Design Life Level: Quantifying risk in a changing climate

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Received 28 September 2012; revised 7 July 2013; accepted 13 July 2013; published 23 September 2013.

[1] In the past, the concepts of return levels and return periods have been standard and important tools for engineering design. However, these concepts are based on the assumption of a stationary climate and do not apply to a changing climate, whether local or global. In this paper, we propose a refined concept, Design Life Level, which quantifies risk in a nonstationary climate and can serve as the basis for communication. In current practice, typical hydrologic risk management focuses on a standard (e.g., in terms of a high quantile corresponding to the specified probability of failure for a single year). Nevertheless, the basic information needed for engineering design should consist of (i) the design life period (e.g., the next 50 years, say 2015–2064); and (ii) the probability (e.g., 5% chance) of a hazardous event (typically, in the form of the hydrologic variable exceeding a high level) occurring during the design life period. Capturing both of these design characteristics, the Design Life Level is defined as an upper quantile (e.g., 5%) of the distribution of the maximum value of the hydrologic variable (e.g., water level) over the design life period. We relate this concept and variants of it to existing literature and illustrate how they, and some useful complementary plots, may be computed and used. One practically important consideration concerns quantifying the statistical uncertainty in estimating a high quantile under nonstationarity.

Citation: Rootzén, H. and R. W. Katz (2013), Design Life Level: Quantifying risk in a changing climate, *Water Resour. Res.*, *49*, 5964–5972, doi:10.1002/wrcr.20425.

1. Introduction

[2] In this paper, we discuss how risk quantification should be altered to meet the challenges posed by global climate change [*Solomon et al*., 2007], and by local climate changes caused by shifts in land use or other anthropogenic influences. We believe this discussion is long overdue, and that it merits much more attention than it has received so far. The setting is adaptation: how should one modify design practice so that it can cope with climate changes?

[3] Risk handling always requires a compromise between risk avoidance and cost. The goal of this paper was to contribute a concept, Design Life Level, which is convenient for quantifying and communicating environmental risk in a changing climate. It can be viewed as an extension to nonstationarity of the concept of "risk of failure" [*Fernandez and Salas*, 1999; *Jakob*, 2013; *Rosbjerg and Madsen*, 1998], often advocated to more effectively communicate the risk of hydrologic extremes under stationarity [e.g., *Kunreuther et al*., 2013; *Michel-Kerjan and Kunreuther*, 2011]. Note that we use the term "risk" in its nontechnical sense to refer to the probability of an extreme event whose consequences would be substantial, and not in its technical sense in risk analysis, where risk commonly refers to expected loss.

[4] At present, design criteria for environmental loads on structures, such as dikes, dams, sewers, or bridges, are based on the (in a stationary climate equivalent) concepts of return levels, return periods, and annual chance of exceedance. To exemplify, *the 10,000 year return flood level*, as used, for example, in dike design in the Netherlands [*Botzen et al*., 2009], is the flood level that on average is exceeded once every 10,000 years; and the corresponding annual chance of exceedance, of course, is 1/10,000. Similarly, *the 300* m^3/s *stream-flow return period* is the time it takes, on average, between the occurrence of two stream flows in excess of 300 m^3/s (as used, e.g., in Swedish regulation, see *Svensk Energi et al*. [2007]).

[5] In a stationary climate, these concepts answer many of the basic questions (see section 2). They have served us well as design tools in the past. However, these concepts are based on the assumption of stationarity and do not apply to a nonstationary environment [see *Milly et al*., 2008].

[6] For example, think of a flood return level, which on average would be exceeded once in 100 years under the hypothetical assumption that climate continues to be as it is in 2015, in a statistical sense (termed in a "2015 climate"). In a nonstationarity world, this level might instead be exceeded on the average once every 90 years in a 2065 climate, and in a 2100 climate, once every 70 years. A construction that is planned to be in service from 2015 to 2064 will necessarily encounter different risks than a similar

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0043-1397/13/10.1002/wrcr.20425

construction did during 1961– 2010, and a construction that is in operation from 2065 to 2114 would encounter still other risks. This behavior cannot be captured using the inherently stationary concept of return levels, and risks cannot be sensibly evaluated if one only knows the length of the projected lifetime of a construction: to assess risks, one must also specify during which years the construction is planned to be in service.

[7] We recognize that current hydrologic risk management under the assumption of stationarity tends to focus on a standard (e.g., in terms of a high quantile corresponding to the specified probability of failure for a single year [*Institute for Water Resources*, 2011]). Nevertheless, implicit in this practice is a risk of failure over a relevant time period much longer than a year (e.g., the probability of at least one flood over the lifetime of a structure). For example, *Institute for Water Resources* [2011, p. vi] states that "... decision-making will ideally occur within a watershed framework and consider lifecycle aspects." Further, a main concern for *Institute for Water Resources* [2011], and for much current work, is the impact of climate change. Nevertheless, methods for risk quantification under nonstationarity are lacking (e.g., as noted by *Cooley* [2013], *Khaliq et al*. [2006], and *Salas and Obeysekera* [2013]).

[8] In a changing climate, risk assessment instead should include both a specification of the period of time when the construction will be in use, the design life period, and of the probability of exceeding a hazardous level during this period. Although some aspects of water resources management could continue to be based on an annual chance of occurring, adapting to its change from one year to the next, this would not be practicable for many design problems (i.e., akin to redefining the flood plain on an annual basis).

