models

By quantifying Kolmogorov-Smirnov (KS) distance the goodness of fitting of the bivariate Gaussian Copula models of the six combinations of the four short-term stocks and the combination of the two long-term stocks will be tested, and compared. The KS distances are shown in Table 4.1.

| | Stock combinations | Parameter $\rho$ | KS distance |
|---|---|---|---|
| Short term | Ericsson-SEB | 0.2752949 | 0.0510 |
| | Ericsson-Volvo | 0.3720650 | 0.0572 |
| | Ericsson-AstraZeneca | 0.3396017 | 0.0370 |
| | SEB-Volvo | 0.5119775 | 0.0464 |
| | SEB-AstraZeneca | 0.2589931 | 0.0446 |
| | Volvo-AstraZeneca | 0.3049567 | 0.0308 |
| Long term | Volvo-AstraZeneca | 0.2777583 | 0.0794 |

Table 4.1: Parameters of the Copula models and their corresponding Kolmogorov-Smirnov distances of the six short-term stock combinations and the long-term stocks combination.

By visually checking, Table 4.1 shows that there are no relationships between the values of the parameters and the KS distances. For the copula models of the short-term stock combinations, the KS distances of the Ericsson-SEB and Ericsson-Volvo combinations are higher than those of other short-term stock combinations. The KS distance of the long-term stock combination is longer than those of the short-term stock combinations. Namely, the fittings of the short-term stock combinations by the copula models are better than the fitting of the long-term stock combination. This is due to the much longer time spans of the two long-term stock data than the short-term stock data, so that the rule of randomness has much more changes over a much longer period of time.

# 5 Portfolio and risk analysis

## 5.1 Portfolio analysis

In this section, I established the portfolios of the six combinations of the four short-term stocks and the combination of the two long-term stocks extracted from the Copula models and Gaussian models respectively. The extractions were done by randomly selecting data with a sample size of 250 (approximately equal to the number of one-year working days) for each of the Copula model and Gaussian model and for 10 times and 100 times.

