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Laboratory work on the topic
Random walks and electric networks

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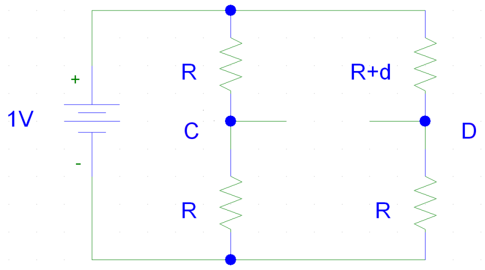
An important family of measurement instruments are based on so called bridge circuits. They are used to measure or compare resistors or impedances. Look up the Wheatstone bridge, Maxwell bridge and Kelvin bridge in encyclopedias for further information. William Thomson was given the title Baron Kelvin due to his achievements in science.

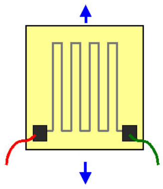
A common measurement technique utilizes the Wheatstone bridge. The potential difference between C and D can be calculated as $V_{CD}=-d/R$.



The resistance (R+d) can be realized as a wire. The electrical resistance of a wire of length L and cross-sectional area A is given by $R = \rho L/A$ where ρ is the resistivity of the wire material.

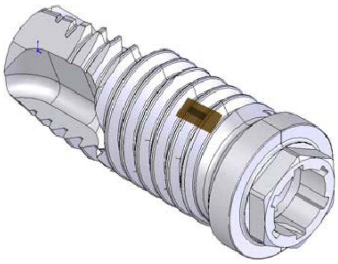
The electrical resistance of a wire changes with strain:
• As strain increases, the wire length L increases thereby increases R.
• As strain increases, the wire cross-sectional area A decreases and R increases.
• For most materials as strain increases the wire resistivity ρ also increases which further increases R.

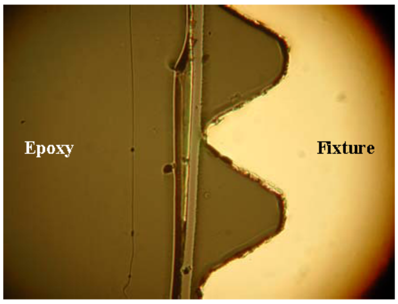
One way to construct a strain sensor is let the strain act on a small diameter wire (actually an etched metal foil) which is attached to a backing material (usually plastic). The wire is looped back and forth several times to create an effectively longer wire. The longer the wire is gives larger the resistance and the change in resistance would be larger when the strain increases. A sketch of a typical strain gauge is shown below.[1]



[1] Simulations and measurements of strains around dental implants using strain gauges and Finite Element Methods (FEM) R. Kumm

A typical case where is strain is of interest is the strain in the titanium screws used for dental surgery. Below the gauge is placed on the surface of the screw, sketch and real microscope image.





Place the gauge as the (R+d) resistor in the Wheatstone bridge, and the strain can be directly measured as the potential difference between C and D.

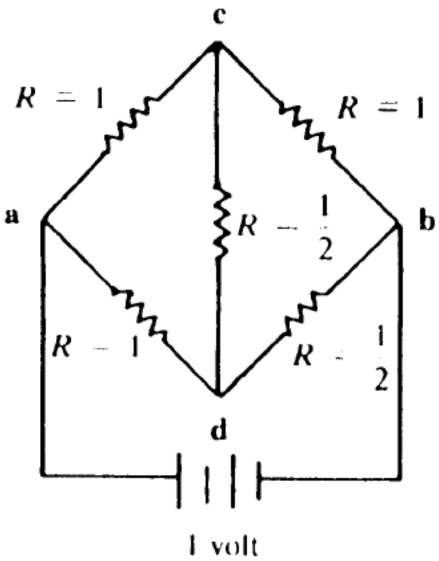


Figure 1, a bridge circuit.

