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Option Pricing on Jump-Diffusion Models

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Abstract

Instead of the well-known Black-Sholes model, We investigate a Jump-Diffusion type of Lévy process for option pricing in this thesis. We try to use Laplace transform and Monte-Carlo simulation to price exotic option. Our conclusion is that Jump-Diffusion model fit the real data very well.

keywords: Jump-Diffusion process, Laplace transform, Monte-Carlo simulation, Exotic option.

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Contents

1 Introduction

The Black-Scholes(BS) model has been widely and successfully used to model the return of asset and to price financial options. Despite of its success the basic assumptions of this model, that is, Brownian motion and normal distribution are not always supported by empirical studies.

Those studies showed the two empirical phenomena[1]: (1) the asymmetric leptokurtic features, (2) the volatility smile. The first means that the return distribution is skewed to the left and has a higher peak and two heavier tails than those of normal distribution, and the second means that if the BS model is correct, then the implied volatility should be flat. But the graph of the observed implied volatility curve often looks like a smile. One of the causes for such phenomena is jumps in assets price processes.

Many models were proposed to explain the two empirical phenomena. For example popular ones are normal jump diffusion model(Merton(1976)), stochastic volatility models(Heston(1993)), ARCHGARCH models(Duan(1993)), etc.

In this master thesis, we try to use Jump-Diffusion model to price Exotic options like Lookback option, to see how well the model works. Among others, we focus on a jump diffusion model consisting of Brownian motion and Compensated Poisson process. This model is simple and has rich theoretical implication as described below: (1) it can explain the two empirical phenomena, that is asymmetric leptokurtic feature and the volatility smile, (2) it leads to analytical solutions to many option-pricing problems.

The thesis is structured as follows:

In Section 2 we introduce necessary knowledge about Lookback option and some pricing formulas for different type of Lookback options.

In Section 3 we first introduce Leévy process, Brownian motion, and then present the Jump-Diffusion model that we are going to investigate, and adapt it to different type of Lookback options.

In section 4 we first introduce Monte-Carlo simulation and Laplace transform ,then evaluate the performance of our Jump-Diffusion model.

Finally, in Section 5 we make conclusions from our work.

2 Exotic options

The Basic European call and put options are sometimes called vanilla or plain vanilla opitons.Their payoffs depend only on the final value of the underlying asset. Options whose payoffs depend on the path of the underlying asset are called path-dependent or exotic options, such as barrier options, Asian options and Lookback options[2].In this chapter, we try to introduce some basic knowledge about Lookback option.

2.1 Lookback option

Lookback option is a contract whose payoffs depend on the maximum or the minimum of the underlying assets price during the lifetime of the options.There exist two kinds of Lookback option : with floating strike and with fix strike.

2.2 Previous researches

Lookback option has been studied under different models.For example, for continuous version lookback options, Under the Black-Scholes model, Goldman,Sosin and Gatto (1979), Xu and Kwok (2005), Buchen and Konstandatos (2005). Under the jump diffusion model, Kou and Wang (2003, 2004). In general exponential Levy models Nguyen-Ngoc (2003).All have given their answers.

But in practice, many contracts with lookback features are settled by reference to a discrete sampling of the price process at regular time intervals. These options are usually referred to as discrete lookback options. In these circumstances the continuous-sampling formulae are inaccurate. The values of lookback options are quite sensitive to whether the extrema are monitored discretely or continuously.

For discrete-version lookback options. Broadie, Glasserman and Kou provided in 1999 developed a technique for approximately pricing discrete lookback options under Black-Scholes model. They use Siegmund¡¯s corrected diffusion approximation, refer to Siegmund (1985).