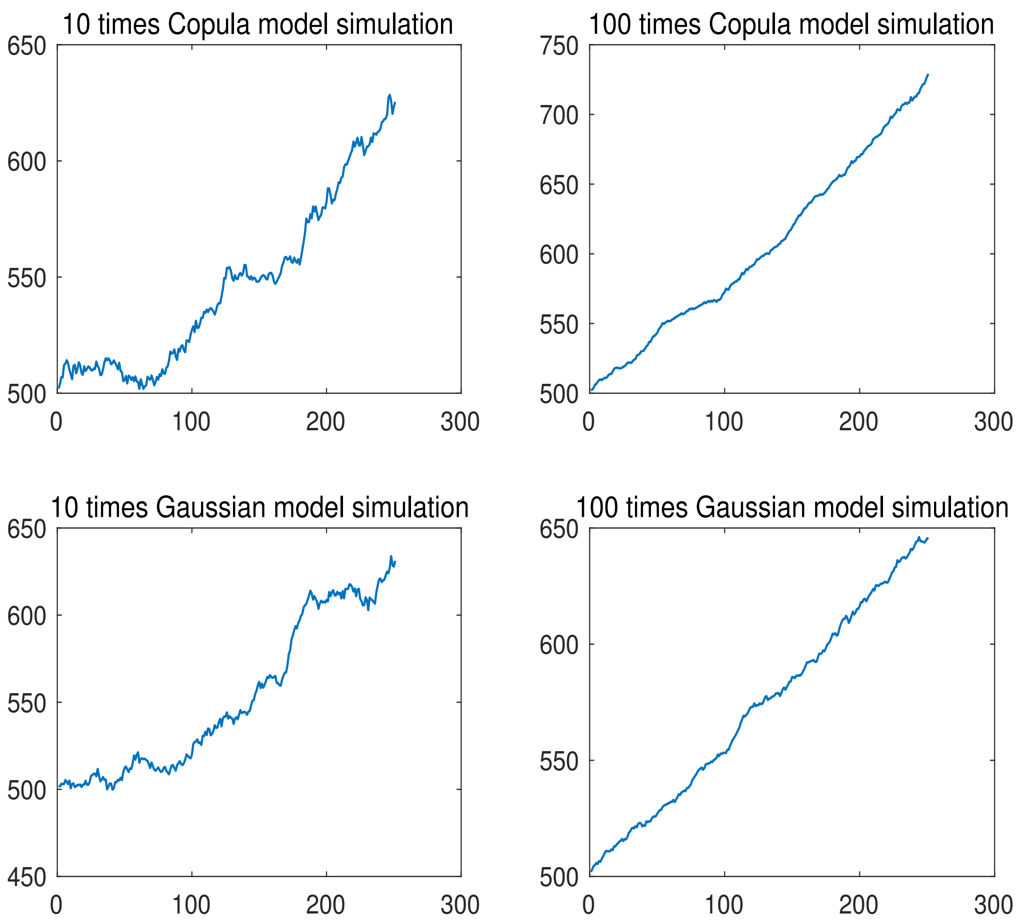


Figure 5.1: Portfolio curves of the 10 times and 100 times simulations of the Copula model and the Gaussian model of the Ericsson and SEB stock combination.

From top-left panel of Fig. 5.1, we can see that the curve has an increasing trend over the time, and the portfolio curve reaches its minimum value at the point t = 60, and at the point t = 245, the portfolio curve reaches its maximum value which is about 625. The curve is increasing from t = 60 to t = 140 and from t = 190 to t = 245. From the top-right panel of Fig. 5.1, we can see that the curve is smoother, but it also has an increasing trend over the time. The portfolio curve reaches its maximum value at the point t=250.

From bottom-left panel of Fig. 5.1, we can see that the portfolio curve reaches its maximum value at the point t = 250. The curve has a radical increase from t = 170 to t = 200. From bottom-right panel of Fig. 5.1, we can see that the curve reaches its maximum value at the point t = 245, and it is smoother than those in 10 simulations but less smooth than that for the Copula model in 100 times simulation. Based on the results in Fig. 5.1 it is difficult to say which model (Copula or Gaussian) is better in simulating the Ericsson and SEB stock combination.

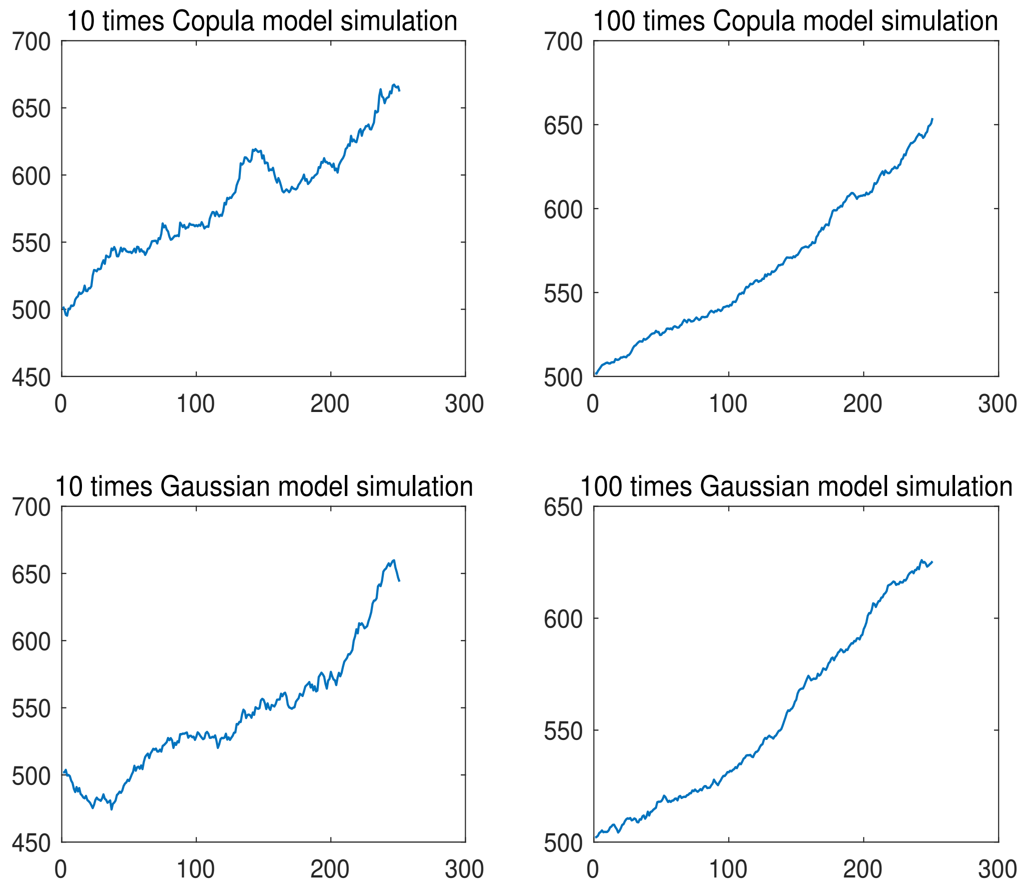


Figure 5.2: Portfolio curves of the 10 times and 100 times simulations of the Copula model and the Gaussian model of the Ericsson and Volvo stock combination.