[9] Besides Design Life Level, we also discuss a variant, termed Minimax Design Life Level, which focuses on the largest exceedance risk for any year of the design life period. Further, the Risk Plots and Constant Risk Plots, as introduced below, may be used to follow how risk changes with time.

[10] These concepts are statistical, but still, in addition to observational data, can also be based on projections of future climate change from the outputs of climate experiments using numerical models of the climate system (provided, of course, the uncertainties inherent in such projections are taken into account). They are general in nature, and different methods and statistical models can be used to compute them, although it would be natural to prefer those based on statistical extreme value theory.

[11] It is not within the scope of this paper to discuss the extent of local or global climate change (although some of the papers reviewed in section 3 do so). The aim is just at what should be done, conceptually, if one believes that risks are influenced by nonstationarity.

[12] Section 2 provides background on return levels and return periods in a stationary environment. Section 3 reviews and discusses some literature on quantifying risk in a nonstationary environment. Section 4 introduces the concepts of Design Life Level and Minimax Design Life Level and the risk plots. In section 5, we use two examples, highest daily rainfall at Manjimup, Western Australia, during the winter wet season, and extremely warm winters (in terms of high minimum winter temperatures) in Fort Collins, Colorado, to illustrate how one can apply Extreme Value Statistics to estimate Design Life Level and the other risk concepts. As are return periods and return levels, these measures are subject to statistical parameter uncertainty due to limited samples, and to structural uncertainty caused by imperfect understanding of how well models fit reality. This issue is discussed in section 6. The final section, section 7, contains a summary and discussion. A more technical subject, how to use the delta method to estimate statistical uncertainty, is treated in Appendix A.

2. Background: Return Levels and Return Periods in a Stationary Climate

[13] Let $F(x)$ be the cumulative probability distribution function (cdf) of the quantity of interest, say, the largest daily rainfall in a year (in a stationary climate, this distribution is the same for all years). The T-year return level for daily rainfall u_T is defined to be the (1–1/T)-th quantile of the distribution of the maximum daily rainfall in a year. Equivalently, on average, one out of T years has at least one daily rainfall that exceeds u_t, so that $T(1-F(u_T))=1$. Again equivalently, the probability is $1/T$ that the annual maximum daily rainfall exceeds u_T in a year.

[14] If one further assumes that the sizes of the largest daily rainfalls in different years are independent, then the probability that at least one rainfall exceeds u_T in a period of N years is

$$1-F(u_T)^N = 1-(1-(1-F(u_T)))^N = 1-\left(1-\frac{1}{T}\right)^N. \quad (1)$$

[15] So, by just specifying T, one can compute the probability of exceeding of u_T in a time period of length N, for *any N*. It is this quantity that has been termed "risk of failure" in the context of hydrologic engineering under stationarity [*Fernandez and Salas*, 1999].

[16] The return period for a rainfall of size u is the expected waiting time (in years) until a daily rainfall larger than u occurs. In a stationary climate, and assuming independence, this waiting time is geometrically distributed with parameter $1-F(u_T)$, so the T-year return level u_T has return period $T=1/(1-F(u_T))$, as intended [*Lloyd*, 1980].

3. Nonstationary Risk Measures

[17] In this section, we review and discuss the quite limited, existing literature on risk in a nonstationary environment.

3.1. Frequency-Based Concept

[18] *Laurent and Parey* [2007] (see also *Parey et al.* [2007, 2010]) define the 100-year return level "as a value reached or exceeded in expectation 1 day over the hot season days of the next 100 years," and, hence, in our terminology, consider the design life period 2001–2100. If we let $F_t(x)$ be the cdf of the maximum temperature in year t, then this definition of the 100-year return level u_{100} hence is the value of u, which solves the equation $(1-F_{2001}(u)) + \ldots + (1-F_{2100}(u)) = 1$.

[19] However, return levels of this kind fix the exceedance probability at the same time as they fix the design life

period. Specifically, assuming independence, the probability that the maximum temperature during this century will exceed u_{100} is

$$
\begin{aligned}
&1 - F_{2001}(u_{100}) \times \cdots \times F_{2100}(u_{100})\,) \\
&\approx 1 - e^{-\left((1-F_{2001}(u_{100}))+\ldots(1-F_{2100}(u_{100}))=1-e^{-1}\approx 0.63,\right.}
\end{aligned}
\tag{2}
$$

where the approximations typically are quite accurate.

[20] Thus, the frequency-based concept always leads to an exceedance probability that is approximately 0.63. However, a 63% risk of an exceedance during the design life period may be quite high in many cases, and is completely unacceptable if an exceedance leads to a catastrophe. For realistic engineering design, one needs a more flexible concept, such as the one introduced in section 4.

3.2. Waiting Time–Based Concept

[21] The aim of *Olsen et al*. [1998] is the same as for the present paper. Continuing the work of *Wigley* [1988], *Olsen et al*. propose that one uses return periods, defined as the expected waiting time until an exceedance, as the risk measure. In a list of three possible design criteria, the authors also in passing write, "The first possible criterion is to ensure that the structure provides protection in all years for at least a 100-year flood (1/probability = 0.01)." Since these 100 year floods are computed under the hypothesis that the climate of the year in question continues indefinitely, this might be close to our Minimax Design Life Level as defined below.

[22] It is tempting to use the concept of return period, defined as expected waiting time, also in a changing climate, since the definition itself carries over without any change. However, this approach has two major drawbacks: (i) for a nonstationary climate, the expected waiting time is a quite imprecise description of the probability distribution of the waiting time (for a stationary climate, as discussed above, the return period, however, determines the distribution), and (ii) return periods depend on the development also after the design life period is over, but this development is less relevant.