2.3 Continuous lookback option

For European floating strike lookback calls and puts respectively:

$$LPC = S_T - \min_{0 \leq t \leq T} S_t.$$

$$LPC = S_T - \min_{0 \le t \le T} S_t.$$

and under the Black-Scholes model, The value process of a Lookback put option (LBP) is given by (continuous case)

$$LBP_0 = S^+ e^{-rt} N(-d_1) - s_0 N(-d_1 + \sigma\sqrt{T}) + A(s_0, s^+, t).$$

where

$$A(s_0, s^+, t) = e^{-rt} \frac{s_0 \sigma^2}{2r} [e^{-rt} N(d_1) - (\frac{s_0}{s_+})^{-\frac{2r}{\sigma}} N(d_1 - \frac{2r}{\sigma}\sqrt{T}).$$

and

$$d_1 = \frac{1}{\sigma\sqrt{T-t}} [\log \frac{x}{K} + (r + \frac{\sigma^2}{2})(T-t)].$$

2.4 Discrete lookback option

For the discrete case, we have the following theorem[4].

Theorem 1. *The price of a discrete lookback at the k^{th} fixing date and the price of a continuous lookback at time $t = k\triangle t$ satisfy*

$$V_m(S_\pm) = \pm[e^{\mp\beta\sigma\sqrt{\Delta t}} V(S_\pm e^{\pm\beta\sigma\sqrt{\Delta t}}) + (e^{\mp\beta\sigma\sqrt{\Delta t}} - 1)S_t] + o(\frac{1}{\sqrt{m}}).$$

where in $\pm$ and $\mp$, the top for puts and the bottom for calls;
the constant

$$\beta = -(\zeta(1/2)/\sqrt{2\pi}) \approx= 0.5862.$$

ζ is the Rimann zeta function.
Otherwise,

$$S_+ = \max_{0 \le U \le t} S_u.$$

$$S_- = \min_{0 \le U \le t} S_u.$$

Set the parameters: $S = 100$, $r = 0.1$, $\sigma = 0.3$,$T = 0.5$, with the number of monitoring points m and the predetermined maximum S_+ varying as indicated. The option in the left panel has a continuously monitored option price of 16.84677, the right panel is 21.06454. We can see the performance of the approximation of this Theorem for pricing a discrete lookback put option:

	$S_+ = 110$			$S_+ = 120$		
m	**True**	**Approx.**	**Error**	**True**	**Approx.**	**Error**
10	**14.12285**	**13.85570**	**-0.26715**	**19.32291**	**19.11622**	**-0.20669**
20	**14.80601**	**14.66876**	**-0.13725**	**19.74330**	**19.63509**	**-0.10821**
40	**15.34459**	**15.27470**	**-0.06990**	**20.08297**	**20.02718**	**-0.05579**
80	**15.754523**	**15.71899**	**-0.03553**	**20.34598**	**20.31747**	**-0.02851**
160	**16.05908**	**16.04117**	**-0.01791**	**20.54389**	**20.52942**	**-0.01447**

Table 1: Performance of theorem 1

3 The model

In this chapter, we give definitions of Lévy processes, Brownian motion, Jump-Diffusion processes and construct the model we will use in our work.

3.1 Lévy process

According to Cont and Tankov(2004)[3], we have

Definition 1 *A cadlag stochastic process* $(X_t)_{t\geq 0}$ *on* $(\Omega, \mathfrak{F}, \mathbb{P})$ *with values in* $\mathbb{R}^d$ *such that* $X_0 = 0$ *is called a Lévy process if it possesses the following properties:*

1.Independent increments: for every increasing sequence of times $t_0 ... t_n$*,the random variables* $X_{t_0}, X_{t1} - X_{t0}, ..., X_{tn} - X_{tn-1}$ *are independent.*

2.Stationary increments: the law of $X_{t+h} - X_t$ *does not depend on* t*.*

3.Stochsatic continuity:$\forall \varepsilon > 0$,$\lim_{h\to 0} \mathbb{P}(|X_{t+h} - X_t|) \geq \varepsilon) = 0$*.*

An important feature of Lévy process is Infinite divisibility.