Figure 5.2 shows similar results with Fig. 5.1. All the four portfolio curves show obviously increasing trends over the whole period, with 10 times simulations showing more variation and 100 times simulations showing less variation over the time.

The portfolio curve in the top-left panel reaches its maximum value at the point t = 245 which value is about 670. The curve shows a radical increasing trend from t = 110 to t = 140, and a decreasing trend from the point t = 140 to the point t = 170. The portfolio curve in the top-right panel reaches its maximum value at the point t=250. The portfolio curve in the bottom-left panel reaches its maximum value at the point t = 245 and minimum value at the point t = 40, respectively. The curve shows a radical increasing trend from t = 205 to t = 245. The portfolio curve in the bottom-right panel reaches its maximum value at the point t = 245. Still, it is difficult to say which model is better in simulating the Ericsson and Volvo stock combination.

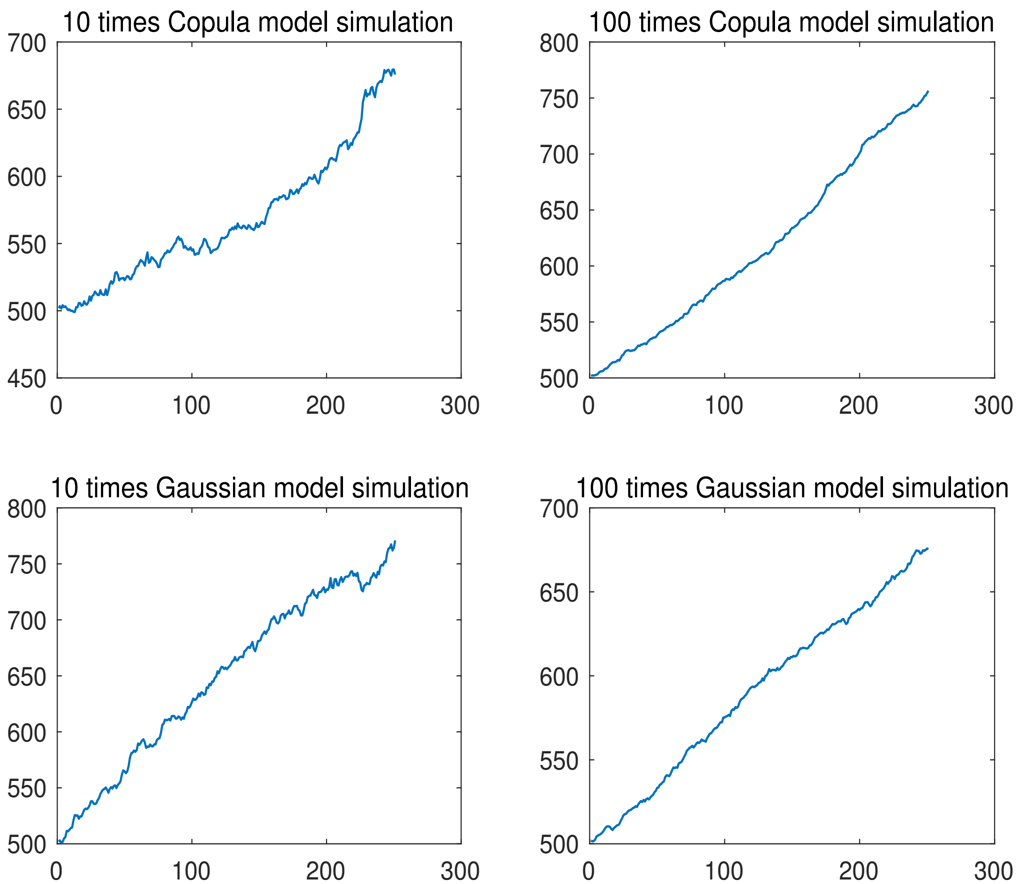


Figure 5.3: Portfolio curves of the 10 times and 100 times simulations of the Copula model and the Gaussian model of the Ericsson and AstraZeneca stock combination.

Figure 5.3 also shows that all the four portfolio curves have obviously positive trends over the whole period, with 10 times simulations showing more variation and 100 times simulations showing less variation over the time. It is also difficult to say, according to the figure, which model is better in simulating the Ericsson and AstraZeneca stock combination.

