MATEMATISKA VETENSKAPER
CHALMERS/GÖTEBORGS UNIVERSITET
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ORAL EXAM
Multivectors, spinors and index theorems
HT 2014

The oral exam

1. You solve all home work problems.
2. When I have marked all of them, you contemplate over the hints for the problems on the course web page and hand in new solutions for all which are marked 0. If you still feel that you have not found the solution, please feel free to discuss with me. Note: Be sure that you answer ALL questions posed in each problem!
3. When you have passed all home work problems (≥ 1), and have studied the course material well enough so that you orally can account for the theory listed below, please contact me to decide on time and date for an oral exam.

At the oral exam I will ask you account for a few of the items listed below (I make the choice at the exam). You should be able to complete each item in around 15-20 minutes. I will stop you after 20 minutes or so. I will pose some follow-up questions in connection to each item, so you are supposed to be able to account for examples and related theory. Please use my lecture notes as well as the course book in your preparations.

1. The construction of $\wedge^k V$: 2.3-2.6
2. The inner simplicity criterion: 2.27-2.28
3. The algebra of interior products and Hodge stars: 2.55, 2.54, 2.60
4. The anticommutation relation: 2.75
5. The algebra for induced homomorphisms: 2.34, 2.68, 2.69
6. The construction and basic properties of the Clifford product: 3.4, 3.12
7. Cartan–Dieudonné's theorem for euclidean spaces: 4.3
8. Equations for the Clifford cone: 4.6
9. Relations between skew matrices, bivectors, rotations and rotors: diagram on p.89, surjectivity and injectivity details
10. Automatic universality for Clifford algebras: 3.24
11. Algorithm for writing up "Pascal's triangle for Clifford algebras": 3.42
12. Definition of the standard representation of spinor spaces: 5.5
13. Uniqueness up to isomorphism of spinor space (only even dimension!): 5.10, 5.15
14. Duality and conjugation for spinors: 5.16, 5.17
15. Definition and duality for d and δ: 7.2, 7.8
16. Definition and duality for pullbacks and pushforwards: 7.11, 7.18
17. The commutation theorem: 7.20
18. Stokes' theorem: 7.37
19. The Cauchy integral formula for the Clifford–Dirac equation: 9.16
20. Levi-Civita covariant derivative on tangent bundle, exterior/Clifford bundle and spinor bundles: 6.32, 11.2, 11.22, 12.23
21. Definition and formula for the Hodge/Clifford–Dirac operator on Riemannian manifolds: 11.31, 11.5, 11.9

22. Formulas for curvature bivectors Ω_{ij}: 11.15, 11.17, 11.18
23. Hodge decomposition on compact Riemannian manifolds: 11.26
24. Poincaré's theorem: 8.2
25. de Rham= Čech: 8.56 (The analogue of this for manifolds, without boundary conditions. See lecture notes 15.) Not the proof, but formulation and necessary definitions and hypotheses.
26. Definition and properties of the Pfaffian: 12.7, 12.8
27. Chern-Gauss-Bonnet: 12.6, formulation and overview of proof.
28. The orientability and spin structure topological constraints for a manifold: 11.13, 12.21
29. Weitzenböck formula for the spin/Atiyah–Singer Dirac operator: 12.26
30. Atiyah–Singer index theorem: 12.27, formulation, hypothesis and definition of $\hat{A}(R)$, not proof.