3.3. Other Concepts

[23] *Vogel et al*. [2011] persuasively argue that there is nonstationarity in peak streamflows in many watersheds in the U.S. due to "a variety of anthropogenic processes including changes in land use, climate, and water use, with likely interactions among those processes making it very difficult to attribute trends to a particular cause" (in fact, once the effects of other processes are removed, *Villarini et al*. [2009] do not find strong evidence of trends due to global climate change). To quantify changes, the authors use "a decadal flood magnification factor, which is defined as the ratio of the *T*-year flood in a decade to the *T*-year flood today" and "obtain flood magnification factors in excess of 2–5 for many regions of the United States." The paper also introduces "a recurrence reduction (RR) as average time between floods in some future year t_f associated with the flood with an average recurrence interval of T_0 in some reference year t_o." Again these concepts seem quite useful as a way to illustrate the effects of nonstationarity, but are not directly relevant for design.

[24] The book chapter *Cooley* [2013] aims at risk communication, and in particular also contains a discussion of the concepts proposed in *Parey et al*. [2010] and *Olsen et al*. [1998]. Consistent with our proposed risk measure, *Salas and Obeysekera* [2013] examined the behavior of the risk of failure under nonstationarity.

4. Risk Measure for a Changing Climate

[25] In this section, we propose Design Life Level as a measure to quantify risk for the purpose of engineering design in a changing climate. We also discuss Minimax Design Life Level, and the use of Risk Plots and Constant Risk Plots. Design Life Level aims to achieve a desired probability of a hazardous exceedance (or risk of failure) during the design life period. Minimax Design Life Level is closely related, and complementary, but instead focuses on the maximal yearly probability of exceedance during the design life period. The Risk Plot specifies the time distribution of risk, whereas the Constant Risk Plot has a somewhat different focus, aiming at situations in which risks are not fixed once and for all by the original design, but rather managed over time, such as water levels in a dam.

[26] We use a hypothetical example, flooding of a dike, to introduce the concepts. In the example, the distribution of the highest water level at the dike during year *t* will be assumed to follow a Generalized Extreme Value (GEV) cdf,

$$
G_t(x) = e^{-\left(1+\xi_t \frac{x-\mu_t}{\sigma_t}\right)^{-1/\xi_t}}, \ \text{for } 1 + \xi_t \frac{x - \mu_t}{\sigma_t} \geq 0. \tag{3}
$$

[27] (Note that for $\xi_t = 0$ this cdf reduces to the Gumbel cdf.) Here, $\mu_t, \sigma_t > 0$, and ξ_t are the location, scale, and shape parameters, respectively, for year number *t* after 2015, the beginning of the first design life period studied below. Further, it is assumed that

$$
\mu_t = 1 + 0.002t, \ \sigma_t = 1 + 0.002t, \ \xi_t = 0.1, \tag{4}
$$

so that the location and shape parameters grow by two tenths of a percent per year, while the shape parameter is constant. For example, the increase in the location parameter could be due to a rise in the mean water level, and the increase of the scale parameter could be caused by an increase in climate variability.

[28] For later use, we note that the expected waiting time until a level *u* is exceeded, denoted $EWT(u)$, is conveniently obtained from equation (3) as

$$
\begin{aligned}
EWT(u) &= \sum_{t=0}^{\infty} \Pr\{\text{waiting time} > \text{t}\} \\
&= 1 + \sum_{t=1}^{\infty} G_1(u) \times \cdots \times G_t(u).
\end{aligned}
\tag{5}
$$

[29] The new concepts now are as follows.

[30] *Design Life Level: The* $T_1 - T_2\, p\%$ *extreme level*. Here T_1 denotes the time of the start of the design life period, T_2 is the end, and *p* is the probability that the level is exceeded during the design life period. With a design life

Table 1. Results for Example (3)[a]

Design Life	Prob.	Design Life Level	Return Level 2015 Climate	EWT	EWT Trend Stopped
2015–2064	0.05	11.5	10.9	251	788
2015–2064	0.01	15.2	14.4	431	3839
2065–2114	0.05	12.6	10.9	262	1008
2065–2114	0.01	16.6	14.4	453	5002

[a]Return levels are for $T = 975$ and $T = 4975$, respectively. EWT is expected waiting time until an exceedance of the Design Life Level. In the first EWT column, parameters are given by equation (4). In the second column, parameters are given by equation (4) up to the end of the design life period and are kept constant after that.

period of 2015–2064 and a risk of 5%, the estimated Design Life Level can be expressed as follows:

[31] The 2015–2064 5% highest water level is 11.5 m.

[32] In nontechnical communication, this could be phrased as "there is a 1 in 20 risk that that the biggest flood during 2015–2064 will be higher than 11.5 m." Technically, the 2015–2064 5% water level is the 95% quantile of the probability distribution function of the maximal water level during the period 2015–2064.

[33] Design Life Level captures risk in a way that is tailored for risk assessment; for example, if the dike is built to withstand a water level of 11.5 m, but not more, then the statement above means that there is a 1 in 20 risk that the dike will be flooded at least once during 2015–2064.

[34] Table 1 shows the Design Life Level for two different design life periods and risk levels, together with corresponding return levels for a stationary climate with the same distribution as in the first year of the design life period (i.e., for the return levels $T = 975$ and $T = 4975$, respectively, which by equation (1) makes the probability of an exceedance in a 50 year period equal to 0.05 and 0.01, respectively.) The table also exhibits the expected waiting times until the first exceedance of the Design Life Levels when the trend is given by equation (4) throughout, and when the trend stops at the ends of the design life periods.