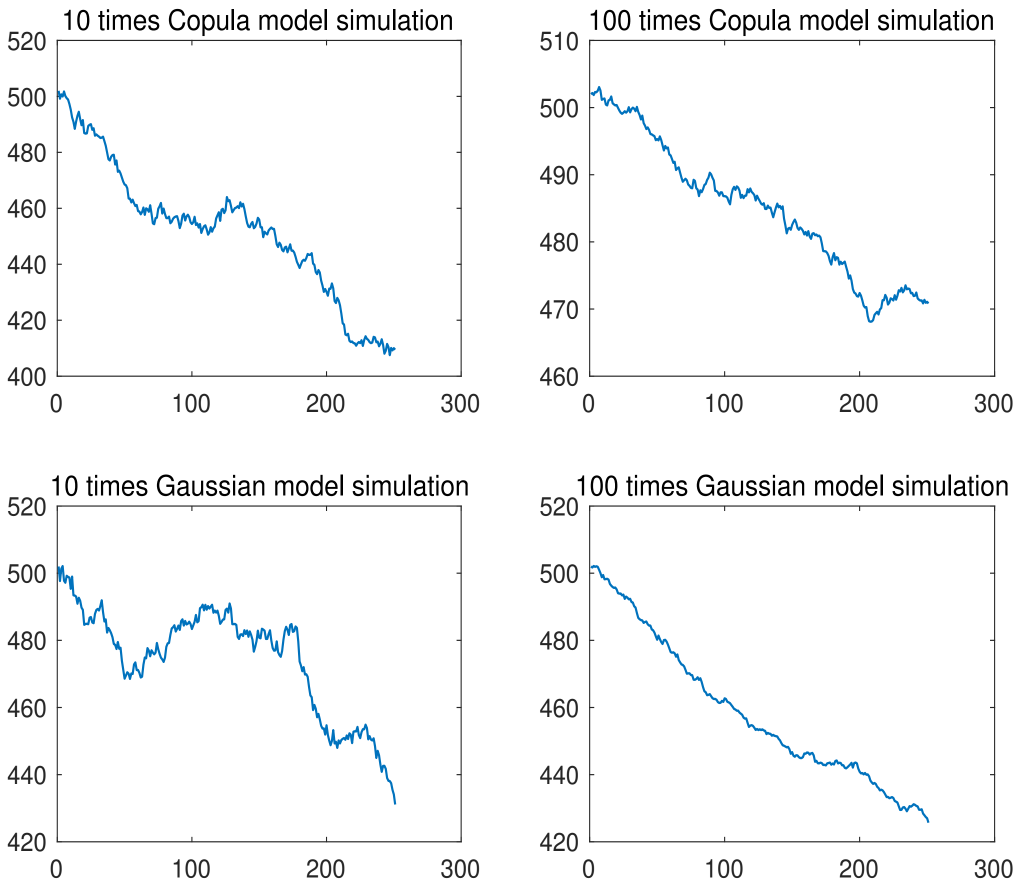


Figure 5.4: Portfolio curves of the 10 times and 100 times simulations of the Copula model and the Gaussian model of the SEB and Volvo stock combination.

Figure 5.4 shows that all the four portfolio curves have decreasing trends over the whole period, with 10 times simulations showing more variation and 100 times simulations showing less variation over the time.

The portfolio curve in the top-left panel reaches its minimum value at the point t = 250, and has an increase trend from t = 110 to t = 130. The portfolio curve in the top-right panel reaches its minimum value at the point t = 205, and has an increasing trend from t = 205 to t = 240.

The portfolio curve in the bottom-left panel shows big fluctuations over the time. The curve has an increase trend from t = 50 to t = 100, and has two obvious decreasing trends from t = 180 to t = 210 and from t = 230 to 250. The curve reaches its minimum value at the point t = 250. The curve in the bottom-right panel reaches its minimum value at the point t = 250.

