TEACHING ELEMENTARY MATHEMATICS: THE BIG MATHEMATICAL ISSUES, THE DIFFICULT TEACHING ISSUES

Teachers of elementary students need to understand the big mathematical ideas which are involved in the mathematics they are expected to teach. This understanding leads to a major paradigm shift in what mathematics teachers teach in their classrooms and the manner in which they enable students to learn.

One task of those who are aiming to improve school mathematics instruction is to help our teachers see and understand this big picture, and to help them convert this understanding into practical terms in the classroom.

In this talk, I will focus on such efforts within my own range of experience, and I will connect
this with the NCTM Standards.

Doris Gluck is presently Mathematics Coordinator for Friends School Haverford (a Pre-School through 6th grade Quaker school outside of Philadelphia, Pennsylvania) She is a frequent speaker at state and national NCTM conferences, and runs workshops for teachers and future teachers in area schools and colleges.

Doris was formerly a classroom teacher in grades 2-4 as well as mathematics teacher in grades 7th -10th and freshman year college. She was also a math supervisor for elementary grades in the Radnor Public School District (near Philadelphia).

Doris has a masters degree in mathematics from Temple University and an Elementary Supervisor certification from Arcadia University.

Her interests are in how children make sense of mathematics and in how to help them understand major mathematical concepts through thoughtful questioning.

OUTLINE

June 12, 2006
Chalmers University
Gotenborg, Sweden

INTRODUCTION

As many of you know, I am the wife of Herman Gluck. Sometimes I travel as the spouse when he talks. At other times he travels as the spouse when I talk. However, this is a first–for the two of us to be speaking at the same university.

My undergraduate and graduate work were in mathematics, but I have spent almost my entire professional life involved in math education. First as a teacher, later as a ‘Math Specialist” or “Math Coordinator” (as I am presently called).

I became interested in how young children learn math when our own children were in elementary school and I felt that there must be a better way to help young students learn, and make sense of the mathematics they were expected to know. Completing workbook pages and mimicking teacher recipes did not seem the best way to do this.

This led me to find others who believed as I did and I spent a number of years volunteering in schools , taking courses, reading and developing a better understanding of how children learn.

After a year of teaching mathematics to blind adolescents (Does anyone know how to calculate on an abacus?) I spent the next 25 years teaching in Radnor Township (which is a public school system in an affluent suburb west of Philadelphia) There I was an elementary classroom teacher and Math Specialist. I worked with students needing help in math and also with some of the most precocious math students in the area. I helped choose, write and implement the math curriculum in the elementary grades

At the present time I am dedicated to working with teachers to help them make math teaching meaningful to them and to their students. I do this through workshops for teachers (or students learning to be teachers), and through demonstration lessons in classrooms. In addition, I am the Math Coordinator in a small Quaker school (Preschool through grade 6).

Today I would like to share my experiences and my beliefs on how we can best help teachers assist young children in learning mathematics. I would also like to have time to answer any of your questions and to learn about elementary math education in Sweden.

AGENDA [ove]

INTRODUCTION

OUR RESPONSIBILITY

NCTM STANDARDS

CONSTRUCTIVISM

ACQUIRING OPERATIONAL SENSE

QUESTIONING TECHNIQUES

STATE OF MATHEMATICS EDUCATION IN UNITED STATES TODAY

QUESTIONS AND DISCUSSION

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NCTM STANDARDS

National Council of Teachers Of Mathematics, 2000

CONTENT STANDARDS

- Number and Operations
- Algebra
- Geometry
- Measurement
- Data Analysis & Probability

NCTM STANDARDS

National Council of Teachers Of Mathematics, 2000

PROCESS STANDARDS

- Problem Solving
- Reasoning and Proof
- Communication
- Connections
- Representation

NCTM STANDARDS

CONTENT STANDARDS

NUMBER AND OPERATION

Instructional programs from pre-kindergarten through grades 12 should enable all students to:

-understand numbers, ways of representing numbers, relationships among numbers, and number systems;

-understand meanings of operations and how they relate to one another;

-compute fluently and make reasonable estimates.

ALGEBRA

Instructional programs from pre-kindergarten through grades 12 should enable all students to:

-understand patterns, relations and functions;

-represent and analyze mathematical situations and structures using algebraic symbols;

-use mathematical models to represent and understand quantitative relationships;

-analyze change in various contexts.