[35] From the table it is seen that the return levels obtained from assuming stationarity are too low and that the expected waiting times are dramatically changed if one changes assumptions about what happens after the design life period. This illustrates that neither one of these concepts is appropriate for this example. (The purpose of the waiting time computations is just to show their sensitivity to assumptions about behavior after the end of the design life period, and not to make physically realistic predictions.)

[36] A variant of Design Life Level is *Minimax Design Life Level: the $T_1 - T_2$ p% bounded yearly risk level*. Here T_1 and T_2 are defined as before, but for this concept, the level is chosen such that the maximal probability of exceedance in any one year in the design life period is at most *p*%. Thus, in the example, "the 2015–2064 0.1% bounded yearly risk water level 12.0 m" could nontechnically be worded as "the risk that there will be a bigger flood than 12.0 m is less than 1 in 1000 for each year in the time period 2015–2064." Technically, the value 12.0 m is obtained by first determining the 0.999 quantile of the distribution of the largest rainfall in 2015, in 2016, . . . , and in 2064, and then taking the largest of these 50 quantiles.

[37] The *Risk Plot* fixes a level and shows how the risk of exceeding this level varies for the different years in the design life period. The *Constant Risk Plot* instead fixes a probability, and for each year in the design life period displays the level that is exceeded with this probability. Summarizing nonstationary risk in terms of changing quantiles (termed "effective return levels" in *Katz et al*. [2002]) is fairly common in the research literature on nonstationary extremes. It informs managers how they should plan in order to obtain the same risk for every year in the design life period but, as already noted, is not feasible in many problems in hydrologic engineering design. Figure 1 displays these plots for the present example. We have chosen the risk level in the constant risk plot, such that it leads to the same overall risk as the one used for Design Life Level.

5. Computation of Design Life Level and Other Risk Measures: Examples

[38] Although the risk measures introduced in section 4 are general, we in this section use Extreme Value Statistics for two examples. The first one is highest daily rainfall during the winter half, May to October, of the year in Manjimup, Western Australia (Figure 2). This region has

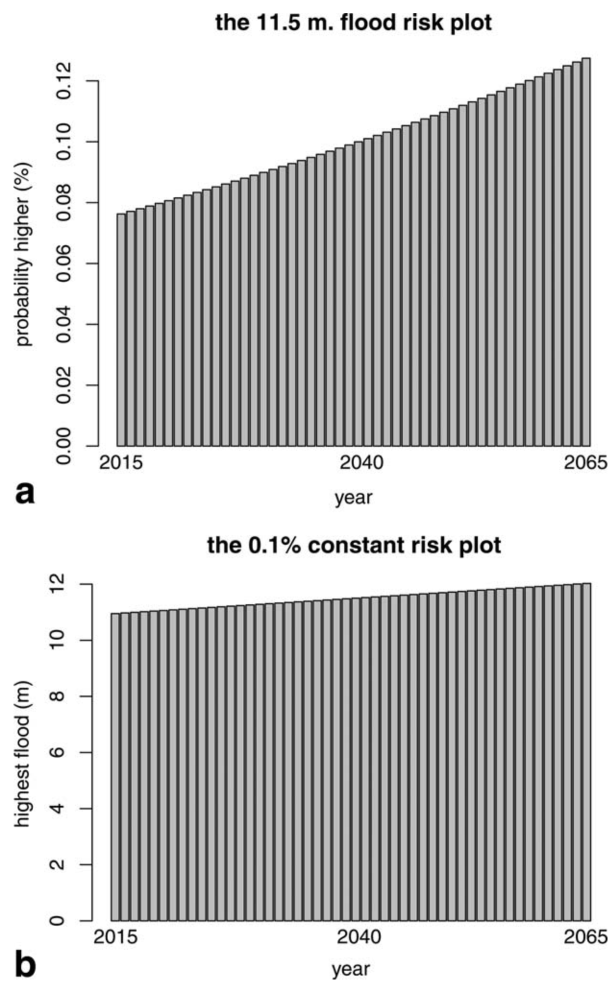

Figure 1. (a) Probability that the largest flood is higher than 11.5 m. (b) The risk that the highest flood in a year is larger than the value shown for this year in plot is 0.1%.

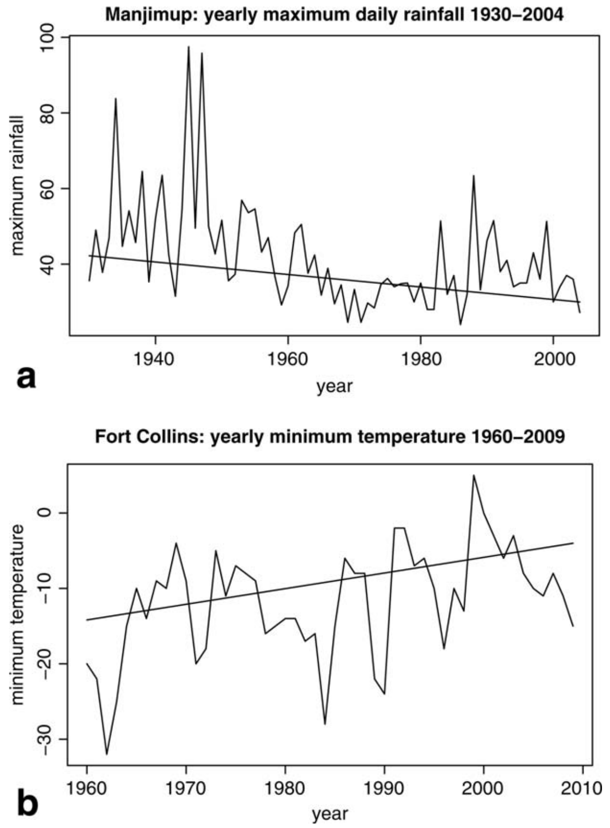

Figure 2. The data. (a) Largest daily rainfall (mm) during winter by year in Manjimup. Line is estimated location parameter (= 37% quantile) in fitted GEV distribution. (b) Lowest daily minimum temperature (°F) by year in Fort Collins. Line is estimated location parameter (= 63% quantile) in fitted GEV distribution for minima.