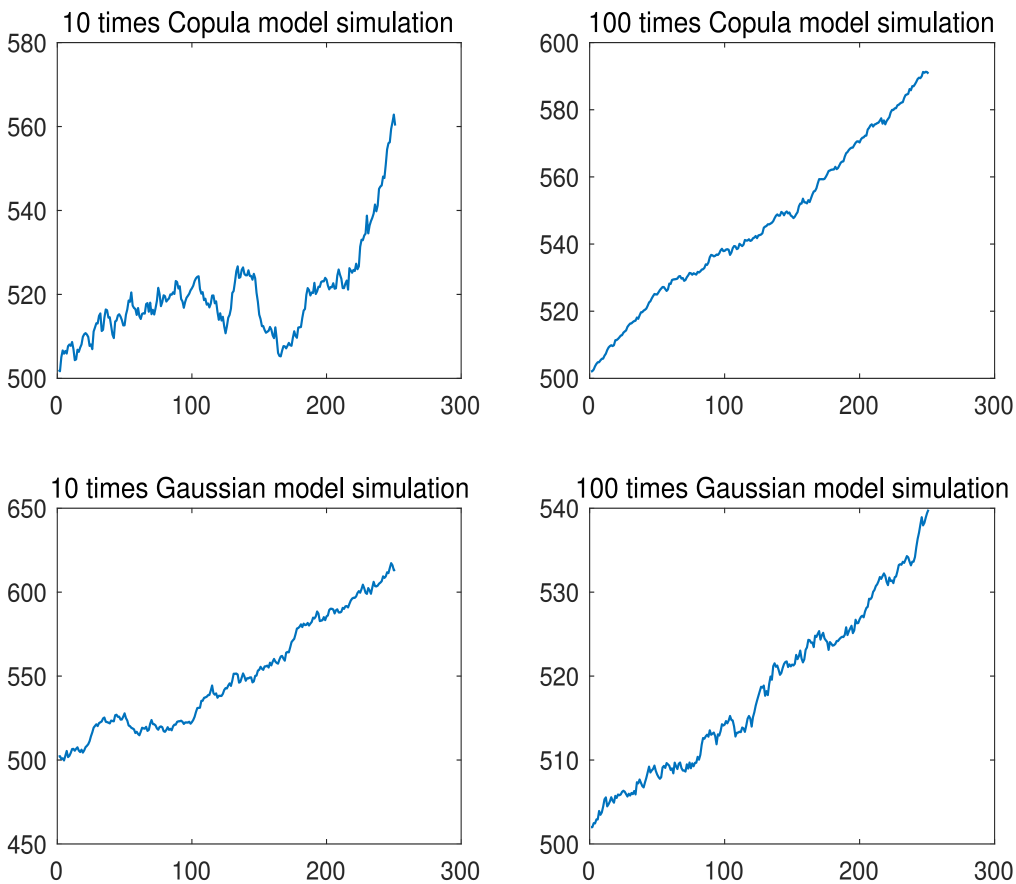


Figure 5.5: Portfolio curves of the 10 times and 100 times simulations of the Copula model and the Gaussian model of the SEB and AstraZeneca stock combination.

All the portfolio curves in Fig. 5.5 show obviously positive trends over the time, with 100 time simulations by Copula model showing least variation over the time.

The curve in the up-left panel reaches its minimum value at the point t = 160 and maximum value at the point t = 250, and has an obvious increase from t = 215 to t=250. The curve in the up-right panel reaches its maximum value at the end point t=250. Both curves in the bottom panels reach their maximum values at the end points.

