

Homework 3: Due February 1

You are strongly encouraged to work together as this maximizes the amount of mathematics that you learn. However, you should write up the final solutions yourselves and tell me who you have discussed the problems with. Solutions should always be clear and concise (and hopefully of course correct).

1. Consider the variant of the 'tribes' function where the function is 1 if there exists some block of length $\log n - \log(\log n)$ containing all 1's. Observe this is monotone.
 - a. Show that this is a noise sensitive sequence. (The point of this example is to obtain another monotone noise sensitive sequence besides 'iterated majority' which we studied in class and besides percolation crossings.)
 - b. Show however that it is impossible to prove noise sensitivity using the algorithm theorem in the notes that were handed out.
2. We showed in class that the 'iterated 3 majority' function is noise sensitive by a recursive argument using elementary facts about different types of fixed points for 1-d mappings. Give 2 other proofs that this (sequence of) functions is noise sensitive using things we have learned (or are about to learn) in class.
3. Let A_n be the event that the first bit is 1 and that the number of

1's in the last $n - 1$ bits is even. We showed in class that this sequence is neither noise-stable nor noise-sensitive. Describe the Fourier spectrum in as precise terms as possible. You should see both (1) a nontrivial amount of spectrum 'near 0 and (2) a nontrivial amount of spectrum 'far from 0.

4. Assume that $\{A_n\}$ is a noise sensitive sequence. (This of course means that the indicator functions of these events is a noise sensitive sequence.)
 - a. Show for each $\epsilon > 0$, we have that $E[I_{\omega \in A_n} | \omega] - P(A_n)$ approaches 0 in probability. (Hint: use the Fourier representation.)
 - b. (Extra credit) Can you show this implication without the Fourier representation?
 - c. Discuss if this implication is surprising.
 - d. Show that the condition in part (a) implies that the sequence is noise sensitive directly without the Fourier representation.