GEOMETRY

Instructional programs from pre-kindergarten through grades 12 should enable all students to:

-analyze characteristics ad properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships;

-specify locations and describe spatial relationships sing coordinate geometry and other representational systems;

-apply transformations and use symmetry to analyze mathematical situations;

-use visualization, spatial reasoning, and geometric modeling to solve problems.

MEASUREMENT

Instructional programs from prekindergarten through grades 12 should enable all students to:

-understand measurable attributes of objects and the units, systems and processes of measurement;

-apply appropriate techniques, tools and formulas to determine measurements.

DATA ANALYSIS AND PROBABILITY

Instructional programs from pre-kindergarten through grades 12 should enable all students to:

-formulate questions that can be addressed with data and collect, organize and display relevant data to answer them;

-select and use appropriate statistical methods to analyze data;

-develop and evaluate inferences and predictions that are based on data;

-understand and apply basic concepts of probability.

NCTM STANDARDS

PROCESS STANDARDS

PROBLEM SOLVING

Instructional programs from pre-kindergarten through grades 12 should enable all students to:

-build new mathematical knowledge through problem solving;

-solve problems that arise in mathematics and in other contexts;

-apply and adapt a variety of appropriate strategies to solve problems;

-monitor and reflect on the process of mathematical problem solving.

REASONING AND PROOF

Instructional programs from pre-kindergarten through grades 12 should enable all students to:

-recognize reasoning and proof as fundamental aspects of mathematics;

-make and investigate mathematical conjectures;

-develop and evaluate mathematical arguments and proofs;

-select and use various types of reasoning and methods of proof.

COMMUNICATION

Instructional programs from prekindergarten through grades 12 should enable all students to:

-organize and consolidate their mathematical thinking through communication;

-communicate their mathematical thinking coherently and clearly to peers, teachers, and others;

-analyze and evaluate the mathematical thinking and strategies of others;

-use the language of mathematics to express mathematical ideas precisely.

CONNECTIONS

Instructional programs from pre-kindergarten through grades 12 should enable all students to:

-recognize and use connections among mathematical ideas;

-understand how mathematical ideas interconnect and build on one another to produce a coherent whole;

-recognize an apply mathematics in contcxts outside of mathematics.

REPRESENTATION

Instructional programs from prekindergarten through grades 12 should enable all students to:

-create and use representations to organize, record, and communicate mathematical ideas;

-select, apply, and translate among mathematical representations to solve problems;

-use representations to model and interpret physical, social, and mathematical phenomena.

ALPHABET MATH

Use the alphabet instead of numbers

to solve these problems.

For example,

Are "C" OBJECTS

Do not **change letters to numbers as you work.**

ALPHABET PROBLEMS

- **Tom has *'H'* fish and he wants to buy *'E'* more fish. How many fish will Tom have then?**

- **Maria had *'M'* cookies. At lunch she ate *'E'* of them. How many cookies did Maria have left?**

- **Karen had *'G'* Kroner. She needs *'K'* Kroner for a candy bar. How many more Kroner does she need?**

- **Vilhelm had 'P' candies. He wanted to give them away to 'C' of his friends. How many did he give to each friend? How many extra were there?**

- **Katrina bought a box of Godiva chocolates. The box had B layers. In each layer there were D rows with C chocolates in each row. How many chocolates were in the box? (If Katrina ate all the**

chocolates, how many kilograms did she gain?)

Meaning

Invention

Efficiency

Power

MY PHILOSOPHY

In the United States, most people who go into teaching young children, do not do so because of a love of mathematics. It could be because of a love of reading or writing, it could be simply because of a love of being with young children. (In addition, most of them are female! [Which affects what and how they teach.)

OUR RESPONSIBILITIES AS TEACHERS OF TEACHERS

Therefore, we as educators of teachers have many responsibilities:

- ↓ We need to help these teachers and future teachers understand the core mathematics behind what they need to teach

Liping Ma [in her research in Chinese and American classrooms] has found that teachers with limited subject matter knowledge are unable to promote conceptual understanding even when they want to teach for understanding. However, she also found in her studies, that a teacher's subject matter knowledge may not automatically produce promising teaching methods.

- ↓ and just **as importantly** we need to help these teachers develop best practices for teaching mathematics in the classroom..