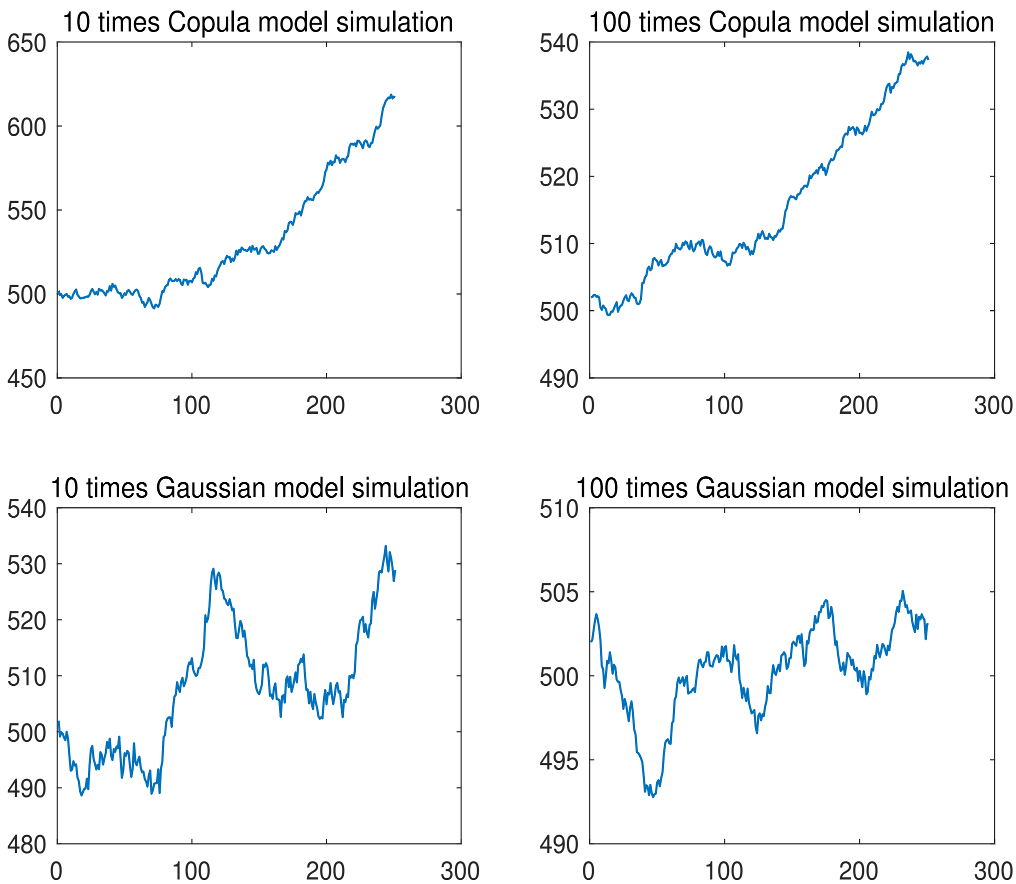


Figure 5.6: Portfolio curves of the 10 times and 100 times simulations of the Copula model and the Gaussian model of the Volvo and AstraZeneca stock combination.

The curves in the first three panels show obviously positive trends for the whole time period, while the curve in the last panel (bottom-right) shows a slightly positive trend for the whole period. The curve in the up-left panel reaches its maximum value at the point t = 250, and the curve in the up-right panel reaches its maximum value at the point t =

230. Seasonal variations are obvious for the curves in the bottom panels. The curve in the bottom-left panel shows two obvious increases from t = 80 to t = 120 and from t = 215 to t = 240, and shows an obvious decrease from t = 120 to t = 160. The curve in the bottom-right panel has three obvious increases from t = 45 to t = 70, from t = 120 to t = 180 and from t = 205 to t = 230, and it also has three obvious decreases from t = 0 to t = 45, from t = 105 to t = 130 and from t = 180 to t = 205.