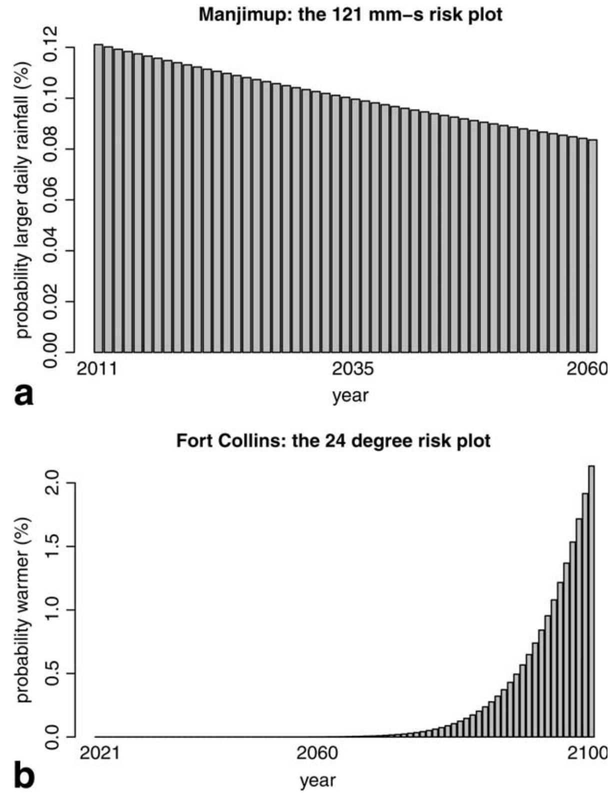

Figure 3. Risk plots. (a) Probability that the largest daily rainfall in Manjimup is greater than 121 mm. (b) Probability that the yearly minimum temperature in Fort Collins is warmer than 24°F.

experienced an overall drying trend in recent decades [*Bates et al.*, 2010], and in particular a decrease in extreme high daily precipitation amounts during the winter wet seasons [*Li et al.*, 2005]. The second example is warm winters in Fort Collins, meaning winters in which the lowest daily minimum temperature is relatively high (Figure 2). We phrase the description of the methods in terms of the first example. The computations for the second example were done in the same way.

[39] The Design Life Levels, for example design life periods and risk levels, are as follows: "the 2011–2060 5% largest daily winter rainfall in Manjimup is 121 mm" and "the 2021–2100 10% highest minimum winter temperature in Fort Collins is 24°F." We also obtained that "the 2011–2060 0.1% bounded yearly risk rainfall in Manjimup is 125 mm" and that "the 2021–2100 0.2% bounded yearly risk highest minimum winter temperature in Fort Collins is 40°F." Risk Plots and Constant Risk Plots are given in Figures 3 and 4. For Manjimup, the risk decreases roughly linearly, whereas for Fort Collins there is an abrupt rise in risk toward the end of the design life period. So, at least for the Fort Collins example, either ignoring the nonstationarity or using one of the alternative frequency- or waiting-time–based concepts of return level would be unrealistic for design purposes. (For example, in 2021 the risk that temperatures stay above 24°F the entire winter is negligible, whereas this risk is 2.1% in 2100. Thus, it clearly is not possible to state a single number that simultaneously captures yearly risk for each of the years in the period 2021–2100.)

[40] The aim of this section is to illustrate how the risk measures can be computed, and also how this introduces questions of uncertainty, to be discussed further in the next section. The choice of trends would benefit from a thorough examination of the physical process underlying and influencing the changes in precipitation and temperature. Nevertheless, assuming a linear trend in the location parameter of the GEV distribution seems natural and useful as a starting point.

[41] Extreme Value Statistics has two main sets of methods, the Yearly (or "Block") Maxima method and the Peaks over Thresholds method [e.g., *Coles*, 2001]. For simplicity, the discussion below is in terms of Yearly Maxima.

[42] We throughout assume that extreme events in the different blocks of time are independent of one another. This in particular means that it sometimes can be suitable to use blocks which are different from calendar years, say for the Manjimup example to use winters instead of years, to avoid cutting up winters into two different years. Since $\min\{x_i, \ldots, x_k\} = -\max\{-x_i, \ldots, -x_k\}$ questions concerning the distribution of minima are immediately transformed to questions concerning the distribution of maxima if one, instead of the original values, analyzes the negated values. If desired, by negating once more, one can then at

