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Modelling the Nordic Electricity Market using Infinite–Dimensional Stochastic PDE:s

Master's Thesis in Engineering Mathematics and Computational Science

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Abstract

The prices of energy futures tend to have a noise structure that cannot easily be described with only a few factors. Therefore an infinite–dimensinoal model has been used to model and simulate the price of energy futures as paths of infinite–dimensional L^2–valued stochastic processes was made. The simulations agreed with previous attempts. The parameters of the model was then fitted to real price data. The results shared some features with the recorded prices.

Acknowledgements

First I would like to thank my supervisor, associate Prof. Patrik Albin. I am also grateful to junior Prof. Andrea Barth for answering questions about her article and Tobias Einarsson for providing data of the price history. Last, but definitely not least I thank everyone giving me useful distraction and moral support. Without you this thesis would not have been possible. Jonas Källén, Gothenburg 2015-01-05

Contents

1 Introduction

1.1 Background

The electricity market is volatile and trading with future contracts is common. It is therefore desirable to have a good model for the futures. Energy differs from most other commodities. In contrast to most commodities electricity is non-storable and the delivery has to be during a period of time rather than instantaneously. Also the prices of the futures seem to have a complicated structure. For some commodities the price fluctuations can be well described with only a few noise components, but Koebakker and Ollmar [1] showed that many components were needed in order to explain the noise structure of electricity prices. Therefore an infinite–dimensional model was developed and simulated by Barth and Benth in [2].

1.2 Aim

The aim of this project is to simulate possible development of prices of energy futures with the model suggested in [2]. Thereafter the parameters of the model will be fitted to data from the real energy market and the results will be compared.

1.3 Limitation

No improvements to the model from [2] will be developed. Also no proper goodness-of-fit test will not be developed or used.

2 Theory

2.1 Banach space valued integration

When integrating a $L^2(\Omega)$-valued process a normal Lebesgue integration is not available. Instead a generalisation to Banach space valued functions is used, called Bochner integration.

Let $(A, \mathscr{A}, \mu)$ be a measure space and let E be a Banach space. We say that a function $f : A \to E$ is Bochner integrable if there exists a sequence of of simple functions f_n such that

$$\lim_{n\to\infty} \int_A ||f - f_n||_E \, d\mu = 0$$

where integration of course is in Lebesgue meaning, as the norm is real-valued. We will see that in our case we get that $E = L^2([0, \tau])$ and $A = [0, T]$, $\tau, T > 0$.

Let $f_n = \sum_{i=1}^n 1_{A_i} x_i$ and $\int f_n = \sum_{i=1}^n \mu(A_i) x_i$. We are now ready to define the integral of any Bochner integrable function as

$$\int_A f \, d\mu = \lim_{n\to\infty} \int_A f_n \, d\mu.$$

The Bochner integral has a property closely related to triangular inequality in the Lebesgue theory, namely that

$$|| \int f d\mu || \leq \int ||f|| d\mu$$

and hence if the integral of the norm is finite, so is the norm of the integral. For further details, see [3].

2.2 Lévy processes

When performing stochastic integration we want the notion of a driving noise to make sense. More precisely we want to able to write $M(t) = \int_0^t dM(t)$. This propertiy is called infinite divisibility.

Definition $M(t)$ is said to be an **infinitely divisible** process if for each choice of $t > 0$ and $n \in \mathrm{N}$ there exists a sequence Y_i of i.i.d. random variables such that

$$M(t) \overset{D}{=} \sum_{i=1}^n Y_i \quad \text{and} \quad Y_i \overset{D}{=} M(\frac{t}{n})$$

The most well-known such distribution is a one dimensional Brownian motion. In that case we have that $Y_i \overset{D}{=} N(0, \frac{t}{n})$. In general this property is had by a rich family of processes, called Lévy processes. Further properties of these processes can be found in [4].

Under reasonable continuity assumptions it is therefore possible on a and b, to define a process by

$$X(t) = \int_0^t a(s, X(s))ds + \int_0^t b(s, X(s))dL(s) \tag{1}$$

for a Lévy process driving noise $dL(t)$. Note that this integral process can be understood as Bochner integrals and that $b : H \to E$, should be be understood as an operator between two, not necessarily equal, Hilbert spaces.

2.2.1 Hilbert valued processes

Let us consider the special case where a Lévy process, $M(t)$, not necessarily equal to $L(t)$, takes values in a separable Hilbert space, H. For any separable Hilbert space, there exists a countable, orthonormal basis and we may write

$$M(t)(x) = \sum_{i=0}^{\infty} (M(t), e_i)_H e_i(x) \tag{2}$$

where the e_i:s form the basis. Note that since the Lévy process is infinitely divisible each inner product will also be a Lévy process.

Let us consider Equation 1 with L defined as in Equation 2, b as an operator, mapping $H \to \mathrm{L}^2([0, \tau])$ and a as an L^2-valued function. With these assumptions we get that $X(t)$ is an L^2-valued process.

2.3 Numerical simulation

Considering the process described in Equation 1. We will focus on the case were $X(t)$ is $\mathrm{L}^2([0, \tau])$-valued. Before simulating a path of the process a discretisation in space of X is needed. For this a method based on e.g., finite differences could be used. Note that no additional noise needs to be added in this step.

After this discretisation has been done we arrive in a, possibly large if the grid is fine, system of coupled ordinary stochastic differential equations.

If this system was deterministic there would have been several different numerical methods available, e.g., Runge-Kutta or Euler. The Euler forward method generalises well to the stochastic case. Some other numerically more efficient and frequently used methods, such as the Runge-Kutta methods do not generalise quite as well. Therefore focus will be on the Euler forward method.

Recall that for a deterministic (vector) valued function and a given partition $0 = t_0 < t_1 < ... < t_n = T$ the Euler forward can be recursively defined through

$$\begin{cases} f(t_{i+1}) \approx f(t_i) + \frac{d}{dt} f(t)|_{t=t_i}(t_{i+1} - t_i) \\ \hfill f(0) = f_0 \end{cases}$$

with global convergence of order $\mathrm{O}(\max_i(t_{i+1} - t_i))$.

The very same principle can be used in the deterministic case. If we consider the process in Equation 1 a path can be simulated by

$$\begin{cases} X(t_{i+1}) \approx a(t_i, X(t_i))(t_{i+1} - t_i) + b(t_i, X(t_i))(M(t_{i+1} - M(t_i)) \\ \hfill X(0) = X_0 \end{cases}$$

with global convergence of order $\mathrm{O}(\sqrt{\max_i(t_{i+1} - t_i)})$. It is worth noting that in the special case where $b(t, X(t)) = b$, i.e., does not depend on neither t, nor $X(t)$ this coincides with the higher order Milstein method which is known to have global convergence of $\mathrm{O}(\max_i(t_{i+1} - t_i))$. For further details, see [].

2.4 Noise generation

The representation in Equation 2 can be used and the inner products can be simulated as independent Lévy processes,

$$M(t) \approx \sum_{i=0}^{N} \sqrt{\lambda_i} e_i M_i(t) \tag{3}$$

where the λ_i:s are the eigenvalues of Q and M_i are i.i.d. real-valued Lévy processes.

A method for choosing N such that the error because of the truncation is smaller then the error due to the discretisation was suggested in [5].

3 Energy market

The energy market can be both volatile and illiquid. It can therefore be desireable for both sellers and buyers to trade with futures in order to increase cash flow and reduce risks.