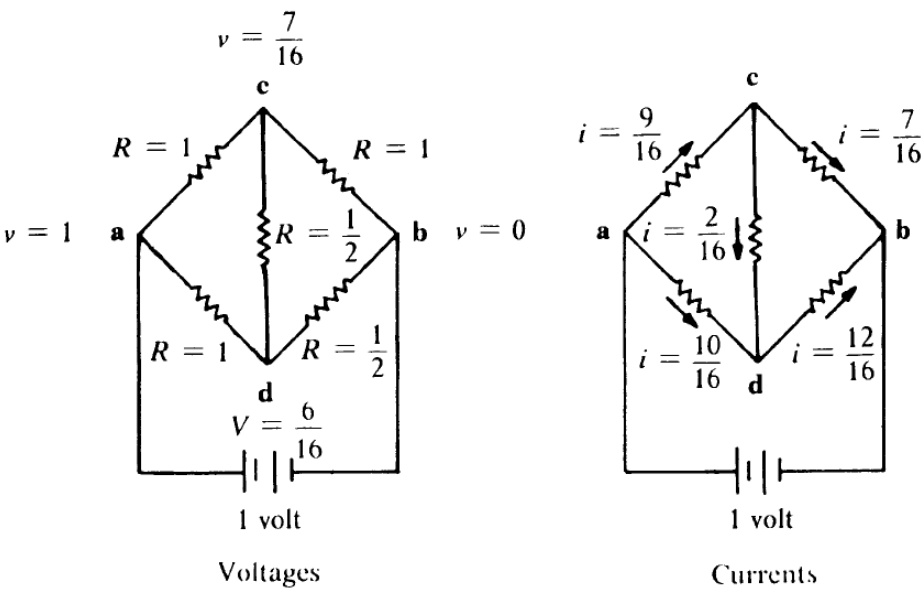


Figure 2, node potentials and currents in the branches.

Realize the circuit on the board and in Pspice, see figure 3.

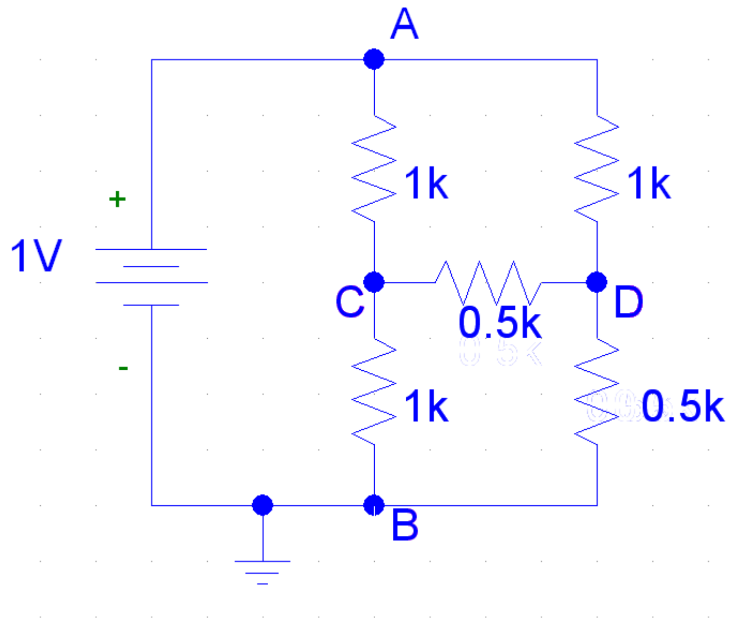


Figure 3, one possible realization of figure 1 in Pspice

Confirm the branch currents and node voltages.

Notes:

Read chapter 1.3.3 Probabilistic interpretation of current

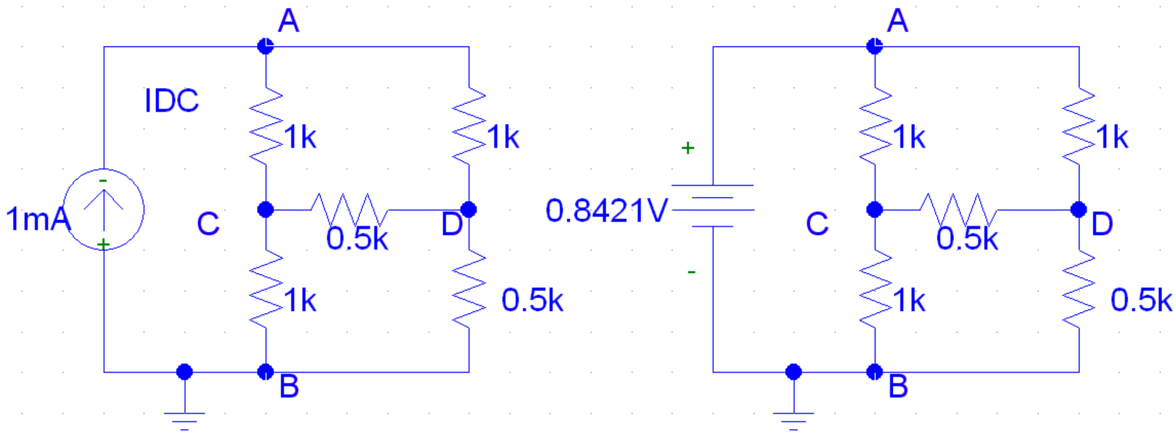


Figure 4, a realization of the unit current flow, using a current source or a voltage source.