The <u>NCTM Standards</u> {2000} states these **5 content areas**: [overhead]

Number and Operation,

Algebra,

Geometry,

Measurement,

Data Analysis & Probability

Although the content of each of these mathematics disciplines IS obvious to you, I would like to spend a moment giving an outline of what they should mean to the classroom teacher. [Description overhead]

NUMBER AND OPERATION

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-understand numbers, ways of representing numbers, relationships among numbers, and number systems;

-understand meanings of operations and how they relate to one another;

-compute fluently and make reasonable estimates.

ALGEBRA

Instructional programs from pre-kindergarten through grades 12 should enable all students to:

-understand **patterns**, relations and functions;

-represent and analyze mathematical situations and structures using algebraic symbols; ***[i.e. generalizing—doubling nos===>2x, function machines]***

-use mathematical models to represent and understand quantitative relationships;

-analyze change in various contexts.

GEOMETRY

Instructional programs from pre-kindergarten through grades 12 should enable all students to:

-analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships;

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-apply transformations and use symmetry to analyze mathematical situations;

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-formulate questions that can be addressed with data and collect, organize and display relevant data to answer them;

-select and use appropriate statistical methods to analyze data;

-develop and evaluate inferences and predictions that are based on data;

-understand and apply basic concepts of probability.

And these 5 **process standards** [overhead]

Problem Solving,

Reasoning and Proof,

Communication,

Connections,

Representation.

These are the core of the ***"how***" where we need to help teachers develop 'best practices'

[Description overhead]

PROBLEM SOLVING [Doing Math!]

Instructional programs from pre-kindergarten through grades 12 should enable all students to:

-build new mathematical knowledge through problem solving;

-solve problems that arise in mathematics and in other contexts;

-apply and adapt a variety of appropriate strategies to solve problems;

-monitor and reflect on the process of mathematical problem solving.

REASONING AND PROOF [Making Sense!}

Instructional programs from pre-kindergarten through grades 12 should enable all students to:

-recognize reasoning and proof as fundamental aspects of mathematics;

-make and investigate mathematical conjectures;

-develop and evaluate mathematical arguments and proofs;

-select and use various types of reasoning and methods of proof.

COMMUNICATION [Discussing or writing about one's thinking]

Instructional programs from pre-kindergarten through grades 12 should enable all students to:

-organize and consolidate their mathematical thinking through communication;

-communicate their mathematical thinking coherently and clearly to peers, teachers, and others;

-analyze and evaluate the mathematical thinking and strategies of others;

-use the language of mathematics to express mathematical ideas precisely.

CONNECTIONS [to other mathematical ideas, and to other disciplines]

Instructional programs from pre-kindergarten through grades 12 should enable all students to:

-recognize and use connections among mathematical ideas;

-understand how mathematical ideas interconnect and build on one another to produce a coherent whole;

-recognize and apply mathematics in contexts outside of mathematics.

REPRESENTATION [modeling, writing,...]

Instructional programs from prekindergarten through grades 12 should enable all students to:

-create and use representations to organize, record, and communicate mathematical ideas;

-select, apply, and translate among mathematical representations to solve problems;

-use representations to model and interpret physical, social, and mathematical phenomena.

When prospective teachers have finished at the university and know the mathematics behind what they will teach and the mathematics they will be expected to teach, they must also know **how to transmit this knowledge to students and help students learn and understand.**

CONSTRUCTIVISM

The NCTM Standards are based on a constructivist philosophy. In other words, they (and I) believe that children construct their own understanding, based on what they already know and understand. And that this understanding is developmental in nature.

For example:

Number and Operation

Subtraction is not the opposite of addition to the 5 or 6 year old.

So why should we emphasize (and have students record) fact families

Thinking multiplicatively does not happen until a child is about 8 years old.

So when should we teach the multiplication facts?

One ten and ten ones are two very different concepts to most 6 and 7 year olds.

How can we teach regrouping in first grade? {Borrowing and carrying]

Geometry

A triangle with the vertex pointing downwards is not a triangle to the young child.

Dina and Pierre van Hiele (Dutch educators) suggested that children and adults go through 5 stages of geometric reasoning and understanding. [ove]

At **LEVEL 0 [VISUALIZATION]**, figures are seen as entire entities, but properties of these figures are not recognized.

At **LEVEL 1 [ANALYSIS]**, students can identify properties of a figure, but not see interrelationships (i.e. a square is a square, not a rectangle.)

