A Monte Carlo simulation

Suppose we are interested in a lognormal mean $\mu = e^{\nu + \tau^2/2}$, and have independent observations $y_1, \ldots, y_n \sim \text{LN}(\nu, \tau)$.

It’s difficult to calculate an uncertainty interval for $\mu$. Instead, we resort to simulation. Here is the Monte Carlo procedure:

1. Calculate $x_i = \log y_i$ for $1 \leq i \leq n$

2. Calculate the log scale summary statistics $\bar{x}$ and $s^2$

3. Repeat the following a suitable number of times
   (a) Draw independent $r^2 \sim \chi^2(n - 1)$ and $z \sim \text{N}(0, 1)$
   (b) Calculate $\tau^2 = \frac{(n - 1)s^2}{r^2}$ and $\nu = \bar{x} + z \frac{\tau}{\sqrt{n}}$
   (c) Calculate and store $\mu = e^{\nu + \tau^2/2}$

4. Analyse the stored $\mu$’s (calculate percentiles, mean, mode, rank correlations, etc)
A Monte Carlo simulation

Suppose we are interested in an accident rate $\lambda$ (per year), the proportion $p$ of accidents in which alcohol is involved and the expected rescue cost $\nu$ of such accidents. The yearly cost has mean

$$\mu = \lambda p \nu$$

We furthermore have data: during the last five years, there has been 13 accidents, and in 3 at least one driver have been under the influence of alcohol. The rescue costs of these 3 accidents were: 416, 349, 328 kSEK ($\bar{x} = 364, s = 46.4$). Calculate a credibility interval for $\mu$.

Solution: We calculate reference posteriors for $\lambda$, $p$ and $\nu$, and simulate the yearly mean $\mu$. The reference posterior for $p$ is beta$(0.5 + f, 0.5 + n - f)$, where $n = 13$, $f = 3$.

The reference posterior for $\lambda$ is given by $\frac{\chi^2(2x + 1)}{2t}$, where $x = 13$, $t = 5$.

The reference posterior for $\nu$ is given by $\bar{x} + t(n - 1)s/\sqrt{n}$, where $n = 3$, $\bar{x} = 364$, $s = 46.4$.

Here is the MC-procedure:

1. Repeat the following a suitable number of times
   
   (a) Draw independently $p \sim \text{beta}(3.5, 10.5)$, $r^2 \sim \chi^2(27)$, $t \sim t(2)$.
   
   (b) Calculate $\lambda = \frac{r^2}{10}, \nu = 364 + t \frac{46.4}{\sqrt{3}}$

   (c) Calculate and store $\mu = \lambda p \nu$

2. Analyse the stored $\mu$'s (calculate percentiles, mean, mode, rank correlations, etc)