The Annals of Statistics
2016, Vol. 44, No. 3, 1360–1361
DOI: 10.1214/15-AOS1415

CORRECTION NOTE

LIMIT THEOREMS FOR EMPIRICAL PROCESSES OF CLUSTER FUNCTIONALS

Ann. Statist. **38** (2010) 2145–2186

BY HOLGER DREES* AND HOLGER ROOTZÉN†,‡

University of Hamburg, Chalmers University† and Gothenburg University‡*

We correct an error in a technical lemma of Drees and Rootzén [*Ann. Statist.* **38** (2010) 2145–2186] and discuss consequences for applications.

In Lemma 5.2(vii), it is stated that under the conditions (B1) and (B3) the length $L(Y_n)$ of the core of a cluster satisfies $\lim_{k\to\infty} \limsup_{n\to\infty} P\{L(Y_n) > k\}/(r_n v_n) = 0$. However, in general, the first inequality in the proof of this part of the lemma is not correct, and it seems likely that the assertion does not hold under the stated conditions. Note that this part of the lemma is used only in Remark 3.7(i); so none of the other results are affected.

The easiest way to correct the error is to replace condition (B3) with the corresponding condition for φ-mixing coefficients

$$\varphi_{n,k} := \sup_{1\le l\le n-k-1} \sup_{B\in\mathcal{B}^n_{n,l+k+1}, C\in\mathcal{B}^l_{n,1}} \bigl|P(B) - P(B|C)\bigr|$$

[with the convention $P(B|C) = P(B)$ if $P(C) = 0$], that is, to assume $\lim_{m\to\infty} \limsup_{n\to\infty} \varphi_{n,m} = 0$. The arguments of the proof of Lemma 5.2(vii) are rectified if $\beta_{n,k}$ is replaced with $\varphi_{n,k}$ everywhere.

However, often the following simpler condition is easier to verify:

($\widetilde{B}3$) For all $n \in \mathbb{N}$ and all $1 \le i \le r_n$ there exists $s_n(i) \ge P(X_{n,i+1} \ne 0|X_{n,1} \ne 0)$ such that $s_\infty(i) := \lim_{n\to\infty} s_n(i)$ exists and $\lim_{n\to\infty} \sum_{i=1}^{r_n} s_n(i) = \sum_{i=1}^{\infty} s_\infty(i) < \infty$.

Since, by stationarity,

$$\begin{aligned}\frac{1}{r_n v_n} P\bigl\{L(Y_n) > k\bigr\} &\le \frac{1}{r_n v_n} \sum_{i=1}^{r_n-k} \sum_{j=i+k}^{r_n} P(X_{n,j} \ne 0|X_{n,i} \ne 0) P\{X_{n,i} \ne 0\} \\ &\le \sum_{j=k}^{r_n} s_n(j),\end{aligned}$$

Received November 2015.
MSC2010 subject classifications. Primary 60G70; secondary 60F17, 62G32.
Key words and phrases. Absolute regularity, core length, weak dependence.

the assertion of Lemma 5.2(vii) follows readily.

To check condition $(\widetilde{B}3)$, typically one bounds $P(X_{n,i+1} \neq 0 | X_{n,1} \neq 0)$ by an expression of the form $s_n(i) = b_n + c_i$ with $b_n = o(1/r_n)$ and $\sum_{i=1}^{\infty} c_i < \infty$. The interchangeability of the limit and the sum is then automatically fulfilled. For example, $(\widetilde{B}3)$ has been verified in Example 8.3 of Drees, Segers and Warchoł (2015) for solutions to stochastic recurrence equations.

Condition $(\widetilde{B}3)$ has the additional advantage that in Remark 3.7(i) it renders condition (3.9) superfluous, that is, condition (C3) is met if $(\widetilde{B}3)$ and (3.8) hold. To see this, check that, for bounded functions ϕ, ψ, using stationarity $E(g_\phi(Y_n)g_\psi(Y_n))/(r_n v_n) = \operatorname{Cov}(g_\phi(Y_n), g_\psi(Y_n))/(r_n v_n) + O(r_n v_n)$ can be represented as

$$\frac{1}{v_n} E\big(\phi(X_{n,1})\psi(X_{n,1})\big) + \sum_{k=1}^{r_n-1} \frac{1}{v_n}\left(1 - \frac{k}{r_n}\right)\big(E\big(\phi(X_{n,1})\psi(X_{n,k+1})\big) + E\big(\psi(X_{n,1})\phi(X_{n,k+1})\big)\big)$$

which tends to $c(g_\phi, g_\psi)$ defined in (3.10) by our assumptions and Pratt's lemma [Pratt (1960)], because the kth summand can be bounded in absolute value by $2\|\phi\|_\infty \|\psi\|_\infty s_n(k)$. Moreover, using the above representation with $\phi = \psi = 1_{E\setminus\{0\}}$, one immediately sees that $(\widetilde{B}3)$ also implies condition (3.5).

Acknowledgment. We would like to thank Johan Segers for pointing out the error discussed in this note.

REFERENCES

DREES, H. and ROOTZÉN, H. (2010). Limit theorems for empirical processes of cluster functionals. *Ann. Statist.* **38** 2145–2186. MR2676886

DREES, H., SEGERS, J. and WARCHOŁ, M. (2015). Statistics for tail processes of Markov chains. *Extremes* **18** 369–402. MR3377814

PRATT, J. W. (1960). On interchanging limits and integrals. *Ann. Math. Stat.* **31** 74–77. MR0123673

DEPARTMENT OF MATHEMATICS
UNIVERSITY OF HAMBURG
SPST
BUNDESSTR. 55
20146 HAMBURG
GERMANY
E-MAIL: holger.drees@uni-hamburg.de

DEPARTMENT OF MATHEMATICAL STATISTICS
CHALMERS UNIVERSITY
412 96 GÖTEBORG
SWEDEN
AND
DEPARTMENT OF MATHEMATICAL SCIENCES
GOTHENBURG UNIVERSITY
412 96 GÖTEBORG
SWEDEN
E-MAIL: rootzen@math.chalmers.se