At **LEVEL 2 [INFORMAL DEDUCTION**], students can see relationships of properties within and among figures. (A square is also a rectangle, a quadrilateral, etc.)

Most high school courses are taught at **LEVEL 3 [DEDUCTION]** where proofs can be understood.

Those of you who are Topologists or Differential Geometers are at **Level 4 [RIGOR]**, but most of the rest of us are not.

So what should geometry look like in the Kindergarten classroom?

ACQUIRING OPERATIONAL SENSE

I would like to spend a few minutes looking at how children acquire a sense of operation. Play along with me in an exercise I use with teachers. If you stay true to the exercise, you will see for yourself some of the developmental levels young children go through as they construct their own understanding of number and operations.

I am going to ask you to use the letters of the alphabet as counting numbers and to think out the problems I will present you with using only those letters, and not translating back into numbers. [overhead] You can use scratch paper, make tally marks, use your fingers, even take out coins or something from your pocket to help you keep track. [2 overheads]

ALPHABET MATH

Use the alphabet instead of numbers to solve these problems.

For example

Are "C" OBJECTS

Do not **change letters to numbers as you work.**

ALPHABET PROBLEMS

□ **Tom has *'H'* fish and he wants to buy *'E'* more fish.**

How many fish will Tom have then?

{ xxxxxxxx=h, xxxxx=e

h+e= xxxxxxxxxxxxx=m}

Did you count on, or count all? How did you keep track?

□ **Maria had *'M'* cookies. At lunch she ate *'E'* of them.**

How many cookies did Maria have left?

{xxxxx///xxxxxxxx = m

xxxxx = e, left xxxxxxxx= h

m - e = h}

These were part of fact family

Did you notice?

□ **Karen had *'G'* Kroner. She needs *'K'* Kroner for a candy bar.**

How many more Kroner does she need?

{xxxxxxx = g

xxxxxxx/xxxx = k d are the extra

G + _____ = k}

What strategy did you use?

Do you need to say each letter for C or D or do you simply recognize them?

□ Vilhelm had 'P' candies. He wanted to give them away to 'C' of his friends. How many did he give to each friend? How many extra were there?

{x x x

x x x

x x x

x x x

x x x

x

E to each of his friends, A for Vilhelm}

□ Katrina bought a box of Godiva chocolates. The box had B layers. In each layer there were D rows with C chocolates in each row.

How many chocolates were in the box?

(If Katrina ate all the chocolates, how many kilograms did she gain?)

{each layer: x x x

x x x

x x x

x x x

B layers : x x x x x x

x x x x x x
x x x x x x
x x x x x x
d * c * b = x}

Did you skip count?

At the same time, think about how you are solving the problems, because this will actually be the way young children make sense of computation.

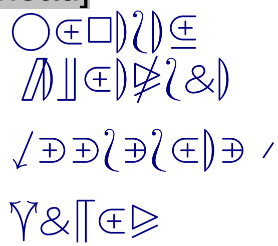
Go over problems and strategies, [overhead]

physical models
counting all
counting on
doubles
skip counting
fact families
finally–recall

I believe it is our job, as teachers of teachers, to help the teachers enable students to make sense of mathematics; to think of mathematics as a 'thinking' curriculum and not just a series of computational tasks to be memorized, or names of figures to be learned.

This can happen when students are taught in a constructivist/developmental.

[overhead]



I believe that using the process standards [overhead again] teachers can help their students achieve this.

Problem Solving,

Reasoning and Proof,

Communication,

Connections,

Representation.

I also believe, that, in order to help teachers do this they must learn how to ask good questions. Questions which will elicit answers that promote a child's thinking.

Vygotsky refers to the 'zone of proximal development'. This is the 'space' between what a child knows and what he is developmentally able to learn.

By good questioning, teachers can encourage (or *gently* push) a child to levels just beyond his present developmental level.

In this 10 minute video you are going to see 2nd grade students are playing a game that they have played before. The students are using base 3 to work with computation and place value. They roll dice and take that number of bottles worth '1'. By trading mentally using individual bottles they accumulate a score.

