Homework 2: Due November 23

You are strongly encouraged to work together as this maximizes the amount of mathematics that you learn. However, you should write up the final solutions yourselves and tell me who you have discussed the problems with. Solutions should always be clear and concise (and hopefully of course correct).

1. Show that for majority, any set of size $n^{1/2+\epsilon}$ has influence approaching 1 while any set of size $n^{1/2-\epsilon}$ has influence approaching 0.

2. Show that the follow natural converse of Friedgut's theorem is FALSE.

If g is very close in L_2 to a function of k variables, then the influence g can't be so much larger than k.

More precisely and more specifically, construct a Boolean function which is within .00001 in L_2 of the dictator function but such that the total influence of the function is larger than 1 million.

3. Recall that if A is a nontrivial set, its spectral measure is a subprobability measure. There are two ways to get a a probability measure instead. One is to look at $I_A - I_{A^c}$ and takes its spectral measure while the other is to simply 'renormalize' the original spectral measure by multiplying it by a constant. How do these things compare?

4. (EXTRA CREDIT). (I don't know the answer to this and haven't

thought about it. So I don't know if easy or hard.) (This is motivated by a question from Sergei.)

Show that in order to get a set with influence close to 1 with $\omega(n)n/\log n$ bits, it does not suffice (or does it?) to take the set of $\omega(n)n/\log n$ bits with the largest influence.