For most commodity markets the goods are delivered instantly at some point in time and are thereafter stored until it is needed. For energy, however, this is typically not the case. Energy commodities, such as electricity are always delivered during a period of time and cannot be stored. This makes the market highly volatile and the price can differ depending on both time of day and season. On top of this there are several stochastic factors affecting the price, such as weather and speculation.

In the Nordic and Baltic countries the spot price is determined at a common market called Nord Pool. At Nord Pool only spot and day ahead prices are recorded. Instead, the trading with futures is available in stock markets, e.g., OMX Nasdaq Stockholm.

3.1 Electricity futures

A future is a contract obliging the owner to buy, or sell, at a predetermined time of delivery at predetermined price. Energy cannot be delivered instantaneously, but rather during a period of time. For example, at OMX Nasdaq Stockholm it is possible to trade electricity forwards with delivery periods ranging from one day to a year.

Thus the price of the future at any given time depends on both the *Start of delivery* and the *End of delivery*.

3.2 Arbitrage

In some markets situations where it is possible to at the same time sell high and buy cheap. This makes it possible to gain money instantaneously and risk free. Often when modelling one assumes this behaviour does not occur. This is often referred to as a no-arbitrage condition.

4 Modelling

The model developed in [2] was used, so rather than going into too much details the reader is referred there. However, there are a few modelling choices and results which I would like to point out.

One might want to view the price as function of time, t¸ start of delivery, T_1 and end of delivery T_2, i.e., the price being a function $F = F(t, T_1, T_2)$. Any reasonable domain for for F would then require that

$$t \leq T_1 \leq T_2$$

and it would be hard defining a reasonable domain for a L^2-process. However, using a so called Musiela parametrisation, we may define a different function, G by

$$G(t, x, y) = F(t, T_1, T_2) \quad \text{with} \quad x = T_1 - t \quad \text{and} \quad y = T_2 - T_1. \tag{4}$$

Under a no-arbitrage condition it was shown that G must fulfil

$$yG(t, x, y) = \int_0^y G(t, x + z, 0)dz \tag{5}$$

and it is therefore sufficient to model $g(t, x) = G(t, x, 0)$, with $t \in [0, T]$ and $x \in [0, \tau]$¸ for some choices of T and τ. In the sequel x will sometimes be referred to as the space variable.

As usual in the scope of financial mathematics it is more natural to consider the log-prices than the actual prices. Therefore, instead of modelling $g(t, x)$ it is natural to model the logarithm instead. To do this a $L^2([0, \tau])$-valued process $X = X(t)$ is defined such that

$$\log g(t, x) = \delta_x X(t)$$

where δ_x, is an evaluation operator.

Again following [2] we arrive in the following L^2-valued stochastic differential equation

$$dX = (\frac{d}{dx}X + a(t))dt + b(t)dL(t),$$

or, equivalently, the process described by the integral

$$X(t) = \int_0^t (\frac{d}{dx}X(t) + a(t))\, dt + \int_0^t b(t)\, dL(t),$$

where the integrals should be understood as Bochner integrals.

As the driving noise, dL two different Lévy process were chosen, first Brownian motion and thereafter normal inverse Gaussian (NIG) processes.

Following [2] the initial and inflow conditions for Brownian motion are presented in Equation 6 and the conditions for the NIG-drift are presented in Equation 7. The modelling choices imply that the process is stationary.

$$\begin{cases} a(t)(x) = \dfrac{\sigma^2}{2}\mathrm{e}^{-2\alpha x} \\ b(t)(x) = \sigma \mathrm{e}^{-\alpha x} \\ X(0,x) = \mathrm{e}^{-\alpha x} + \dfrac{\sigma^2}{4\alpha}(1-\mathrm{e}^{-2\alpha x}) \\ X(t,\tau) = \mathrm{e}^{-\alpha\tau} + \dfrac{\sigma^2}{4\alpha}(1-\mathrm{e}^{-2\alpha\tau}) \end{cases} \tag{6}$$

$$\begin{cases} a(t)(x) = \mathrm{e}(-\alpha x)/2 \\ b(t)(x) = K_0(\tilde{\alpha}x)/(\pi)\mathrm{e}^{-\alpha x} \\ X(0,x) = \mathrm{e}^{-\alpha x} + \dfrac{K_0(\tilde{\alpha})}{\pi\alpha}(1-\mathrm{e}^{-2\alpha x}) \\ X(t,\tau) = \mathrm{e}^{-\alpha\tau} + \dfrac{K_0(\tilde{\alpha})}{\pi\alpha}(1-\mathrm{e}^{-2\alpha\tau}) \end{cases} \tag{7}$$

4.1 Correlation of the driving noise

Any effect on the electricity price (due to drought, heavy rains, etc.) will most likely not only affect the future price at one single maturity. It seems more reasonable that is affects the price at maturities close to it maximum and then decaying with distance to its maximum. This was taken care of by inferring a correlation in space. For simplicity it will be assumed to be time independent.

Let us consider an arbitrary, but fix time, t. The driving noise, $dL(t)$ may be correlated with respect to the space variable, x. Considering two different points in space, x_1 and x_2 the driving field was correlated with

$$q(x_1, x_2) = \mathrm{e}^{-\kappa|x_1-x_2|^2}.$$

To this end an uncorrelated field $M(t)$ can be correlated with q and dL can be defined by

$$dL(t)(x) = \int_0^\tau dM(t)(y)q(y, |y-x|)\, dy.$$

4.2 Description of parameters

The parameters σ in Equation 6 and $\tilde{\alpha}$ in Equation 7 influence to the volatility and jump intensity respectively.

The parameter κ decides the range of the correlation; a large value of κ means a fast decay in correlation. The interpretation of α is related to the cost of capital – a high value of α simulates a high cost of capital. Furthermore, since the quantity of κ is time^{-2} and the quantity of α is time^{-1}, a simultaneous decrease or increase of κ and α can be used for scaling.

5 Simulation

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5.1.1 Choice of parameters

For comparability with the results in [2] the same parameter values were chosen, i.e., $\sigma = \frac{1}{2}$, $\tilde{\alpha} = 10$, $\kappa = 2$ and 10, $\alpha = 0.2$ and 4 and $T = \tau = 1$.

5.1.2 Effect of parameters

Especially in the Gaussian plots we see a lot of humps. These appear as a consequence of the high dimensionality and the correlation structure. The smaller κ is, i.e., the larger the correlation is, the wider and smoother humps are obtained.

From a large cost of capital, i.e., large α, we obtain a steep decay of the price. Also the volatily at the large values of x becomes smaller. Since α can be used for scaling in time this can be interpreted as for long maturities the volatility smaller than it is close to the time of maturity.

We can clearly see that the simulations share a lot of properties with the ones performed in [2]. The humps that appear are a consequence of the high dimensionality and the spatial correlation. With a large value of κ, as in e.g., Figure 4 the humps becomes steeper.

The parameters in equations Equation 6 and Equation 7 were chosen in accordance with the chioces made by Barth and Benth in [2]

5.2 Numerics

The process was simulated in a way described in Section 2.3

For simplicity a equidistant grid in both time and space was chosen and for discretisation in space a finite difference method was used. This is known to have a convergence of $\mathrm{O}(\Delta x)$.

In [6] it was suggested to use a Fourier basis $(\cos \frac{n\pi}{\tau} x)_{n=0}^{\infty}$ as eigenvectors when generating M in Equation 2.