Definition 2 *A prbability distribution* F *on* $\mathbb{R}^d$ *is said to be infinitely divisible if for any integer* $n \geq 2$ *,there exists* n *i.i.d. random variables* $Y_1, ..., Y_n$ *such that* $Y_1 + ... + Y_n$ *has distribution* F*.*

Some common examples of infinitely divisible laws are: the Gaussian distribution, the gamma distribution, α-stable distribution and the Poisson distribution.A random variable having any of these distributions can be decomposed into a sum of n *i.i.d.* parts having the same distribution but with modified parameters.

3.2 Brownian motion

From Steven(2004)[2], we have

Definition 3: *Let* $(\Omega, \mathfrak{F}, \mathbb{P})$ *be a probability space. For each* $\omega \in \Omega$*, suppose there is a contiunous function* $W(t) of t \geq 0$ *that satisfies* $W(0) = 0$ *and that depends on* ω*.Then* $W(t), t \geq 0$*, is a Brownian motion if for all* $0 = t_0 < t_1 < ... < t_m$ *the increments*

$$W(t_1) = W(t_1) - W(t_0), W(t_2) - W(t_1), ..., W(t_m) - W(t_{m-1}).$$

are independent and each of these increments is normally distributed with

$$\mathbb{E}[W(t_{i+1}) - W(t_i)] = 0.$$
$$Var[W(t_{i+1}) - W(t_i)] = t_{i+1} - t_i.$$

Brownian motion with drift is a Lévy process $X(t)_{t\geq 0_{t\geq 0}}$ that has Gaussian increments. Specifically, $X(t)$ is $N(\mu t, \sigma^2 t)$-distributed where $\sigma > 0$ and $\mu \in \mathbb{R}$ are parameters.

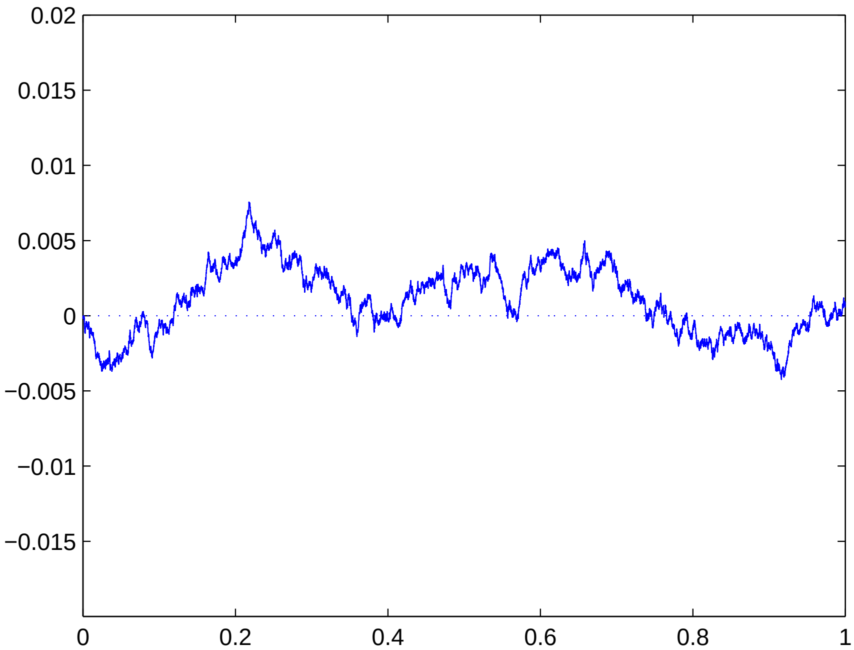

Figure 1: Sample of Brownian motion

3.3 Jump-Diffusion process

A lévy process of Jump-Diffusion type has the following form:

$$X_t = \gamma t + \sigma W_t + \sum_{i=1}^{N_t} Y_i.$$

where $(N_t)_{t \geq 0}$ is the Poisson process counting the jumps of X and Y_i are jump sizes(i.i.d. variables).To define the parametric model completely, we must now specify the distribution of jump sizes $\nu_0(x)$.It is especially important to specify the tail behavior of $\nu_0(x)$ correctly depending on one's beliefs about behavior of the jump measure determines to a large extent the tail behavior of probability density of the process.

