

Boolean functions: Homework 1: due September 14

For the homework, the best way to learn is to both (1) think things over yourself and (2) discuss with others. Feel free obviously to discuss with me also. Write down whom you have discussed the problems with.

1. Compute the exact influence for each variable for iterated majority of depth k .

2. Compute a reasonable upper bound (in terms of n) of the influence for each variable for tribes which is $f_{\log_2 n - \log_2 \log_2 n, \frac{n}{\log_2 n}}$.

Remark: The above together with what we did in class essentially gives the influences for 5 of our 6 key examples of Boolean functions. The computation of the influences for the last example (percolation crossings) is *highly!!!* nontrivial.

3. Prove precisely that dictator is not noise sensitive.

4. Prove precisely that parity is noise sensitive. Here there is a nice trick to keep the computations a little less messy. Instead of “flipping” each bit with probability ϵ , it is equivalent to “rerandomize” each bit with probability 2ϵ , where rerandomize means that you then choose, independently of everything else, for the bit to be 1 or 0 each with probability $1/2$. Convince yourself this is the same thing and use it to show noise sensitivity fairly easily.

Remark: We will show (much later on) in class that iterated 3-majority is noise sensitive and that majority is not; the latter is even noise stable, another concept which will be introduced. Of course, you can always also think about how one might prove these things.

5. (a). Find an example of a Boolean function where the events “bit 1 is pivotal” and “bit 2 is pivotal” are independent.
- (b). Find an example of a Boolean function where the events “bit 1 is pivotal” and “bit 2 is pivotal” are strictly positively correlated.
- (c). Find an example of a Boolean function where the events “bit 1 is pivotal” and “bit 2 is pivotal” are strictly negatively correlated.