Examination in Statistical Image Analysis, May 26, 2010
Course code Chalmers: TMS016, Gothenburg University: Statistisk Bildbehandling
Written examination May 26, 2010, 8.30-12.30
Literature and notes may be brought for this written examination. All types of pocket calculators are allowed but not computers. In the written examination there are two pages and two problems. You are supposed to answer both problems, and in the judgement they have the same weight. Answers may be given in English or Swedish.

## Problem 1.

Figure 1 shows six consecutive images from an experiment used for tracking fluorescent particles and for determining their diffusion properties. The object is first to find particles in each image and to determine the positions of these particles. Particles in the focal plane are sharp and bright while particles away from the focal plane get less and less bright and less and less sharp.


Figure 1: Six consecutive microscope images showing fluorescent particles. The images in the upper row were (from the left) obtained at times $0, \Delta t, 2 \Delta t$ and the images in the second row at times $3 \Delta t, 4 \Delta t, 5 \Delta t$ with $\Delta t$ $=0.04 \mathrm{sec}$.
a) Suggest a method for finding bright particles in the images and to determine their centres as accurately as possible. Suppose for simplicity that each image is treated separately without using the fact that most of the particles in one image can also be found with a minor displacement in the consecutive images.
b) Discuss how one could find the centres also of less bright particles. Again the six images should be treated separately.
c) Discuss how one can check that the particles in one image have centres that are distributed as a Poisson point process.

## Problem 2.

Suppose that we have localized centres of particles in the six images in Figure 1 as described in Problem 1.
a) Suggest a method for constructing trajectories for particles from the positions in the consecutive images. Discuss problems that might occur.
b) Suppose that we have a two-dimensional coordinate system in the plane where the diffusing particles are observed. (Disregard the motion in the verticle direction.) According to diffusion theory the position change of a diffusing particle from one image to the consecutive image in the $x$ - and $y$-directions should be independent and normally distributed with mean zero and variance $2 D \Delta t$, where $D$ is the diffusion coefficient and $\Delta t$ is the time interval between the images. Consider first the trajectory of only one particle. Formulate a statistical model for the data and describe how the diffusion coefficient can be estimated from the data.
c) Assume now that we have observations of $n$ particles for which the trajectories have the same numbers of steps. How should then the diffusion coefficient be estimated? (All particles are assumed to have the same diffusion coefficient).
d) Assume now that particles have been observed with different numbers of steps. How could then the diffusion coefficient be estimated? (All particles are again assumed to have the same diffusion coefficient).