The advantages of Jump-Diffusion models are that they can capture Empirical Phenomena features like Asymmetric leptokurtic and Volatility smile.

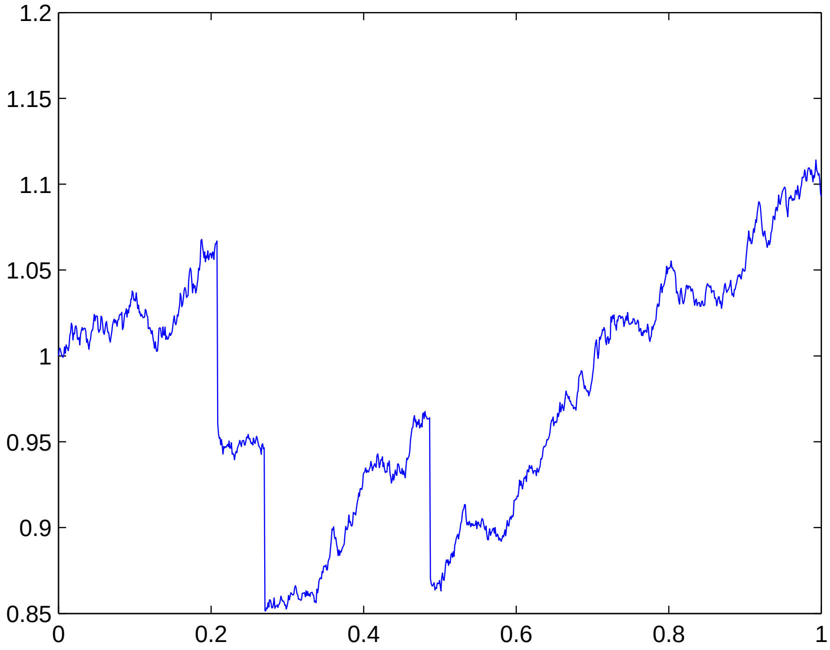

Figure 2: Sample of Jump-Diffusion process

3.4 Continuous lookback option

Before apply the Jump-Diffusion model, we need to change the measure from original probability to a risk-neutral probability measure.According to Steven (2004)[2]

To construct a risk-neutral measure, we can do as follows

Letθ be a constant and λ be positive number. Define

$$Z_1(T) = \exp[-\theta\overline{W}(t) - \frac{1}{2}\theta^2 t].$$

$$Z_2(t) = \exp[(\overline{\lambda} - \lambda)t](\frac{\lambda}{\overline{\lambda}})^{N(t)}.$$

$$Z(t) = Z_1(t)Z_2(t).$$

Then the new measure is defined as follows,

$$P^*(A) = \int_A Z(t)dP, \forall A \in F_t.$$

Under the probability measure P^*, the process

$$W(t) = \overline{W}(t) + \theta t.$$

is a Brownian motion. $N(t)$ is a Poisson process with intensity λ. $W(t)$ and $N(t)$ are independent.

Under the original measure P, we have

$$\begin{aligned} dS(t) =& \alpha S(t)dt + \sigma S(t)d\overline{W}(t) + (\delta - 1)S(t-)d\overline{M}(t) \\ =& [\alpha - (\delta - 1)\overline{\lambda}]S(t)dt + \sigma S(t)d\overline{W}(t) + (\delta - 1)S(t-)dN(t). \end{aligned} \tag{1}$$

where

$$M(t) = N(t) - \overline{\lambda}t.$$

is the compensated Poisson process and is a martingale.

