Homework 3: Due April 2

Same rules as on all homeworks.

Part I (General problems)

1. Let A and B each be two 2×2 matrices given as follows.

31

42

and

26

14

Determine the value of $pA + (1 - p)B$ for each p. Determine the value of this game if player I is told which matrix was chosen at random (and they only play one time). If they play n times (n is large), the matrix is determined at the beginning of the game once and for all, player I is told which matrix is chosen, and the moves but not the payoffs are revealed each game, what would be the approximate value of this game? If I plays optimally, are there any values of p for which player II would eventually be almost certain of which matrix was chosen? For which values of p would II obtain no information at all concerning which matrix was chosen if I plays optimally?

2. For a three person game, a Nash equilibrium is called *strong* if no two players have an incentive to deviate together from the Nash equilibrium. Give an example of a Nash equilibria which is strong and one which is not strong.

3. The game of "chicken" is the following.

$$\begin{array}{cc} (2,2) & (0,3) \\ (3,0) & (-1,-1) \end{array}$$

Determine all Nash equilibrium. Find a convex combination of Nash equilibria which yields an expected payoff of 1.5 to each player. Are there correlated equilibria which yield even a higher payoff to each player? Can you determine the maximum value of s such that there exists a correlated equilibrium which gives each player an expected value of s?

4. Given two $n \times m$ matrices A and B, show that the function $v(p) = v(pA + (1-p)B)$ is a continuous function of p. Any guesses on where it is differentiable?

5. Find all Nash equilibria for the following non-zero-sum 3×3 game.

$$\begin{array}{ccc} (2,-1) & (0,1) & (1,0) \\ (-2,3) & (4,-3) & (4,-3) \\ (-2,3) & (-2,3) & (2,-1) \end{array}$$

Part II (A possibly more difficult problem)

1. Is it the case that given any continuous function $f : [0,1] \to R$, there exist two matrices A and B (of the same size) such that

$$f(p) = v(pA + (1-p)B)?$$