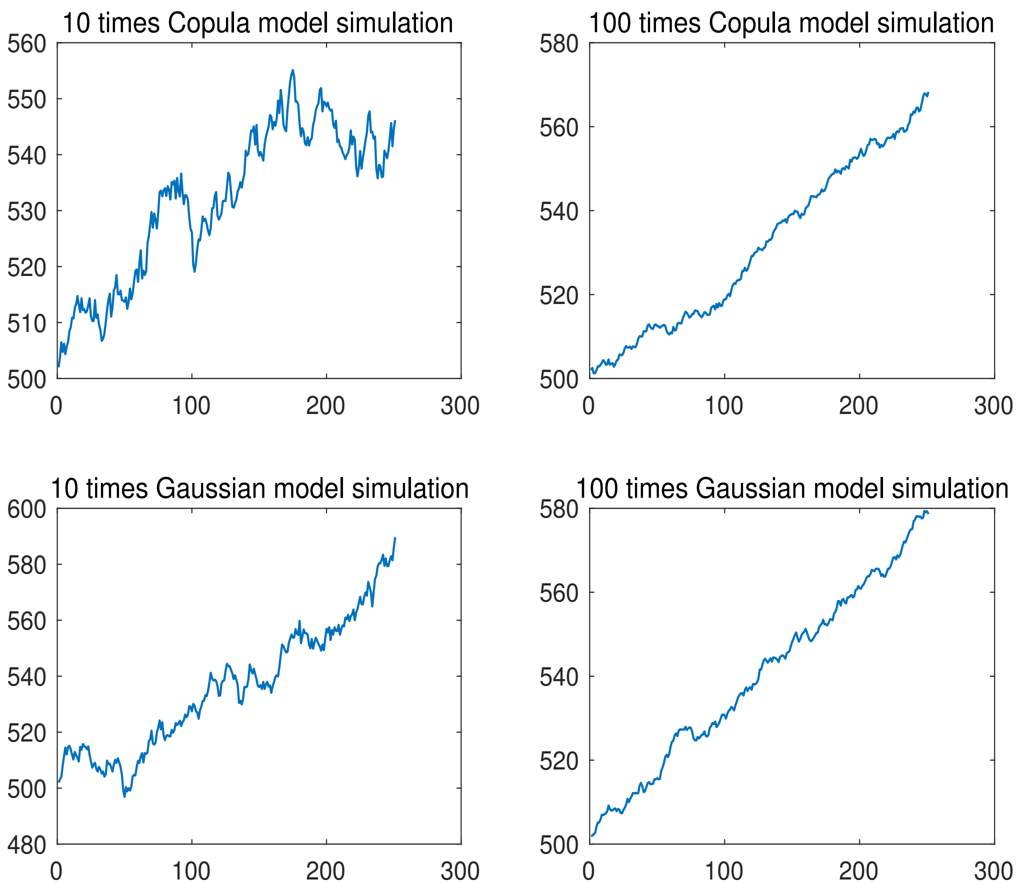


Figure 5.7: Portfolio curves of the 10 times and 100 times simulation of the Copula model and the Gaussian model of the long-term Volvo and AstraZeneca stock combination.

All the four portfolio curves in Fig. 5.7 show obviously increasing trends for the whole period, with 10 times simulations showing more variation and 100 times simulations showing less variation over the time.

The curve in the top-left reaches its maximum value at the point t = 180. The curves in the top-right and bottom panels reach their maximum values at the end point.

According to the results from the portfolio analysis (Fig. 5.1-5.7) it is difficult to say which model is better in simulating the stock combinations. Therefore, risk analysis will be done in the next section.

## 5.2 Risk analysis

In this section, in order to make more accurate comparisons between the Copula models and Gaussian models, I simulated using each of the Copula model and the Gaussian model 100 times and calculated the portfolios. The 100 values at the end points (t=250) calculated based on each model were saved. Then I calculated the probability density of the 100 values for each model. The results are shown in Fig. 5.8 and 5.9.

From Fig. 5.8 and Fig. 5.9, we can see that the risks of the Copula models $C_{eric-seb}$, $C_{eric-astra}$, $C_{seb-volvo}$, $C_{seb-astra}$, $C_{volvo-astra}$ and $C_{l-volvo-astra}$ are smaller than the risks of the Gaussian models $G_{eric-seb}$, $G_{eric-astra}$, $G_{seb-volvo}$, $G_{seb-astra}$, $G_{volvo-astra}$ and $G_{l-volvo-astra}$. And the risk of the Copula model $C_{eric-volvo}$ is bigger than the risk of the Gaussian model $G_{eric-volvo}$.