As we can see in Equation 6 and Equation 7 b is independent of both t and X (but of course not x) and hence the total global convergence rate is $\mathrm{O}(\Delta t + \Delta x)$

The truncation of the Lévy process Equation 2 was chosen in accordance with the recommendations in [5].

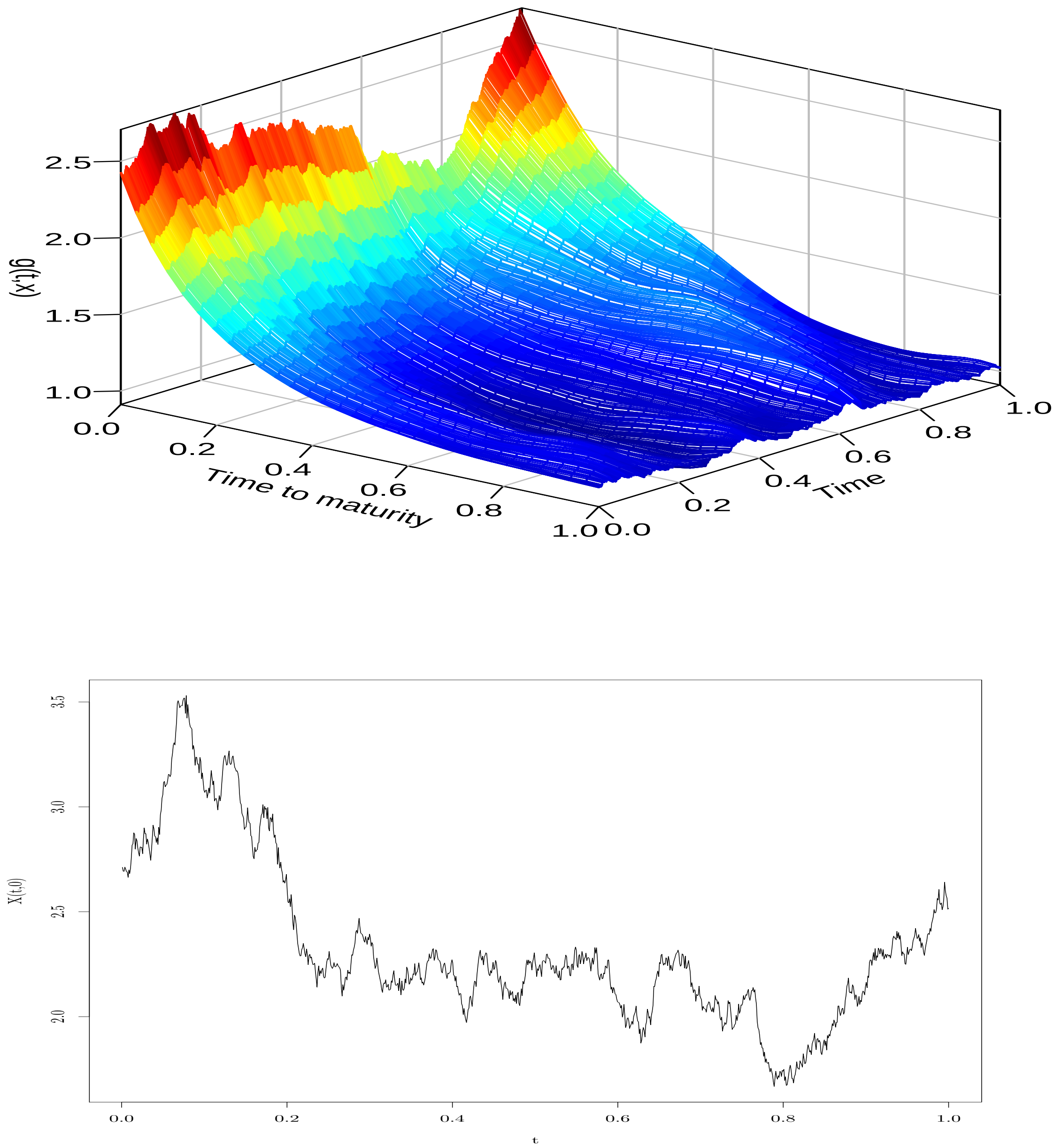


Figure 1: Future simulation and spot curve for $\alpha = 0.2$ and $\kappa = 2$ with Brownian drift.

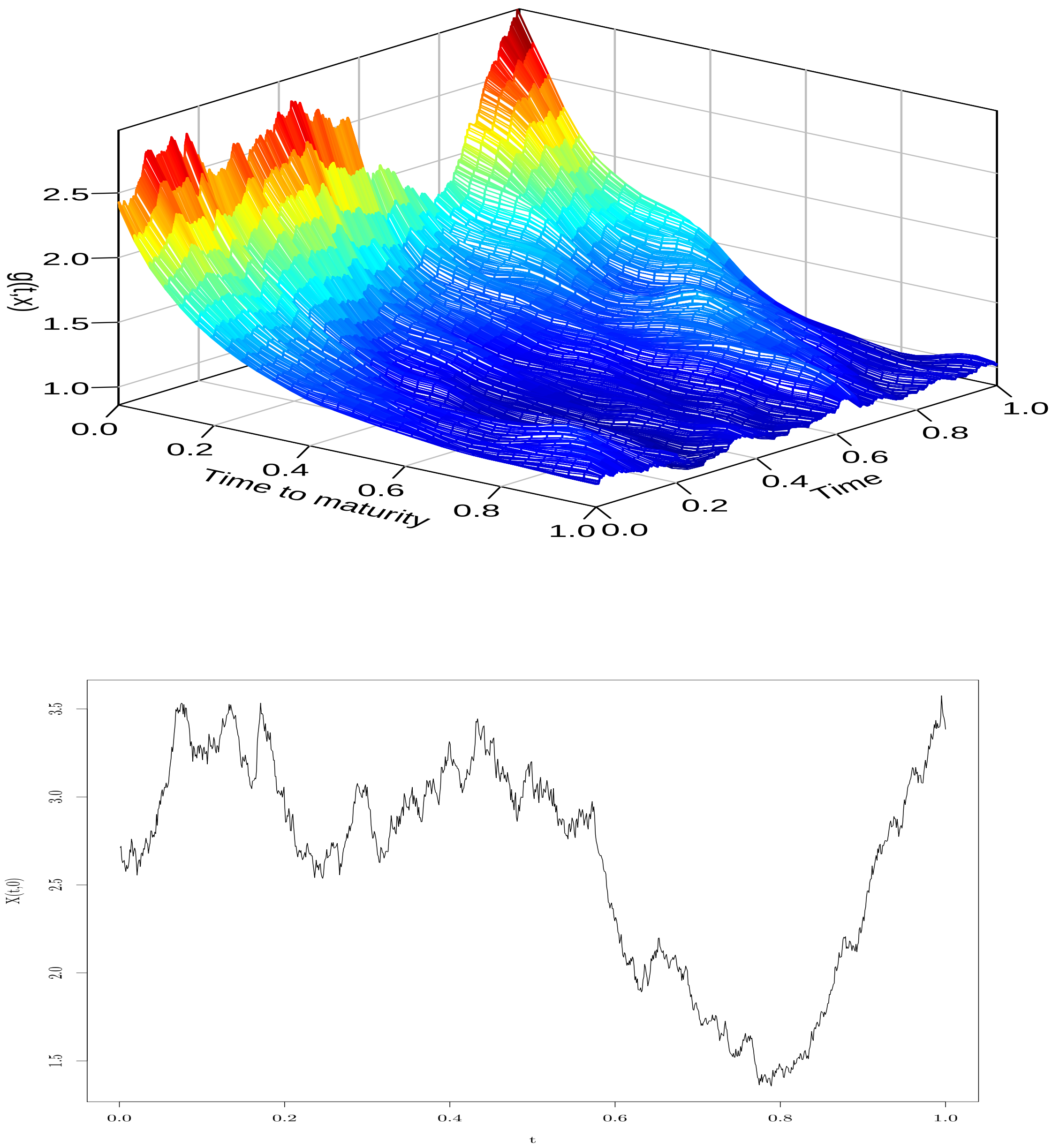


Figure 2: Future simulation and spot curve for $\alpha = 0.2$ and $\kappa = 10$ with Brownian drift.

