Boolean functions: Homework 2: due September 28

For the homework, the best way to learn is to both (1) think things over yourself and (2) discuss with others. Feel free obviously to discuss with me also. Write down whom you have discussed the problems with.

1. Show that for majority, any set of size $n^{1/2+\epsilon}$ has influence approaching 1 while any set of size $n^{1/2-\epsilon}$ has influence approaching 0.

2. Show there is a Boolean function which is within .00001 of the dictator function in L_2 (or L_1) but such that the total influence of the function is larger than 1 million.

3. Exercise 1.8 in GS.

4. Let A_n be the event that the first bit is 1 and that the number of 1's in the last $n-1$ bits is even. Show that this sequence is neither noise-stable nor noise-sensitive. Describe the Fourier spectrum in as precise terms as possible. You should see both (1) a nontrivial amount of spectrum 'near 0 and (2) a nontrivial amount of spectrum 'far from 0.

5. Exercise 3.6 in GS.

6. Exercise 3.7 in GS.