And P^* is risk-neutral if and only if

$$\begin{aligned} dS(t) =& rS(t)dt + \sigma S(t)dW(t) + (\delta - 1)S(t-)dM(t) \\ =& [r - (\delta - 1)\lambda]S(t)dt + \sigma S(t)d\overline{W}(t) + (\delta - 1)S(t-)dN(t). \end{aligned} \tag{2}$$

By contrast, we can get the relation

$$\alpha - (\delta - 1)\overline{\lambda} = r - (\delta - 1)\lambda + \sigma\theta.$$

Since there are one equation and 2 unknowns,θ and λ, there are multiple risk-neutral measures. Extra stocks would help determine a unique risk-neutral measure.

On the probability space $(\Omega, \mathfrak{F}, P^*)$,

$$\begin{aligned} S(t) = & s_0 \exp(\sigma W(t) + \mu t + (\log \delta) N(t)) \\ & = s_0 e^{\sigma W(t) + \mu t} \delta^{N(t)} \\ & = s_0 \exp X(t). \end{aligned} \tag{3}$$

where $\mu = r - \frac{1}{2}\sigma^2 - \lambda\delta$ and $\delta > 0, \delta \neq 1$.

Let$\tau(h, S) \equiv \inf(t > 0 : S_t > h)$ and $S_+(t) = \max_{u \in [0,t]} S(u)$ for some fixed time t.

According to Steven(2004)[2]

The price of a continuous floating strike lookback put (LBP) option at arbitrary time$0 < t < T$ is given by $t = k \bigwedge t$.

$$V(t) = e^{-r\tau} E^*[\max_{t \in [0,T]} S_t - S_T | \mathfrak{F}_t].$$

where $\tau = T - t$.

Rewrite $\max_{t \in [0,T]} S_t$ as $\max(S_+, \max_{u \in [t,T] S_u})$, and then

$$V(t) = e^{-r\tau} E^*[S_+ + (\max_{u \in [t,T]} S_u - S_+)^+ - S_T | \mathfrak{F}_t].$$

Then we can use the fact that

$$\{\max_{u \in [t,T]} S_u \geq b\} = \{\tau(b, s_0 e^X) \leq T\}.$$

to get the continuous value process as follows,

$$\begin{aligned} V(t) = & e^{-r\tau} S_+ - S_t + e^{-r\tau} E^*[(\max_{u \in [t,T]} S_u - S_+)^+ | \mathfrak{F}_t] \\ & = e^{-r\tau} S_+ - S_t + e^{-r\tau} \int_a^\infty s_0 e^x P^*(s_0 e^M \geq s_0 e^x) dx. \end{aligned} \tag{4}$$

where $a = \ln \frac{S_+}{s_0}$ and denote $M(t) = \max_{u \in [t,T]} X(u)$.

Define $V^p(S_+)$ as the floating strike lookback put, $F^c(S_+)$ as the fixed strike lookback call.The relation between them at an arbitrary time t satisfies

$$\begin{aligned} F^c(S_+) &= e^{-r\tau}E^*[(\max_{u\in[0,T]} S_u - S_+)^+|\mathfrak{F}_t] \\ &= e^{-r\tau}E^*[(\max(S_+, \max_{u\in[t,T]} S_u) - S_+)^+|\mathfrak{F}_t] \\ &= e^{-r\tau}E^*[(\max_{u\in[t,T]} S_u - S_+)^+|\mathfrak{F}_t] \\ &= V^p(S_+) - e^{-r\tau}S_+ + S_t. \end{aligned} \tag{5}$$

3.5 Discrete lookback option

For the discrete case, fix $T > 0$, let $\Delta t = \frac{T}{m}$,for $n = 1, 2, ..., m$, define

$$S_n = s_0 \exp\{\sum_{i=1}^{n}(\sigma\sqrt{\Delta t}Z_i + (\mu + \lambda \log\delta)\Delta t + M_i \log\delta)\} \equiv s_0 \cdot \exp X(n).$$

where $Z_i \sim N(0, 1)$ and $N_i \sim Poisson(\lambda\Delta t)$.
$M_i = N_i - \lambda t$ is the compensated Poisson process.