By asking questions of the group and of individual students the classroom teacher and I are attempting to have them make sense of what they are doing. Notice the different ability levels in the classroom by how the students approach the tasks. Observe the students **'communicating**' [explain their strategies], **'representing'** the numbers [in base 3], '**making connections'** with previous problems, **'problem solving**, Listen to how they **'reason and prove'** their results. [video]

There was a tremendous range of student conceptual understanding and factual knowledge, yet, hopefully they all were able to make sense of the questions being asked.

The first student needed to count by ones and trade each group of three. Alex, the last student you saw was one of the most mathematically precocious children I have ever worked with. His number sense was far superior to the others and he was perfectly able to explain his thinking.

Some samples of what I would call ‘good questions’–open-ended questions are: [overhead] Tell me what you are doing?

Why did you do that?

What would you do if______?

Can you explain___________?

How would you do ____________?

How could you write that?

How could you show/represent that?

I wonder _______________?

How can you be sure that______________?

So you are saying ______________? [revoicing]

Can you explain what __________ was saying in your own words?

How will you begin?

I am not sure I understand. Could you tell me again?

What did you mean when you said/did ___________?

Do you agree with ___? Why?

Would you like to add on to what ______ was saying? [piggy-back]

BE SURE TO USE SUFFICIENT WAIT-TIME.

WHAT IS HAPPENING IN MATH IN THE UNITED STATES TODAY?

What I have shown you today is what I am doing and what I believe in, but I am sure you also would like to know what is happening throughout the United States today.

□ The **NCTM Standards is in its 2nd edition, the first being published in 1989**

□ There is the **No Child Left Behind Act**

< States are required to set up state testing and schools must comply and show improvement.

< These tests are often written quickly and are not statistically reliable.

< They are based on state standards which are not necessarily the same as NCTM standards

< They create a pressure on teachers and administrators to teach to the test and to teach for short term learning.

□ There exists what is commonly called the **Math Wars**

That is between those who feel mathematics education should be about computation, recall and efficiency, vs those who feel math should be a thinking subject, that the process is also important, that math should make sense to each and every child.

- There exists the **‘Common Ground Initiative**’.
  - This is a group that was organized by the Mathematics Association of America (MAA) less than two years ago.
  - Six members, some from each camp in the Math Wars debate
    - Including Jeremy Kilpatrick who has been a guest professor here since 1993.
- There is a newly established National Mathematics Advisory Panel set up to advise Bush and Secretary of Education Margaret Spelings.

(References [ove])

Questions, discussion.

I would love to know more about what elementary math education is like in Sweden and how you are working with teachers here.

VAN HIELE LEVELS
OF
GEOMETRIC THOUGHT

LEVEL 0 [VISUALIZATION]

Figures are seen as entire entities
Properties of these figures are not recognized.

LEVEL 1 [ANALYSIS]

Students can identify properties of a figure
Students can not see interrelationships
(i.e. a square is only a square, but not a rectangle.)

LEVEL 2 [INFORMAL DEDUCTION]

Students can see relationships of properties within and among figures.
(i.e. a square is also a rectangle, a quadrilateral, etc.)

LEVEL 3 [DEDUCTION]

Most high school courses are taught at this level
Proofs can be understood.

LEVEL 4 [RIGOR]

Those of you who are Topologists or Differential Geometers are at this level
Most of the rest of us never reach this level.

STAGES FOR SOLVING COMPUTATION PROBLEMS:

1. **Direct modeling**
   (using physical objects to represent quantities)

2. **Counting all**

3. **Counting strategies**
   - **Counting on**
   - **Counting back**

4. **Derived facts**
   - **i.e. doubles**
     - **skip counting**
     - **fact families**

5. **Recall**

SAMPLE QUESTIONS

Tell me what you are doing?

Why did you do that?

What would you do if______?

Can you explain___________?

How would you do ____________?

How could you write that?

How could you show/represent that?

I wonder _______________?

How can you be sure that______________?

So you are saying ______________? [revoicing]

Can you explain what __________ was saying in your own words?

How will you begin?

I am not sure I understand. Could you tell me again?

What did you mean when you said/did ____________?

Do you agree with ___? Why?

Would you like to add on to what ______ was saying? [piggy-back]

BE SURE TO USE SUFFICIENT WAIT-TIME.

Stages for solving computation problems:

1. **Direct modeling**
   (using physical objects to represent quantities)

2. **Counting strategies**

   Counting all
   Counting on
   Counting back

3. **Derived facts**
   Doubles
   Skip counting
   Fact families

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