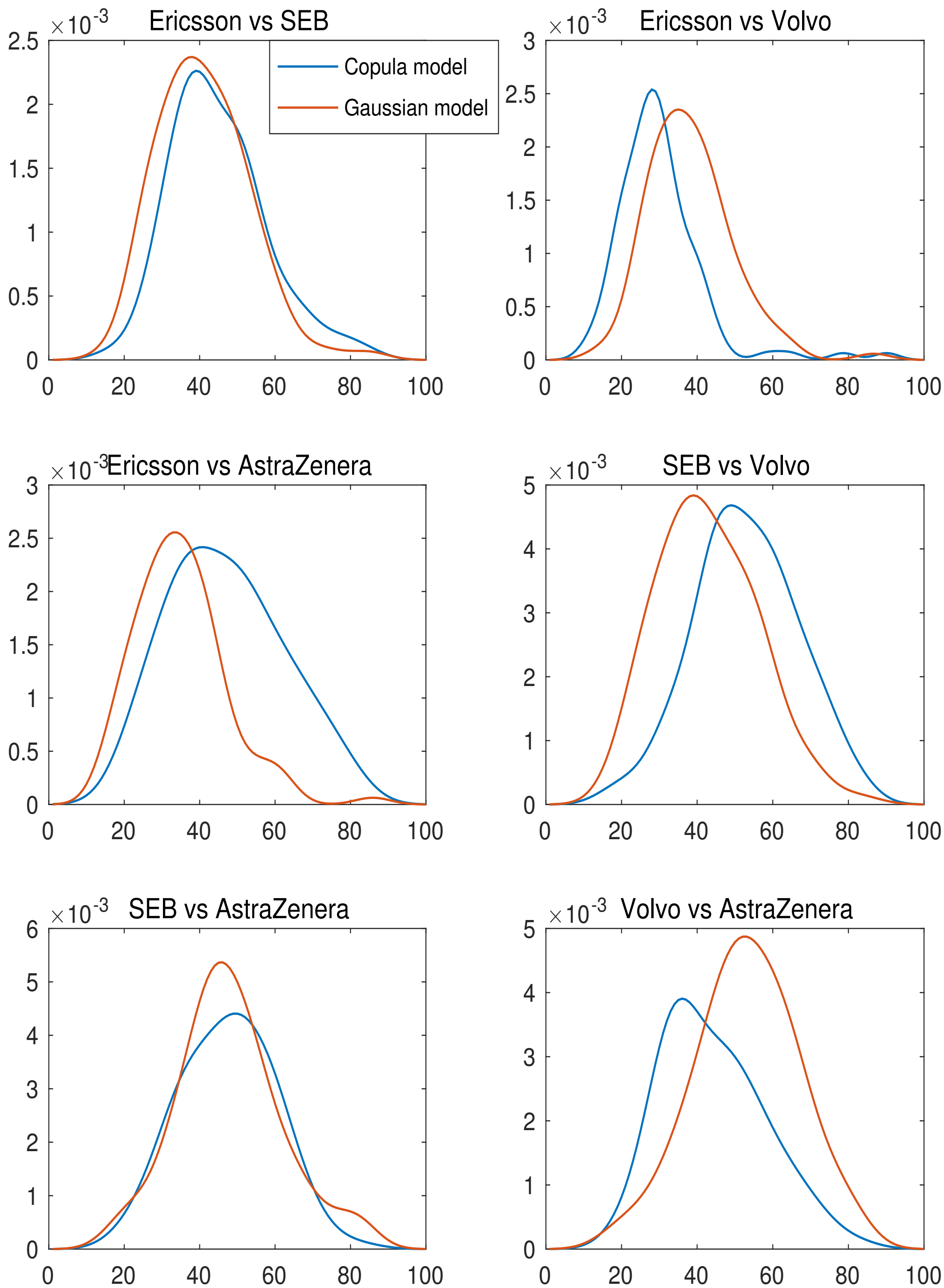


Figure 5.8: Risks comparisons of the Copula models and the Gaussian models of the six short-term stock combinations.

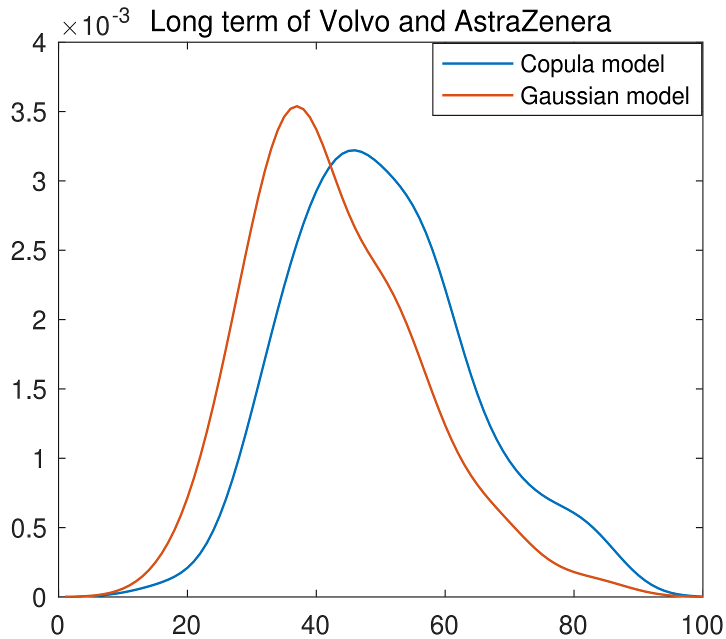


Figure 5.9: Risks comparisons of the Copula models and the Gaussian models of the long-term stock combinations.

# 6 Conclusion

The Copula method was applied to the traditional bivariate Gaussian model to improve the dependence analysis of stock returns within several bivariate portfolio. It was concluded that strong positive dependences existed in ACPs of the stocks between Ericsson and Volvo, and Ericsson and AstraZeneca, and Volvo and AstraZeneca, and the strongest positive dependence among these stocks appeared between the stock SEB and the stock Volvo. In the stocks market, it is good to execute a portfolio if there is a strong positive dependence between the two stocks.

According to the portfolio analysis, both of the Copula and Gaussian models showed the same trend for the same stock combination. For the same stock combination, the maximum values of the portfolio curves in most of the Copula models with 100 times simulations were bigger than those in the Gaussian models if the curve trend was increasing, while the minimum value of the portfolio curve in the Copula model with 100 times simulations was smaller than this in the Gaussian models if the curve trend was decreasing. Together with the figures of the risk analysis, we conclude that most of the Copula models have the smaller risk than the Gaussian models.

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