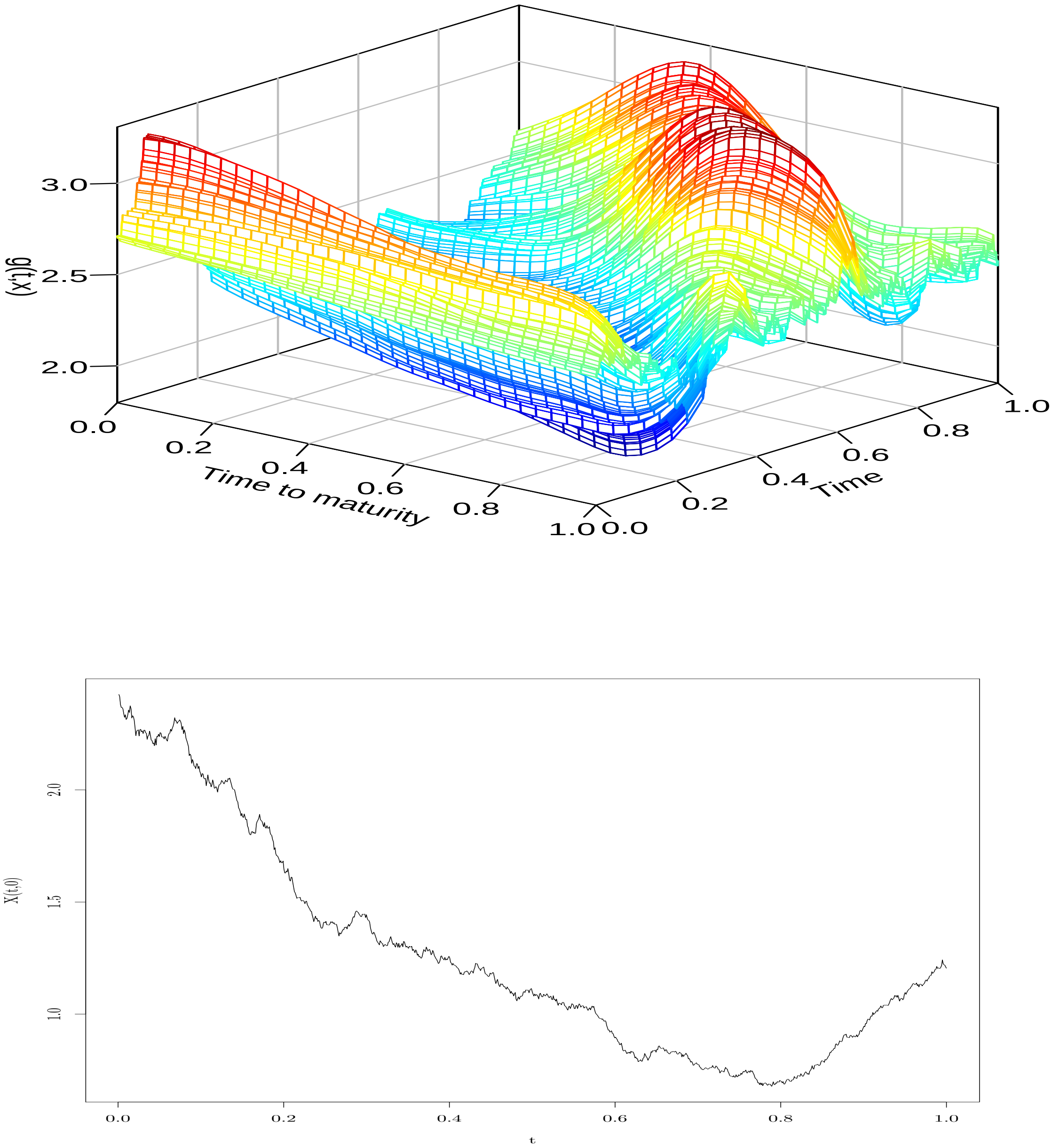


Figure 3: Future simulation and spot curve for $\alpha = 4$ and $\kappa = 2$ with Brownian drift.