Let $\tau'(h, S) := \inf\{n \geq 1 : S_n \geq h\} = \inf\{n \geq 1 : X(n) \geq \ln \frac{h}{s_0}\}$
and $S_+ = \max_{0\leq n\leq k}$ at the k^{th} fixing date(known).

The price of a discrete floating strike LBP option at the k^{th} fixing date is given by

$$V_m(t) = e^{-r\tau}E^*\{\max_{n\in[0,m]} S_n - S_m|\mathfrak{F}\}.$$

where $\tau = T - t$.
Rewrite $\max_{n\in[0,m]} S_n$ as $\max\{S_+, \max_{n\in[k,m]} S_n$
then

$$V_m(t) = e^{-r\tau}E^*\{S_+ + (\max_{n\in[k,m]} S_n - S_+)^+ - S_m|\mathfrak{F}\}.$$

Similarly, we can use the fact that

$$\{\max_{n\in[k,m]} S_n \geq b\} = \{\tau'(b, s_0e^X) \leq T\}.$$

to get the discrete value process as follows,

$$
\begin{aligned}
V_m &= e^{-r\tau}S_+ - S_t + e^{-r\tau}\int_a^{\infty} s_0 e^x P^*(s_0 e^{M_m} \geq s_0 e^x)dx \\
&= e^{-r\tau}S_+ - S_t + e^{-r\tau}\int_a^{\infty} s_0 e^x P^*(\tau'(x,X)\Delta t \leq T)dx.
\end{aligned} \tag{6}
$$

where $a = \ln \frac{S_+}{s_0}$.

So, for the Discrete case, we have

$$V_m = e^{-r\tau}S_+ - S_t + e^{-r\tau}\int_a^{\infty} s_0 e^x P^*(\tau'(x,X)\Delta t \leq T)dx.$$

and for the continuous case, we have

$$V(t) = e^{-r\tau}S_+ - S_t + e^{-r\tau}\int_a^{\infty} s_0 e^x P^*(\tau(x,X) \leq T)dx.$$

For relation between the distributions of $\tau'(x,X)\Delta T$ and $\tau(x,X)$. we have this theorem:

Theorem 2. *For a fixed constant $b > 0$ we have*

$$\tau'(b,X)\Delta t - \tau(b + \beta C\sqrt{\Delta t}, X) \rightarrow^d 0.$$

where$\rightarrow^d$ means converging in distribution, moreover

$$P^*(\tau'(b,X)\Delta t \leq T) = P^*(\tau(b + \beta C\sqrt{\Delta t}, X) \leq T) + o(\frac{1}{\sqrt{m}}).$$

3.6 Continuity correction

We need to extend the results from fixed constant b to $r.v.$ $y \in [a = \ln \frac{S_+}{s_0}, \infty)$ That is, we have to discuss the uniform convergence of the distribution of stopping time when the constant b is a variable number.

Theorem 3. *Suppose that y is a flexible number,$a = \ln \frac{S_+}{s_0}$ and $\Delta t = \frac{T}{m}$,Then we have that as $n \rightarrow \infty$*

$$P^*(\tau'(b,X)\Delta t \leq T) = P^*(\tau(b + \beta C\sqrt{\Delta t}, X) \leq T) + o(\frac{1}{\sqrt{m}}).$$

holds for all $y \in [a, \infty)$.

Theorem 4. *For $0 < \vartheta < 1$ the discrete-version at k^{th} and continuous-version at time $t = k\Delta t$ floating strike LBP option satisfy*

$$V_m(S_+) = e^{-\beta C\sqrt{\Delta t}} V(S_+ e^{\beta C\sqrt{\Delta t}}) + (e^{-\beta C\sqrt{\Delta t}} - 1)S_t + o(\frac{1}{\sqrt{m}}).$$

and for $\vartheta > 1$, floating strike LBC have the approximation

$$V_m(S_-) = -e^{\beta C\sqrt{\Delta t}} V(S_- e^{-\beta C\sqrt{\Delta t}}) + (e^{\beta C\sqrt{\Delta t}} - 1)S_t + o(\frac{1}{\sqrt{m}}).$$