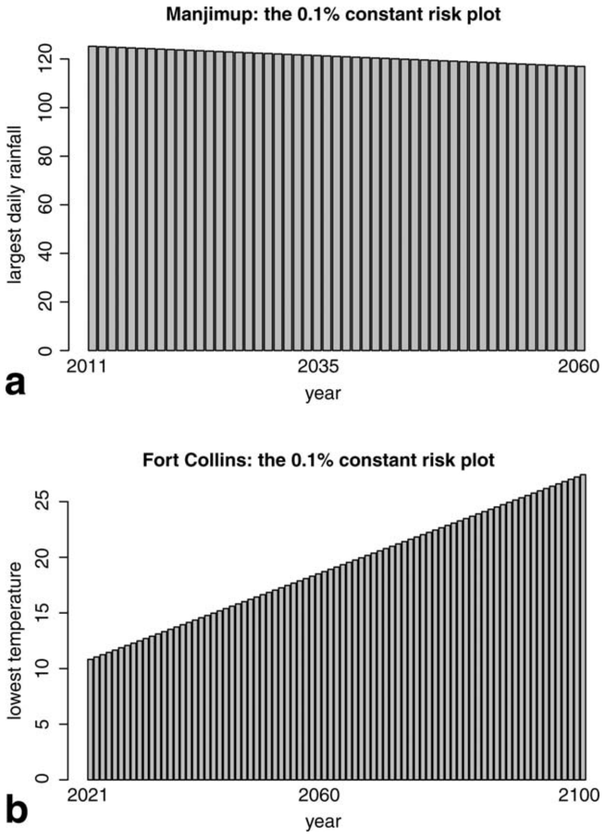

Figure 4. Constant risk plots. (a) The probability that the largest daily rainfall in a year in Manjimup is greater than the value shown for this year in plot is 0.1%. (b) The probability that the minimum temperature in a year in Fort Collins is higher than the value shown for this year in plot is 0.1%.

the end of the analysis return to the original values, and present the results in terms of them. This was how we handled the second example.

[43] The analysis can conveniently be made in four steps: (1) obtain the distribution of Yearly Maxima, (2) derive the distribution of the maximum over the design life period, (3) compute Design Life Level and other risk measures, and (4) find estimates of the statistical uncertainty of the risk measures.

[44] ***(1) Compute the distribution of Yearly Maxima.*** Let M_t be a random variable which describes the probability distribution of the largest (i.e., maximum) daily rainfall in year t, so that we have at our disposal one observation, m_{1930} of M_{1930}, one observation, m_{1931} of M_{1931}, and so on until the last observation which is available to us, m_{2004}.

[45] The nonstationary version of the Yearly Maxima method consists of assuming that block maxima follow the cdf (3). One then uses, for example, standard maximum likelihood techniques [*Coles*, 2001, Chapter 3] to estimate the parameters in equation (3) from the data $m_{1930}, \ldots m_{2004}$, and to choose between different candidate functional forms for the trends in the parameters.

[46] In the analysis of the Manjimup data, we tried out models with a linear trend in all three parameters and in the end, guided by plots and likelihood ratio tests, chose the model in which σ and ξ did not change with time but with a linear trend $\mu_t = a + b(t - 1929)$ in the location parameter. For this model, we obtained the estimates and standard errors $\hat{a} = 42.4 \pm 2.2, \hat{b} = -0.17 \pm -0.05, \hat{\sigma} = 8.0 \pm 0.8$, and $\hat{\xi} = 0.15 \pm 0.09$ (for a more detailed analysis, see *Katz* [2013]). These parameter values were then used in the remaining steps of the analysis. We used the open source R-program extRemes [*Gilleland and Katz*, 2011; *R Development Core Team*, 2011] for the computations. For the negated yearly minimum temperatures in Fort Collins, the corresponding values were $\hat{a} = 14.4 \pm 5.6, \hat{b} = -0.21 \pm 0.06, \hat{\sigma} = 6.05 \pm 0.68$, and $\hat{\xi} = -0.16 \pm 0.10$ (note that the negative trend, $b < 0$, for negated minima corresponds to a warming trend in terms of minimum temperature).

[47] ***(2) Compute the distribution of the maximum over the design life period.*** Let $M_{2011-2060}$ denote the size of the largest daily rainfall that occurs in the time period 2011–2060. Using the results from the previous step, we can compute estimates

$$\begin{aligned}\hat{G}_t(x) &= e^{-\left(1+\hat{\xi}_t \frac{x-\hat{\mu}_t}{\hat{\sigma}_t}\right)^{-1/\hat{\xi}_t}} \\ &= e^{-\left(1+\hat{\xi}\frac{x-(\hat{a}+\hat{b}(t-t_0))}{\hat{\sigma}}\right)^{-1/\hat{\xi}}}, \quad \text{for } 1+\hat{\xi}\,\frac{x-\hat{\mu}_t}{\hat{\sigma}} \geq 0\end{aligned} \tag{6}$$

of the distribution of M_t, for each level x and year $t = 2011, \ldots, 2060$ of interest. Using the assumption that the rainfall maxima in different years are independent, the cdf $F_{2011-2060}(x)$ of $M_{2011-2060}$ is obtained as $F_{2011-2060}(x) = \Pr\{M_{2011-2060} \leq x\} = G_{2011}(x) \times G_{2012}(x) \times \cdots \times G_{2060}(x)$.

