Homework 3: Due April 10

(a). You are strongly encouraged to work together as this maximizes the amount of mathematics that you learn.

(b) However, you should write up the final solutions yourselves.

(c) Solutions should (of course) always be correct, clear and concise.

(d) Please also tell me with whom you have worked to figure out the problems.

1. (a). Read Proposition 4.38 on page 116. Let $d = 1$ and let A be the zero set intersected with $[0, 1]$. Explain what the proposition says in this case.

2. Let C be an infinite cone with apex at the origin in 3 dimensions. Fix a ray L starting from the origin disjoint from C (other than the origin). Now, consider the probability that the point on L distance r from the origin hits the cone before hitting the sphere of radius $10r$. Show that this probability doesn not depend on r.

3. Consider our solution for the Dirichlet problem

$$g(x) = E^x[f(B_T)]$$

where f are the boundary values and T is the hitting time of the boundary. Show that g is continuous on the domain.

One possible way to do this is to define a coupling of two BM's starting at near by points.

4. Show that a 2 dimensional BM path is dense in the plane a.s.

5. To show that a BM does not have double points, show why it suffices to show that for any 2 disjoint intervals I and J,

$$P(\exists s \in I, \exists t \in J : B_t = B_t) = 0.$$

6. Read the statement of Theorem 9.23.

(a). Consider a 3 dimensional BM. Let A be the the range of the BM up to time 1 intersected with the x, y-plane. Compute the Hausdoff dimension of A.

(b). Consider a 2 dimensional BM. Let A be the the range of the BM up to time 1 intersected with the x-axis. Compute the Hausdoff dimension of A.

7. (a). Do exercise 1.12 (page 39) but don't hand in.

(b). Challenge problem. Exercise 1.13 (page 39).