

I will give a PhD course on *Multivectors, Spinors and Index Theorems* in Gothenburg during the first half of the autumn term 2014. The course literature to be used is a third version of a monograph with the title *Geometric Multivector Analysis* that I am writing. You can find more information on my web page

<http://www.math.chalmers.se/~rosenan/>

The main addition to this third version is a treatment of index theorems for Dirac operators on compact manifolds, and the main goal of the course will be to work through a very down-to-earth proof of these theorems for the Hodge-Dirac (de Rham) operator, and the Atiyah-Singer (spinor) Dirac operator. We will take this as an excuse to properly study some fundamental algebraic and geometric constructions, for which a strange dichotomy exists. On the one hand, they have never entered the basic mathematics curriculum properly despite being 100-200 years old by now. For this reason they are surprisingly unknown to most students of mathematics. On the other hand, these constructions of Grassmann, Clifford and Cartan which we refer to, are used in a rather abstract way in some fields of advanced mathematical research. We will try to bridge the gap between these two extremes. The first and basic construction is the exterior algebra of multivectors: The higher dimensional bivectors and k -vectors beyond the boring one dimensional vectors that everyone knows. We are not going to follow Cartan in focusing algebraically on the dual alternating forms, but build our geometric intuition about multivectors properly. A second construction is the Clifford product of multivectors in an inner product space, for example euclidean space or Minkowski spacetime. This constitutes the real non-commutative higher dimensional analogue of the complex algebra in the euclidean plane. However, algebraically the Clifford product behaves like a matrix product. This leads us to a third basic construction, namely that of spinors, the square roots of multivectors, which are needed for the Atiyah-Singer Dirac operator in particular.

After our study of the algebra and geometry of exterior algebra, Clifford algebra and spinors, we proceed to multivector analysis. For the index theorems, we will need some basic differential and integral calculus for multivectors on manifolds. The Atiyah-Singer index theorem is usually regarded as one of the main achievement of 20th century mathematics, and it is easy to get lost in seemingly abstract mathematics when studying this. As indicated above, we will we have a very concrete goal in this course: For two different Dirac operators we want to prove a formula for the index of this Fredholm operator, which involves an integral over the manifold with an integrand calculated locally from the curvature tensor. My main source of inspiration is the beautiful book "The index theorem and the heat equation method" by Yanlin Yu. Hopefully the course will inspire and well equip you for further studies in this area. Remember: The human being is an expert on generalisation and abstraction, it is the basic idea and the concrete examples which are hard to get!

If you are interested in following my course, please email me at andreas.rosen@chalmers.se your name, email address and position as soon as possible. I will put further information about the course on my web page. Please also feel free to forward this information to friends and colleagues who you think may be interested in participating.