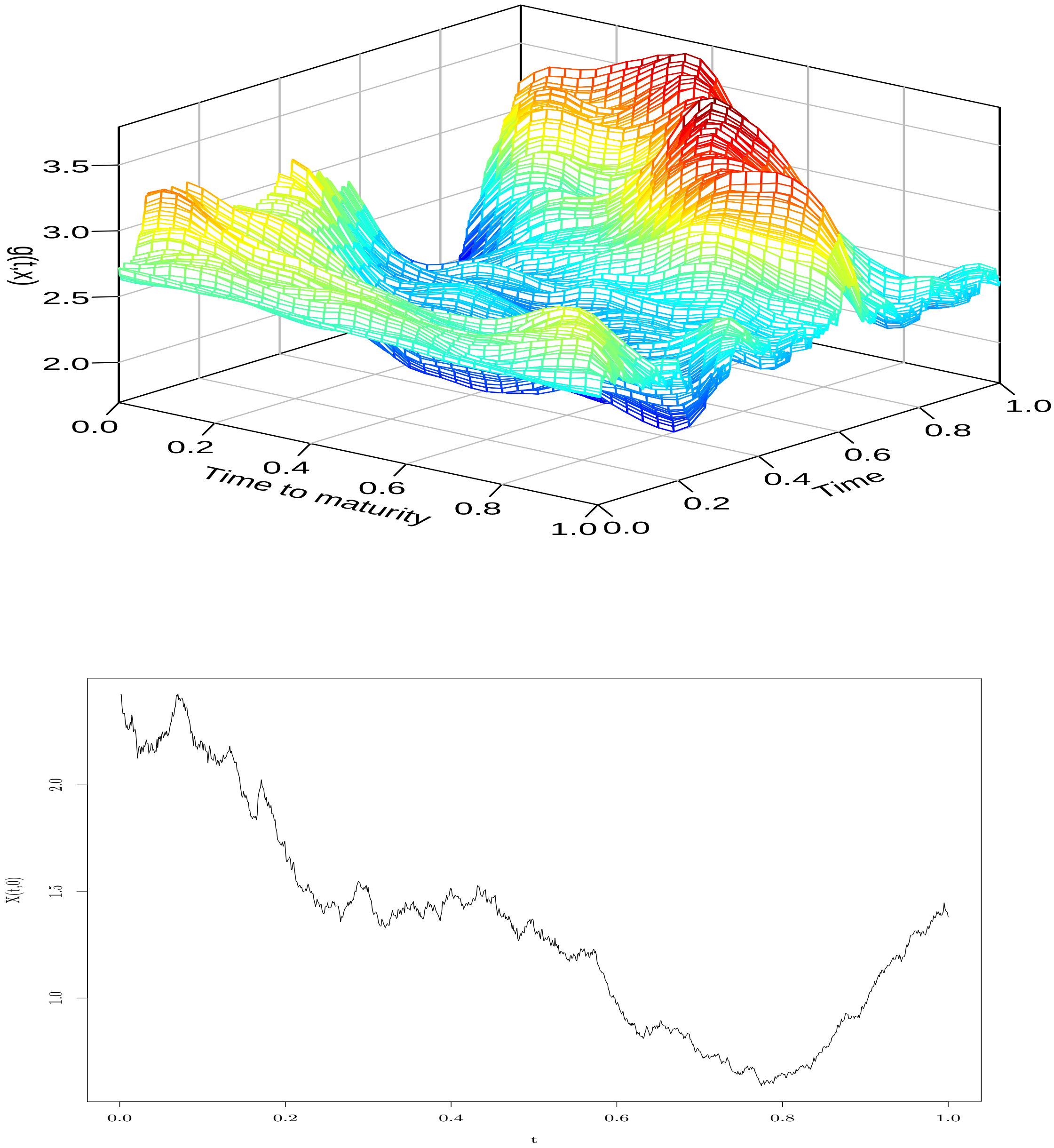
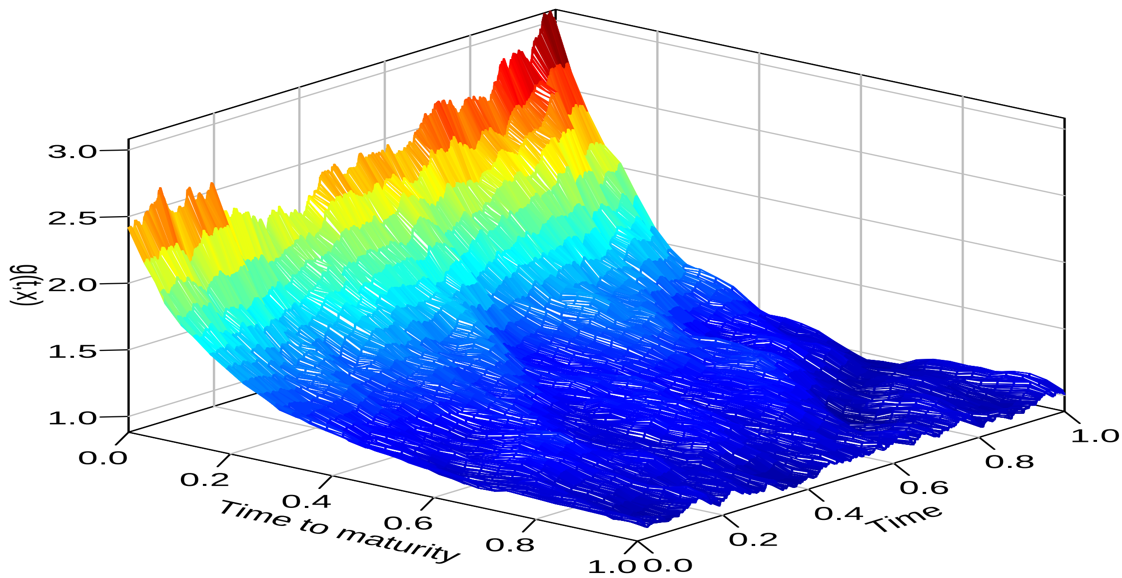


Figure 4: Future simulation and spot curve for $\alpha = 4$ and $\kappa = 10$ with Brownian drift.



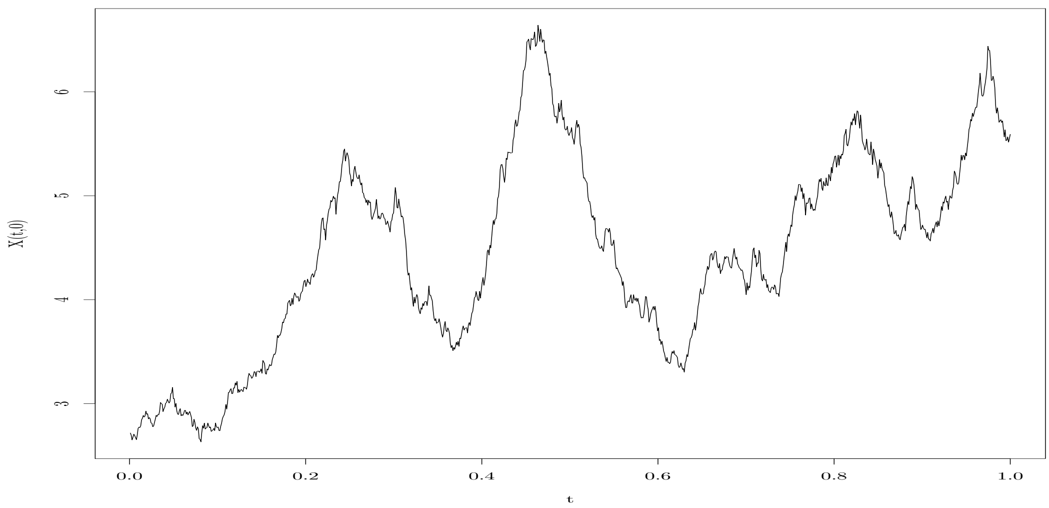
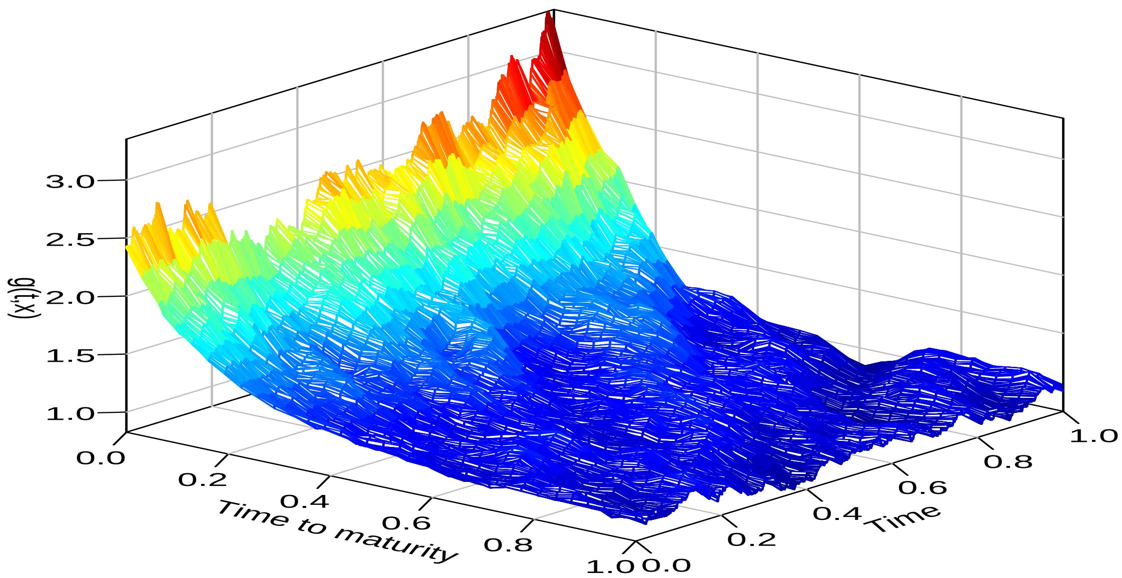


Figure 5: Future simulation and spot curve for $\alpha = 0.2$ and $\kappa = 2$ with NIG drift.



