GAUSSIAN COPULA
What happens when models fail?

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November 23, 2012

Division of labour

Both authors have contributed to all parts of the report. Conclusions where made through discussion.

1 Introduction

During the late 1990's a new financial derivative called Collateralized Debt Obligations or CDOs appeared on the market. These derivatives allowed banks to form securities out of different types of debt, e.g. mortgages. In order to price these securities it was necessary to model the correlation between defaults. David X. Li, once called "the worlds most influential actuary" by the Financial Times, presented a model to do just that in [9]. A variation of Li's approach called the Gaussian copula has been accused of increasing the severity of the financial crisis [10].

Li's copula approach to model default correlation proved to be a major breakthrough in the field of credit risk, as it was a quick and mathematically elegant way to model a quite complex problem. As the formula gained traction within the financial industry, the market for derivatives exploded. In 2000, when Li published his influential article, the total issuance of CDOs where worth less than USD 70 billion. At the height of the CDO bubble in 2006 and 2007, global issuance had grown to about USD 500 billion per year. As the crisis hit, the CDO market collapsed and in 2010, global issuance was about USD 10 billion [3].

Using Gaussian copula based methods banks was able to bundle together high risk credits into CDOs that appeared to carry low risk. These was consistently given investment grade ratings from the credit rating institutes. Investors with various risk tolerance invested in these securities, including mutual and pension funds from all over the world.

This report contains a short description of the model presented by Li and how this model is flawed. Further, risks related to overuse of any model is examined. Finally suggestions on how to reduce similar risks are discussed.

1.1 What is a Collateralized Debt Obligation (CDO)?

A Colleteralized Debt Obligation is a type of asset backed security, created in order to allow the issuer to use cash flows from different types of debt assets to back bonds, which in turn can be sold to investors. The issued bonds are "tranched", which means that there are several categories of bonds, each with different levels of risks and returns.

The creation and structure of a CDO is outlined in [4]. The process is quite complex; generally it starts with some financial institution, for example a bank, that owns a pool of debt assets. Due to several reasons (further explained below) the bank might want to move these assets of their balance sheet, which they can do by transferring the asset pool to a so called Special Purpose Vehicle (SPV). The SPV is a company started by the bank, which issues bonds (CDOs) to investors and the money acquired from this bond issue is then used to purchase the pool of debt assets from the bank. The pool could be a portfolio of house mortgages, car loans or credit card loans for example.

If the pool of assets backing the CDOs is for example a portfolio of mortgages, then the cash from the mortgage payments will in turn be paid out from the SPV to the holders of the CDOs in the form of coupons (interest). The size of the coupons are determined by which tranche it belongs to. The least risky tranches are called the senior tranches, though they are also paid the smallest coupon. The tranches with medium risk are called the mezzanine tranches, which are rewarded with a higher coupon as compensation for the higher level of risk exposure. The riskiest tranche, as well as the one with the highest return, is called the equity tranche.

The reason why the different tranches have different risks, has to do with what happens if the mortgage payers start to default on their mortgage payments. The cash flows from the mortgage payments are first used to pay coupons to the most senior tranches, then to the mezzanine and lastly to the equity tranche. If the mortgage payers start to default, the holders of the equity tranche bonds might not get their full coupons. If enough people defaults on their mortgages, the equity tranche might get wiped out and the most junior classes of mezzanine bonds will begin to occur losses. The senior tranches are shielded by all the more risky tranches and thus will take losses last.

There are a few reason for banks to create SPVs and CDOs. For example, it allows them to move certain assets such as mortgage loans from their balance sheets. Thus, the banks have to hold less regulatory capital which can be lent out to other people and companies. Further, it creates a new income stream for the bank, from the fees they charge by selling CDOs from the SPV.

2 The copula approach

In [9] David X. Li suggests a method to model default correlation. A random variable is introduce to signify the time-until-default, denoted T_i. The default correlation is defined as the correlation between these random variables. For two credit risks A and B, with time-until-default T_A and T_B, the default correlation is defined as follows

$$\rho_{AB} = \frac{Cov(T_A,T_B)}{\sqrt{VarT_A VarT_B}} = \frac{E(T_A T_B) - E(T_A)E(T_B)}{\sqrt{VarT_A VarT_B}} \tag{1}$$

In order to estimate ρ_{AB}, information on distribution of T_A and T_B is required. Basically we need the distribution for the survival time for a certain credit. This can be estimated by relying on historical default data from rating agencies, by using Black-Sholes methodology suggested by Merton in [1] or by using market observable information.

David X. Li's approach is to use market information instead of historical data for a number of reasons. Market data is quick to react to changes. Market data should take default severity into account, whereas most rating agencies only handles default frequency. A method for estimating the distributions of the survival time from market information is presented in [8].

Lets consider a set of credits. Under the assumption that there is no correlation between the individual survival times it is trivial to calculate the risk for the set. However, historical data indicates that default rates are higher in recessions than in periods of strong economic growth. This implies some positive correlation between survival times.

Under the assumption that the survival times are correlated the joint distribution for the survival times is required to draw conclusions about the portfolio of credits. This method takes the approach of using a copula function to derive the joint distribution from the marginal distributions.

$$C(F_1(x_1),...,F_n(x_n)) = F(x_1,...,x_n) \tag{2}$$

In [11], Sklar shows that for any multivariate distribution F there is a copula function C, such that equation 2 holds. Further, if the marginal distributions are continuous C is unique.