The constant

$$C = \sqrt{\sigma^2 + \lambda(\log \vartheta^2)}.$$

Theorem 5. *For $0 < \vartheta < 1$ the discrete-version at k^{th} and continuous-version at time $t = k\Delta t$ fixed strike LBC option satisfy*

$$F_m(S_+) = e^{-\beta C\sqrt{\Delta t}} F(S_+ e^{\beta C\sqrt{\Delta t}}) + o(\frac{1}{\sqrt{m}}).$$

and for $\vartheta > 1$, fixed strike LBP have satisfy

$$F_m(S_-) = e^{\beta C\sqrt{\Delta t}} F(S_- e^{-\beta C\sqrt{\Delta t}}) + o(\frac{1}{\sqrt{m}}).$$

The constant

$$C = \sqrt{\sigma^2 + \lambda(\log \vartheta^2)}.$$

4 Numerical results

4.1 Monte-Carlo simulation

If no closed formulas are at hand, we can try to find a good indication of the price of the option by doing a huge number of simulations. The accuracy of the final estimate depends on the number of sample paths used.

The method is basically as follows. Simulate,say m, paths of stock-prices process and calculate for each path the value of the payoff function V_i,$i = 1, ..., m$. Then the Monte Carlo estimate of the expected value of the payoff is

$$\hat{V} = \frac{1}{m} \sum^{m} -i = 1V_i.$$

The final option price is then obtained by discounting this estimate: $\exp(-rT)\hat{V}$.

In our case, the Monte Carlo Pricing procedure goes as follows[5]:

1.Calibrate the model on the available vanilla option prices in the market and find the risk-neutral parameters of the model;

2.Simulate N trajectories of the calibrated Lévy process based models;

3.Calculate the payoff function p_i for each trajectory,$i = 1, ..., N$;

4.Calculate the sample mean payoff to get the estimated payoff $\hat{p} = \sum_{i=1}^{N} p_i/N$;

5.Discount the estimated payoff at the risk-free and get the derivative price $e^{rT}\hat{p}$.

4.2 Laplace transform

Laplace transform is one of the most widely used integral transforms. It is used to produce an easily solvable algebraic equation.

Definition 4 *Let $\psi_t(\theta) = \log E^*[e^{\theta X(t)}]$ be the cumulant generating functions of $X(t)$. And then it is given by*

$$\psi_t(\theta) = [\frac{1}{2}\sigma^2\theta^2 + \mu\theta + \lambda(e^{\theta \log \delta} - 1)] \cdot t \equiv G(\theta) \cdot t.$$

Denote $g(\cdot)$ as the inverse function of $G(\cdot)$.

Theorem 6. *For α such that $\alpha + r > 0$ the Laplace transform w.r.t. T of the LBP option is given by [6]*

$$\begin{aligned} Lp(\alpha) &= \int_0^\infty e^{-\alpha T} V(T) d(T) \\ &= \frac{S_+ e^{rt}}{\alpha + r} - \frac{S_t}{\alpha} + \frac{s_0 e^{rt+\alpha[1-g(\alpha+r)]}}{(\alpha+r)[g(\alpha+r)-1].} \end{aligned} \tag{7}$$

We can use the Gaver-Stehfest algorithm for Inverse Laplace transform:[7]

$$f(t) \simeq [\ln(2)/t] \sum_{j=1}^{J} d(j, J) F[j \ln(2)/t].$$

where J is an even integer whose optimal value depends upon the computer word length,$M = J/2$, and m is the integer part of $(j+1)/2$ and

$$d(j, J) = (-1)^{j+M} \sum_{k=m}^{\min(j,M)} \frac{k^M (2k)!}{(M-k)!k!(k-1)!(j-k)!(2k-j)!}.$$

Set the LBP parameters as: $m = 250$, $s = 90$,$S_+ = 90$,$r = 0.1$,$\sigma = 0.3$,$\delta = 0.9$, $\lambda = 1$. $T = 1$ and $t = 0.8$.

