Homework 2: Due March 14

You are strongly encouraged to work together as this maximizes the amount of mathematics that you learn. However, the working together stage should only include discussions where you figure out the solutions. The final "writing up of the solutions" should be done alone.

I. Problems for everyone (although 2 might take some work!; in fact, problems 1 and 2 in part II might be easier than this one).

1. Problem 3 on page II-41 of Ferguson's note.

2. Let I and II both have $[0, 1]$ as their strategy spaces. Let the payoff matrix be given by $K(x, y)$ where

$$K(x, y) = -1 \text{ if } x < y < x + 1/2$$

$$= 0 \text{ if } x = y \text{ or } y = x + 1/2$$

$$= 1 \text{ otherwise .}$$

Compute the upper and lower values for this game. Does the game have a value? If you want a hint (and I might think must people would), there is a hint sheet.

II. Further optional problems (which I hope people will think a little bit about). Some of them might require background which you don't have in which case of course skip them.

1. Develop a method for solving the "duel game" in which each player has 2 bullets instead of one. Assume again that the guns are "silencers" so that you can't hear the gun shots.

2. Redo the "duel problem" (with one bullet for each of the players) where the guns are not "silencers" and so you can hear the shot of your opponent. (I have NOT looked at this myself and so have no idea if it is trivial, hard or whatever).

3. Does there exist an example of a game with state space $[0, 1]$ for both players such that there is a unique optimal strategy which is continuous and singular? (Continuous means that every point as 0 weight (i.e., no atoms) and by singular, I mean singular with respect to Lebesgue measure.) (I also have NOT looked at this myself and so have no idea if it is trivial, hard or whatever).

4. This one is somewhat of a tangent but it should be connected to some interesting things. (I also have NOT looked at or thought about this myself at all and so you should also view this with a very open mind and see if there is anything interesting to say). Let player 1's moves be the compact space X (perhaps [0,1]) and let player I1's moves be the compact space Y (perhaps [0,1]). Let G_1 be the set of homeomorphisms f taking X to itself such that there exists a homeomorphism g taking Y to itself such that $A(x, y) = A(f(x), g(y))$ for all $x \in X, y \in Y$. Let G_2 be defined analogously. A mixed strategy μ (i.e., a prob. measure on X) is called invariant (for player I) if for all $f \in G_1$, $\mu f = \mu$ meaning that for all (Borel) sets B $\mu(f(B)) = \mu(B)$. (This is the analogue of an optimal strategy as defined in class for compact spaces.)

It is a fact that invariant mixed strategies may not exist. (Any ideas of an example?). It is also true that if G_1 is (isomorphic to) the integers, then invariant mixed strategies (for player I) do exist. (This is not so easy and related to ergodic theory).

Question: If there exist invariant mixed strategies, is there necessarily one which is optimal?