There are a number of different copula functions. The Frank Copula

$$C(u,v) = 1/\alpha \log(1 + \frac{(e^{\alpha u} - 1)(e^{\alpha v} - 1)}{e^{\alpha} - 1}), -\infty \leq \alpha \leq \infty$$

The bivariate Gaussian copula

$$C(u,v) = \Phi_2(\Phi^{-1}(u),\Phi^{-1}(v),\rho)$$

where Φ^{-1} is the inverse of the cdf for the normal distribution. Φ_2 is the bivariate normal cdf. ρ is the correlation parameter. For a multivariate Gaussian copula ρ would be replaced by a correlation matrix Σ describing the correlation between the different variables.

3 Risks related to the Gaussian copula approach to default correlation

The crisis exposed significant weaknesses in Li's model. The underlying assumption of the model that correlations between different assets where constant over time and could be combined into one number, had proved to been an serious misjudgement. In the financial industry and academia, people had realized that this assumption was not particularly realistic. For example, [13] points out that correlations are notoriously unstable and questions if a theory should be built on such unpredictable parameters. Though these warning signs where in general disregarded by the practitioners, as they simply made to much money from selling these securities [10].

The most popular approach to find the distributions for the time to default, was to use observed market data from another type of credit default swaps (CDS). This approach builds on the assumption that the market for credit default swaps is efficient and correctly prices these derivatives. Though the CDS is a relatively new financial innovation, hence there were only data available from a short period of time, which had been characterized by rising house prices and low default correlations [10]. The method was however believed to be unbiased among many financial professionals, but lately for example [6] argues that this approach does in fact result in biased estimates of the default probabilities and should not be used at all.

4 General overconfidence in models

The value of models are strongly dependent on how well they model reality. If predictions fail, the model is to blame not reality. For more complex financial products estimating the feasibility of a risk measure becomes more difficult. Risk measures derived from model that has not been validated by historical data, has limited value.

For regulators to demand risk measures derived from certain model, validates the legitimacy of the model. This further conformity among models and leads investors to trust the established models to a higher degree.

5 Risks related to conformity

In the case of pricing CDOs and other credit derivatives the model of choice has been Gaussian copula. Conformity presents a increased risk to the system since the risk measures are built on the same assumptions. When those assumptions turns out to be false the compound losses can be catastrophic. A more diverse range of models decreases the risk to the system.

6 Conclusions

The causes of the financial crisis and the part the Gaussian copula played is widely debated. In hindsight, it is apparent that the practitioners put to much confidence in David X. Li's model and its ability to model default probabilities.

Though some of the problems with the models assumptions was not unknown it was too profitable for the CDO issuers to keep on selling these securities. At the same time, the CDOs was also looked upon very favorable by investors such as pension funds, since the perceived risk was very small for senior CDO tranches. The bonds issued in these tranches were consistently given very high credit ratings from rating agency such as Standard & Poor's and Moody's, further fueling the illusion of low risk and spreading the securities through out the global financial system [10].

Ultimately, its safe to say that the financial meltdown was not the outcome of one single factor, but rather of a combination of several. The answer to which these factors were and how big of a part they played depends on who you ask. Deregulation of the financial industry [7] as well as predatory lending by banks [2] are often pointed out as reasons to why the sub-prime mortgage bubble was created. Other explanations are for example government intervention by keeping interest rates artificially low in order to promote home-ownership [12] and overleveraged financial institutions and households [5].

6.1 How can similar risks be reduced in the future?

Today, the flaws of the Gaussian copula are well documented. Thus, the challenge of the future lies in minimizing the damage caused by failing models.

The flaws in a model is generally not fully exposed until it is put under stress, such as during a financial crisis. Depending on the popularity of the model in question, the breakdown of it will have different levels of impact on the financial system. When a model, as popular as the Gaussian copula was before the global financial crisis, breaks down the consequences can be severe. Evidently, the industries ability to identify flawed models is not sufficient, therefore further precautions should be taken.

For example, if the sellers of securities provided more transparency regarding underlying assets, it would be possible for the buyers to better make their own judgement whether a price is fair or not. An effect of this would hopefully be that the buyers could more easily detect mispricings and bring the market value of securities closer to their intrinsic value. Further, they would not have to put as much blind trust into the credit rating agencies to determine credit risks.

When considering how bonds are to be rated, it is important that the incentives are to give a correct risk rating. In the case of CDOs, the credit rating agencies were hired by the issuers to rate the products. This resulted in an incentive for the agencies to give high credit ratings, otherwise they would lose business to a competitor willing to give a better rating. If the buyers were the ones soliciting the ratings, it would shift the incentives to provide a more accurate risk evaluation.

In order to minimize model risks, financial institutions could set up functions within their risk management department with the sole purpose of continuously evaluating their models. To limit model risks it could also be beneficial to use multiple models based on fundamentally different assumptions. If different entities use different models, the risk of a systemic break down decreases. How to stimulate diversity of models, is in it self a problem. The complexity of financial models in general make them difficult to understand for the average investor. Overwhelmed investors will probably turn to best-practices. This could in part be circumvented if regulators required risk measures from several different models.

7 Reading list

Recommended reading for those who would like to know more.

- Recipie for disaster: The formula that killed Wall street[10].
  A good intruduction to the copula and its role in the financial crisis.
- On Default correlation: A copula approach[9].
  David X. Li's original paper on default correlation. A technical report on the Gaussian copula approach.
- Problems with using cds to infer default probabilities[6].
  A paper on why using market data from cds is problematic when determining default probabilities

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