[48] We thus immediately obtain an estimate of the cdf $F_{2011-2060}(x)$ of the size of the largest daily rainfall in the period 2011–2060 as

$$\hat{F}_{2011-2060}(x) = \hat{G}_{2011}(x) \times \hat{G}_{2012}(x) \times \cdots \times \hat{G}_{2060}(x). \tag{7}$$

[49] ***(3) Compute the Design Life Level and other risk measures.*** Let $F^{-1}(y)$ denote the inverse of the cdf $F(x)$ (called the "quantile function"). An estimate $\hat{q}_{2011-2060}(y)$ of the quantile function $q_{2011-2060}(y) = F^{-1}_{2011-2060}(y)$ corresponding to the cdf $F_{2011-2060}(x)$ is then obtained by numerical inversion of $\hat{F}_{2011-2060}(x)$ (perhaps easiest by just computing $\hat{F}_{2011-2060}(x)$ on a grid of x-values). Now, $\hat{q}_{2011-2060}(0.95)$ is an estimate of the Design Life Level for the risk $p = 0.05$.

[50] Similarly, the Minimax Design Life Level is easy to compute from $\hat{G}_{2011}(x), \ldots, \hat{G}_{2060}(x)$: just compute these on a grid of x-values and choose the x-value that makes their maximum equal to 0.999. The 2011–2060 Risk Plot is a bar plot of 1-$\hat{G}_{2011}(121), \ldots,$ 1-$\hat{G}_{2060}(121)$, 1-$\hat{G}_{2011}, \ldots,$ 1-$\hat{G}_{2060}$, and the 2011–2060 Constant Risk Plot is a bar plot of $\hat{G}^{-1}_{2011}(0.999), \ldots, \hat{G}^{-1}_{2060}(0.999)$. Since the inverses are straightforward to compute, the Constant Risk Plot is also easy to construct.

[51] ***(4) Estimate the statistical uncertainty.*** One standard method to estimate the statistical uncertainty is the delta method. The method consists of using a Taylor series expansion to find the standard deviation of Design Life Level, a nonlinear function of the estimated parameters. A more detailed description of the method is given in Appendix A. As one example, for Manjimup, the delta method

estimate of the standard error of Design Life Level is 36 mm (recall that the estimated 2011–2060 5% Design Life Level is 121 mm).

6. Statistical Uncertainty and Uncertainties of Model Choice

[52] Risk measures have to be based on data: the data could be past climate observations or output from climate change experiments using numerical models of the climate system. Statistical methods are then used to obtain estimates of the risk measures from the data. This invariably leads to a statistical parameter uncertainty in the estimates, as seen in the previous section, for example.

[53] Further, even in a stationary climate, the choice of statistical model can have substantial influence on the value of a risk estimate: typically, a model is used to extrapolate from the observed values to the more extreme values that have not yet been experienced, but that pose real threats for the future. Different models for this extrapolation can then lead to different results. Still, it is by now fairly well understood how to handle this uncertainty.

[54] However, a nonstationary climate involves a second round of model choice, which entails further uncertainty: typically statistical extrapolation into the future involves choosing a functional form of one or several trends. But, should one use a linear trend, or a quadratic one, or perhaps something quite different? The different forms of the trends might be almost indistinguishable for the observational period, but lead to rather different future behavior.

[55] Finally, of course, if a climate model is used, it involves a third set of choices of spatial resolution, of differential equation models, of initial values, and of parameter values, and many more model choices. A further important, and uncertain, choice is to select a scenario for the development of human activities which influence the climate. This last consideration of course is important also for the choice of the functional form of trends in statistical extrapolation.

[56] Thus, in design, major efforts may have to be directed at reducing these uncertainties. Possibilities include (a) borrowing strength across space to estimate a common or smoothly varying trend [see, e.g., *Hanel et al.*, 2009; *Westra and Sisson*, 2011], (b) combining observed historical trends with projections from (perhaps an ensemble of) climate models, and with historical experience of other similar situations (e.g., via a Bayesian approach), and (c) use of Peaks over Threshold instead of Yearly Maxima; in stationary situations this often does not improve precision much, but there is some evidence that it may do so in nonstationary situations.

[57] These last two sets of model uncertainties may in some situations have a major influence. How should they be handled? This is difficult to do quantitatively. Instead, serious nonquantitative consideration of these is of basic importance for good design practice. We have no general rules for how this should be done, but still list some possibilities:

[58] 1. Sensitivity studies: change some of the model assumptions and see how this affects the risk estimates.

[59] 2. Take smaller risks with designs if there is a large model uncertainty. (But then, how much smaller?)

[60] 3. Already in the design phase plan for later modification to make the construction more resistant, if need should arise.

[61] 4. Plan for regular adjustment of rules for managing the construction.

[62] 5. Plan for regular updating of risk measures as experience and knowledge increases.

[63] The issue of uncertainty about the future has led to a reluctance to abandon the stationarity assumption. In fact, some have even argued that it would be better just to incorporate a "safety factor" into the estimates based on stationarity [*Olsen*, 2006]. Nevertheless, as climate change trends are anticipated to accelerate in the future, a risk-based approach would clearly be preferable for engineering design [e.g., *Zevenbergen et al.*, 2013].

7. Summary and Discussion

[64] Our main assertion is that, in a changing climate, to quantify and communicate risks, one should specify both a period of time, the design life period, and a probability of failure. This probability should correspond to the desired risk of an extreme event; say, a water level that will lead to the flooding of a dike, or a rainfall that exceeds the capacity of a sewer system over the design life period.

[65] We propose a concept, Design Life Level, which does this. In it one simply specifies the design life period and the probability of exceeding an extreme/hazardous level during this period. A complementary variant, the Minimax Design Life Level, instead specifies the maximal risk of failure during any one year in the design life period. In addition, Risk Plots, which show how the probability of failure changes over the design life period, are often useful, and sometimes also Constant Risk Plots, which for each year in the design life period show the level that is exceeded with a given specified probability. Nevertheless, the probability of failure over the design life period, as specified to determine the design life level, remains the most informative single quantity for risk characterization and communication.