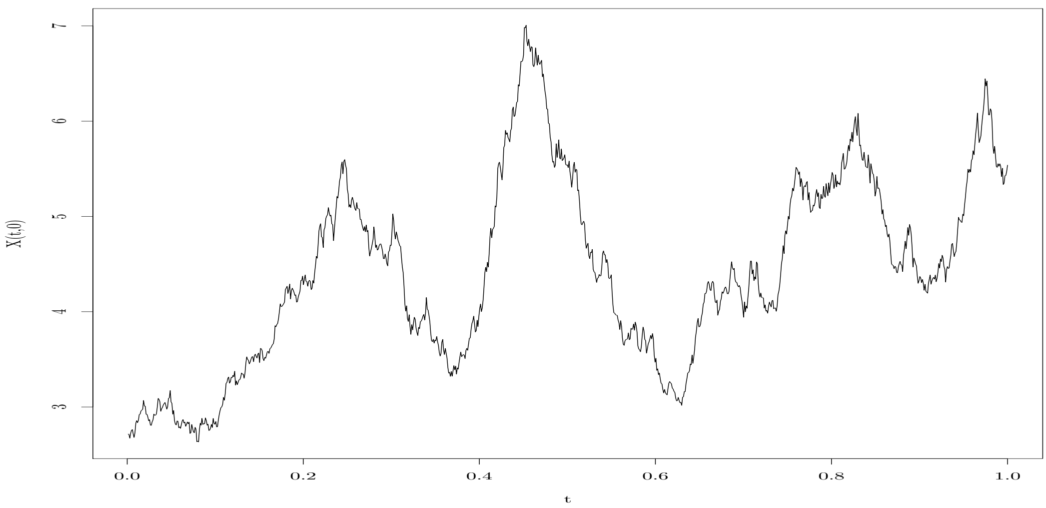


Figure 6: Future simulation and spot curve for $\alpha = 0.2$ and $\kappa = 10$ with NIG drift.

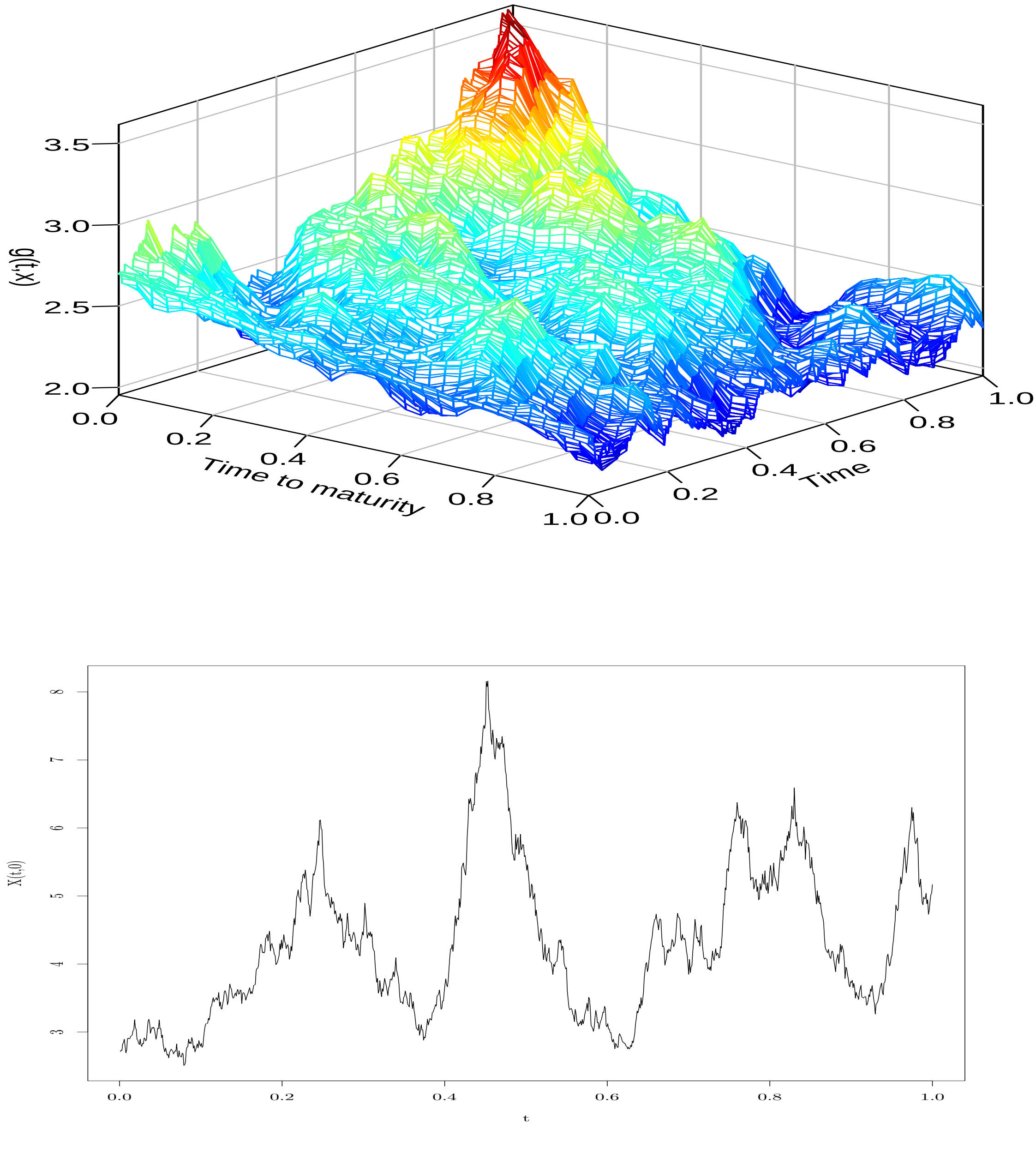


Figure 7: Future simulation and spot curve for $\alpha = 4$ and $\kappa = 2$ with NIG drift.