Then for the discrete case, by using the Monte-carlo simulation with 10^5 replications, we get the value is 8.4029; For the continuous case, by Mathematica, we get 9.9901. The absolute error is 0.0571, and relative error is 0.67 percent.

4.3 Results

A)Set option parameters as: $s = 95$,$S_+ = 110$,$r = 0.1$,$\sigma = 0.3$,$\delta = 0.9$,$T = 1$ and $t = 0.8$.Assume 250 trading days per year,$m = 250$ monitoring points corresponds to daily monitoring of the extrema.(The Approx.(1) is obtained by theorem 4). We can get the table below

J	$\lambda=1$ V (S_+)	Approx. (1)	$\lambda=2$ V (S_+)	Approx. (1)	$\lambda=4$ V (S_+)	Approx. (1)
4	13.3177	12.2128	14.6740	13.4831	17.2905	15.9330
6	12.2639	11.1722	13.5771	12.3999	16.1151	14.7726
8	12.2804	11.1886	13.5926	12.4153	16.1287	14.7863
10	12.2827	11.1907	13.5951	12.4176	16.1316	14.7890

Table 2: Continuous LBP option, varing λ anf J

B) Set option paremeters as: $s = 95$,$S_+ = 110$,$r = 0.1$,$\sigma = 0.3$,$\delta = 0.9$,$\lambda = 1$, $T = 1$ and $t = 0.8$. Assume 250 trading days per year.The continuous LBP $V(S_+) = 13.3177$ and price (1) is the approximative value in theorem 4. we have

m	Discrete lookback V_M (S_+)	Corrected contiuous lookback (1)	Absolute error AE	Relative error RE (%)
10	9.9874	8.2146	-1.7728	-17.75
20	10.5995	9.6022	-0.9973	-9.41
40	11.2436	10.6359	-0.6077	-5.40
80	11.8076	11.3937	-0.4139	-3.51
160	12.0339	11.9433	-0.0906	-0.75
250	12.2346	12.2118	-0.0218	-0.18

Table 3: Lookback Put Option Price, S_+=110, varing m

C) Set $s = 95$,$r = 0.1$,$\sigma = 0.3$,$\delta = 0.9$,$\lambda = 1$, $T = 1$ and $t = 0.2$. Assume 250 trading days per year.We have

S_+	m	Discrete Lookback $V_m(S_+)$	Corrected Contiunous Lookback (1)	Absolute error AE	Relative error RE (%)
	10	6.2338	4.3655	1.8683	29.97
	40	8.7566	7.2850	1.4716	16.80
105	80	9.8155	8.3611	1.4544	14.82
	160	9.5242	8.0714	1.4528	15.25
	250	10.1299	9.4194	0.7105	7.01
	10	9.9874	8.2146	1.7728	17.25
	40	11.2436	10.6359	0.6077	5.40
110	80	11.5467	11.3937	0.1530	1.33
	160	11.9873	11.9433	0.0440	0.37
	250	12.2346	12.2128	0.0218	0.18
	10	14.1987	13.1950	1.0037	7.00
	40	14.8044	14.6757	0.1287	0.87
115	80	15.0148	15.2099	0.1951	1.29
	160	15.5154	15.6172	0.1018	0.66
	250	15.8215	15.8229	0.0014	0.00

Table 4: Lookback Put Option Price, varing m and S_+

5 Conclusion

In the thesis, we separate Lookback option into two types: Continuous case and Discrete case. We construct a risk-neutral measure, under the new measure, we present the Jump-Diffusion model, and adapted it to each case of Lookback option.Then we use Laplace transform and Monte-Carlo simulation to test the performance of the pricing formulas.

From the numerical testing results, we can conclude that this Jump-Diffusion type Lévy process ,which consisting of Brownian motion and Compensated Poisson process, matches the real data quite well.

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