[66] If one is not aiming primarily at design, but just wants to illustrate the extent of changes, simpler concepts may sometimes suffice. In particular, *Laurent and Parey* [2007] use a nonstationary version of the 100-year return level, and *Vogel et al.* [2011] introduce a "flood magnification factor," which quantifies how the distribution of extreme events shifts from decade to decade. This is discussed in section 3.

[67] An always important aspect of risk measurement is quantification and handling of uncertainties in the risk measurements. This is discussed in sections 5 and 6. One (obvious) conclusion is that in a changing climate, it, already at the design stage, is important to plan for later modifications of constructions and for recurring reevaluation of risks.

[68] To facilitate use of the new concepts, in section 5 we exhibit how one can use Extreme Value Statistics to calculate the risk measures. However, it should be emphasized that the concepts in no way are tied to extreme value methods, and that there are many other ways to compute them. The examples assume independence between years, but again, the concepts are equally useful for dependent

extremes. How to make direct use of climate model output remains an open question, especially concerning extremes. In particular, realistic projections of extremes are not yet available at the required spatial and temporal scales. But, with the continuous increase in resolution and realism of general circulation models and with advances in downscaling technology, we expect that many such examples soon will appear.

[69] We recognize that the adoption of the proposed concept of Design Life Level would require a shift from more common standard-based to less common risk-based engineering design. But such a shift would be desirable even under a stationary climate [e.g., *Kunreuther et al*., 2013]. At present, the design of water resource structures does sometimes take into consideration future conditions (e.g., projections of increased demand), but not typically any projected changes in climate. Determining the optimal level of the probability of failure may not be straightforward; in particular, requiring an integration of expected costs and benefits over the design life period. Further, engineering design is constrained by political and legal systems that might well hinder adoption of the proposed solution. Nevertheless, the probability of failure is one important input into the very complex processes that are used in the design of water resources structures.

[70] Finally, this paper does not provide the complete and final solution to the problem of risk quantification in a changing climate. Instead, we hope that it will be one starting point for a long overdue discussion.

Appendix A

[71] We here describe how the delta method can be used to estimate the statistical uncertainty in Design Life Level. For this, we change to more general notation and write $\hat{F}(x)$ instead of $\hat{F}_{2011-2060}(x)$; and $\hat{q}(y)$ instead of $\hat{q}_{2011-2060}(y)$; and $\hat{G}_1(x)$ instead of $\hat{G}_{2011}(x)$; $\hat{G}_2(x)$ instead of $\hat{G}_{2012}(x)$, and so on; and also write N instead of 50. Thus, with this notation, $\hat{F}(x) = \prod_{i=1}^{N} \hat{G}_i(x)$ and $\hat{q}(y) = \hat{F}^{-1}(y)$. We further assume that the estimates $\hat{G}_i(x)$ are computed from a vector of d parameter estimates $\hat{\theta} = (\hat{\theta}_1, \ldots \hat{\theta}_d)$ (thus in the Manjimup example $d=4$ and $\hat{\theta} = (\hat{a}, \hat{b}, \hat{\sigma}, \hat{\xi})$) so that $\hat{G}_i(x) = G_i(x; \hat{\theta})$; that the θ-estimates are (approximately) normally distributed; and that we have an estimate $\hat{\sum} = (\hat{\sigma}_{i,j}; 1 \leq i,j \leq d)$ of the covariance matrix of $\hat{\theta}$ at our disposal (for our examples $\hat{\sum}$ was one of the outputs of extRemes).

[72] With this notation, the delta method [see, e.g., *Coles*, 2001, p. 33] consists of estimating the variance $\sigma^2(y)$ of $\hat{q}(y)$ by

$$\hat{\sigma}^2(y) = \left(\frac{\partial}{\partial \theta_i} q(y;\theta)|_{\theta=\hat{\theta}}; 1 \leq i \leq d\right) \hat{\sum} \left(\frac{\partial}{\partial \theta_i} q(y;\theta)|_{\theta=\hat{\theta}}; 1 \leq i \leq d\right)^t. \tag{8}$$

[73] It is straightforward to compute the derivatives in the expression above numerically using a computer algebra program. For this paper, we used Maple, to first compute $\hat{F}(x)$ and then $\hat{q}(y)$ by numerical inversion, by just plotting $\hat{F}(x)$ for a grid of x-values. Derivatives were then computed numerically by making small perturbations of the parameter values, one at a time, and computing approximate derivatives from the corresponding changes of $\hat{q}(y)$.

[74] **Acknowledgments.** We thank four reviewers, an Associate Editor, and an Editor for their comments. We thank the participants of the National Oceanic and Atmospheric Administration/National Climatic Data Center 2008 workshop "Precipitation intensity Estimates in a Changing Climate," the Banff International Research Station 2010 workshop "Extreme Events in Weather and Climate," and the Ascona 2011 workshop "Environmental Risk and Extreme Events" for inspiring comments. Francis Zwiers suggested the concept we term Minimax Design Life level, and Henning Rust and Dan Rosbjerg provided helpful input. We thank Yun Li (Commonwealth Scientific and Industrial Research Organisation, Perth, Australia) for providing the Manjimup data, and Marcus Walter for some preliminary data analysis. Research was supported by the Wallenberg Foundation, the Swedish Foundation for Strategic Research, and the National Center for Atmospheric Research (NCAR) Weather and Climate Impacts Assessment Program. NCAR is sponsored by the National Science Foundation.

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