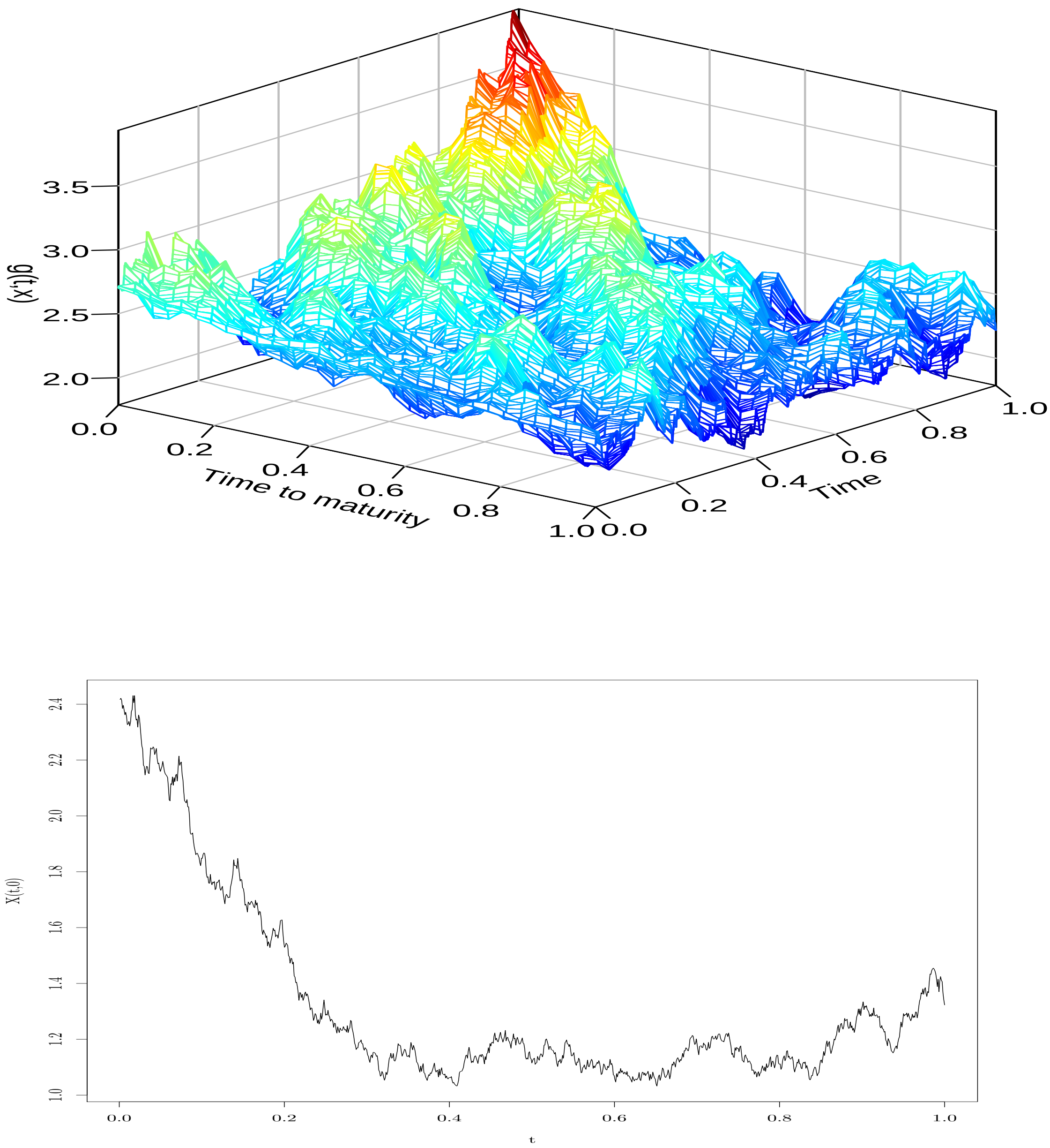


Figure 8: Future simulation and spot curve for $\alpha = 4$ and $\kappa = 10$ with NIG drift.

6 Result

The effect of a large value of α is evident from Figures 1 to 4 – the decay in time is substantially faster with higher value and the humps become smaller for large values of x. This can be interpreted as both the price and the volatility being small for long maturities. Also the effect of κ is clear. A high value gives steeper humps and somewhat higher maxima. A smaller value on the other hand makes the humps smoother and wider.

These effects are present also in the NIG driven processes. The main difference between them and the Brownian processes is the smoothness of the curves. As NIG-processes have jumps, this behaviour is also present in the future curves.

The simulations performed in [2] show similar results.

7 Market data

At OMX Nasdaq Stockholm trading with electricity futures is available. The contracts are only available a certain time before the maturity. Futures with delivery period one month become available six months before start of delivery and they have delivery period being from first to last of the actual month..

At Nord Pool, spot prices are recorded and publicly available. Such a set has been used for parameter estimation.

7.1 Description of data

The data set consisted of closing prices from OMX Nasdaq Stockholm of electricity forwards.

Using the Musiela parametrisation described in Equation 4 this corresponds to $G(t, x, y)$ with:

$$\begin{cases} 0 \leq x \leq 6 \text{ months} \\ y = 1 \text{ month} \\ 0 \leq t \leq 70 \text{ months} \end{cases}$$

The future prices and corresponding monthly average spotprice are shown in Figure 9 and Figure 10

7.2 Fitting the model to market data

Recall that the model does not simulate $G(t, x, y)$ directly, but rather $g(t, x)$ (or $G(t, x, 0)$). An integration, as described in Equation 5, had to be performed before comparing simulation and recorded prices.

7.2.1 Estimating α

The simulated process is a martingale and hence $E[\log g(t, x)] = E[X(t, x)] = X(0, x)$, which was specified as the initial condition, see Equation 6 and Equation 7. Furthermore, using the mean value theorem on Equation 5 we see that there exists a $z, 0 \leq z \leq 1$ such that $g(t, x + z) = G(t, x, 1)$. If we approximate that this z is the same for all values of t and x we can estimate α as

$$\min_{\alpha} \int_0^T \int_0^\tau (\log(\frac{\tilde{G}(t, x, 1)}{\tilde{G}(t, 0, 1)}) - X(0, x))^2 \, dx \, dt$$

where $\tilde{G}$, is the market data.

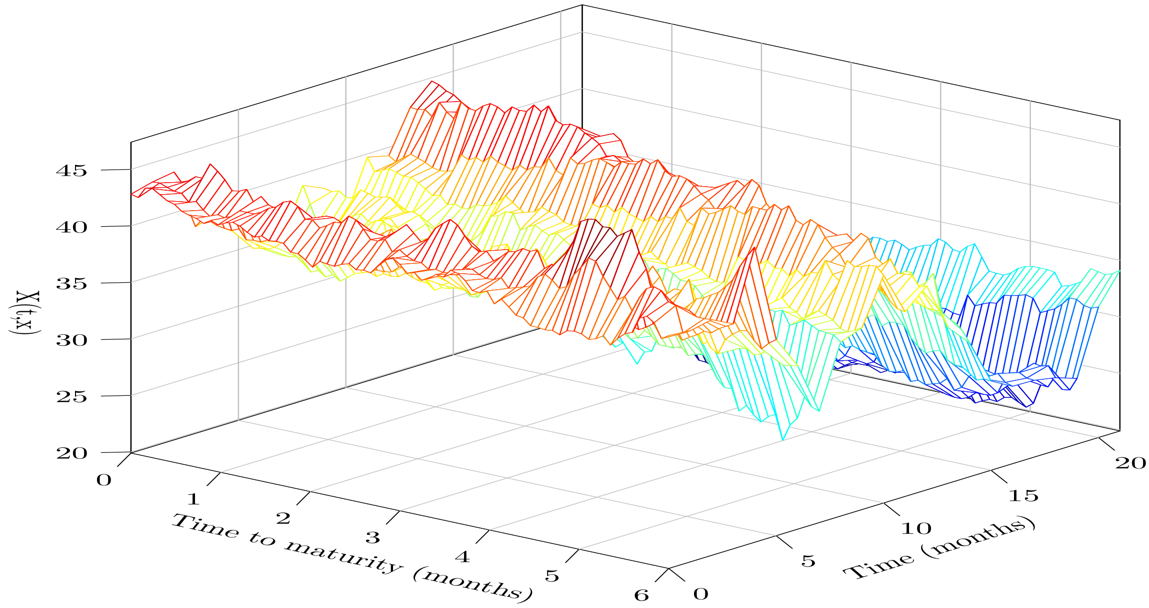


Figure 9: Real prices of futures for the period 2012-01-01 to 2013-12-31.

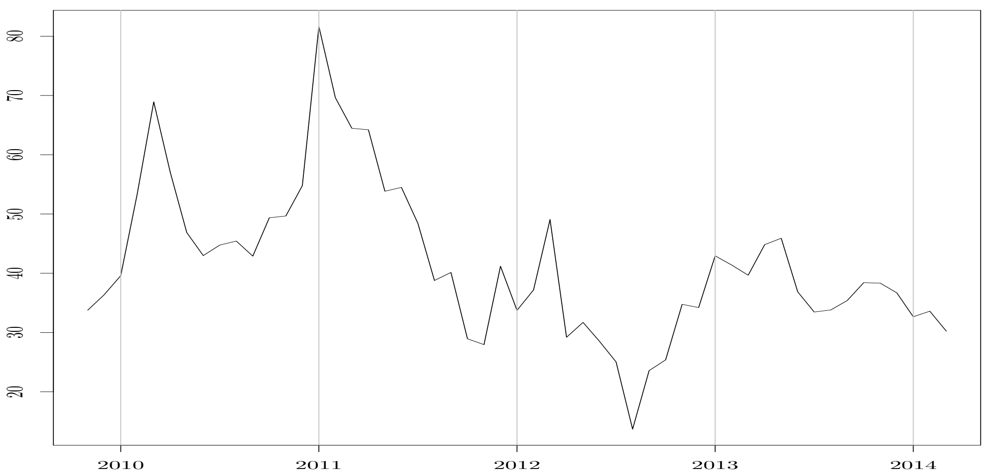


Figure 10: Monthly averages of spot price for the period 2009-10-01 to 2014-03-01.

7.2.2 Estimating κ

Since the market data is of the form $G(t, x, 1)$ it is nontrivial to estimate the covariance of $G(t, x, 0)$, which was simulated. The idea of measuring the wideness of the ridges has been used as a small value will give long ridges.

A number of paths were simulated for different values of κ. Then a value of κ that gave reasonably sized ridges was chosen.

7.2.3 Estimating σ and $\tilde{\alpha}$

For any fix x_0 the proces $X(t, x_0)$ will be a real valued Brownian motion or NIG-process. For these processes the variance will be proportional to $\sigma^2 t$ and $\frac{t}{\tilde{\alpha}}$ respectively. The proportional constant will depend on the values of α and κ and can be computed by averaging over a large number of simulations. The point $x_0 = 0$ was chosen and the empirical value of the standard deviation was computed from the Nord Pool spot data.

A more thorough investigation of the behaviour of Nordic electricity prices was performed in [1] and [7]. The estimates of variance and α are in line with what was estimated in [7].

7.3 Comments on the fitted simulation

The fitted parameters for the process with Brownian drift were

$$\alpha = 2.8 \text{ year}^{-1}$$
$$\kappa = 18 \text{ year}^{-2}$$
$$\sigma = 0.32 \text{ year}^{-\frac{1}{2}}$$

A plot of the simulated prices can be seen in Figure 11 and Figure 12.

As expected the integration makes the the surfaces smoother in x than was the case when only simulating $g(t, x)$. Also the humps observed get squeezed out to ridges as they get integrated. These ridges appear also in Figure 9.

The simulated surfaces are a lot smoother than the market prices. Both the covariance operator and the integration make the simulations rather smooth, much smoother than the recorded prices.

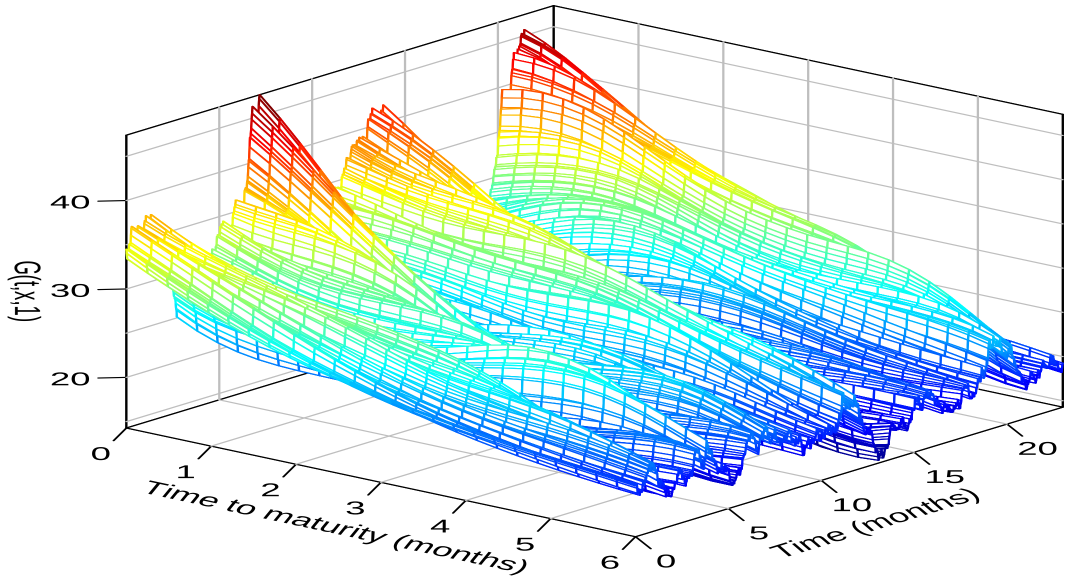


Figure 11: Brownian curve fitted to the real futures.

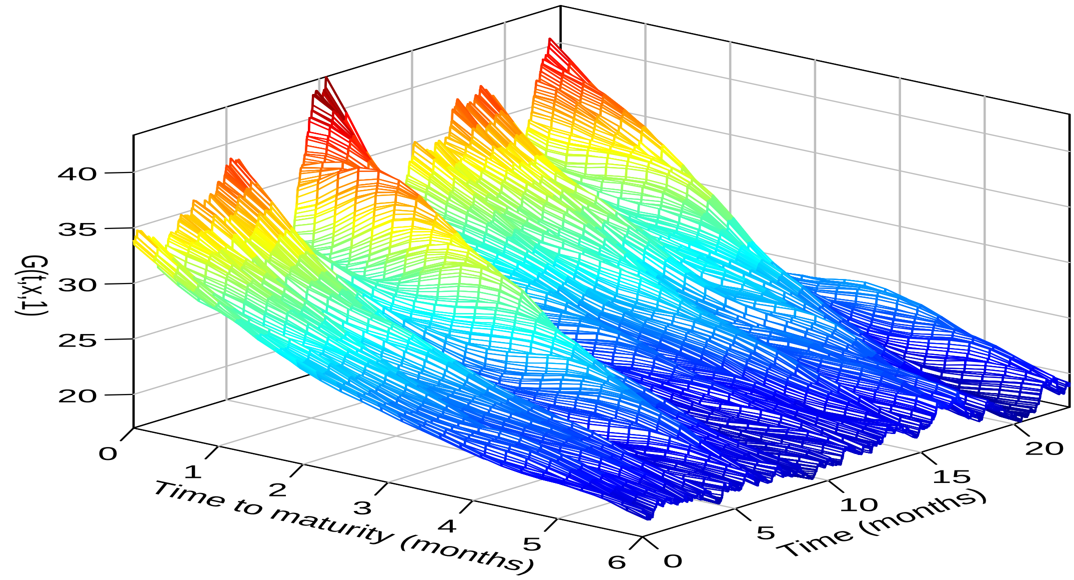


Figure 12: NIG curve fitted to the real futures.

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