1mA current source or voltage source of 16/19V gives a unit current flow (unit is here 1mA).

Notes:

1.2.1 An example

We turn now to the more complicated problem of a random walk on a two-dimensional array. In Figure 5 we illustrate such a walk. The large dots represent boundary points; those marked E indicate escape routes and those marked P are police. We wish to find the probability p(x) that our walker, starting at an interior point x, will reach an escape route before he reaches a policeman. The walker moves from x = (a, b) to each of the four neighboring points (a + 1, b), (a − 1, b), (a, b + 1),(a, b−1) with probability 1/4 . If he reaches a boundary point, he remains at this point. The corresponding voltage problem is shown in Figure 6. The boundary points P are grounded and points E are connected and fixed at one volt by a one-volt battery. We ask for the voltage v(x) at the interior points.

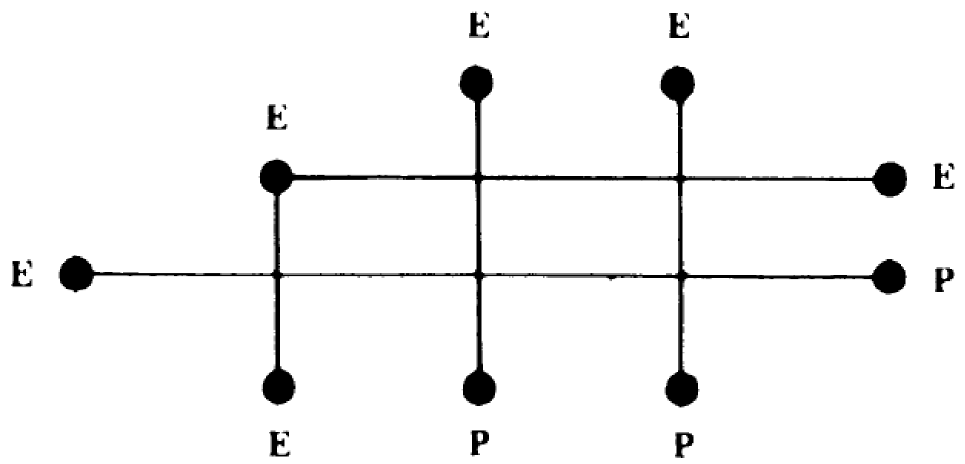


Figure 5

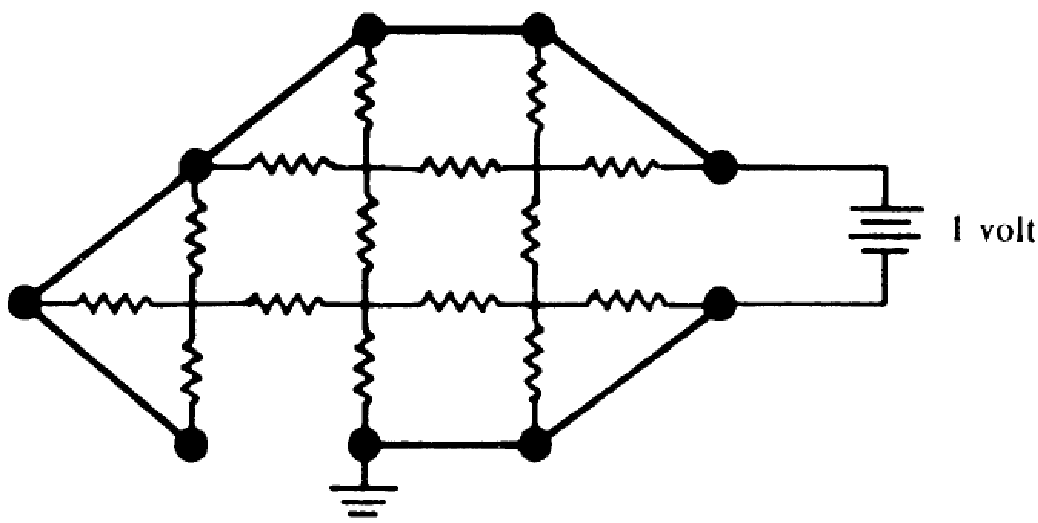


Figure 6

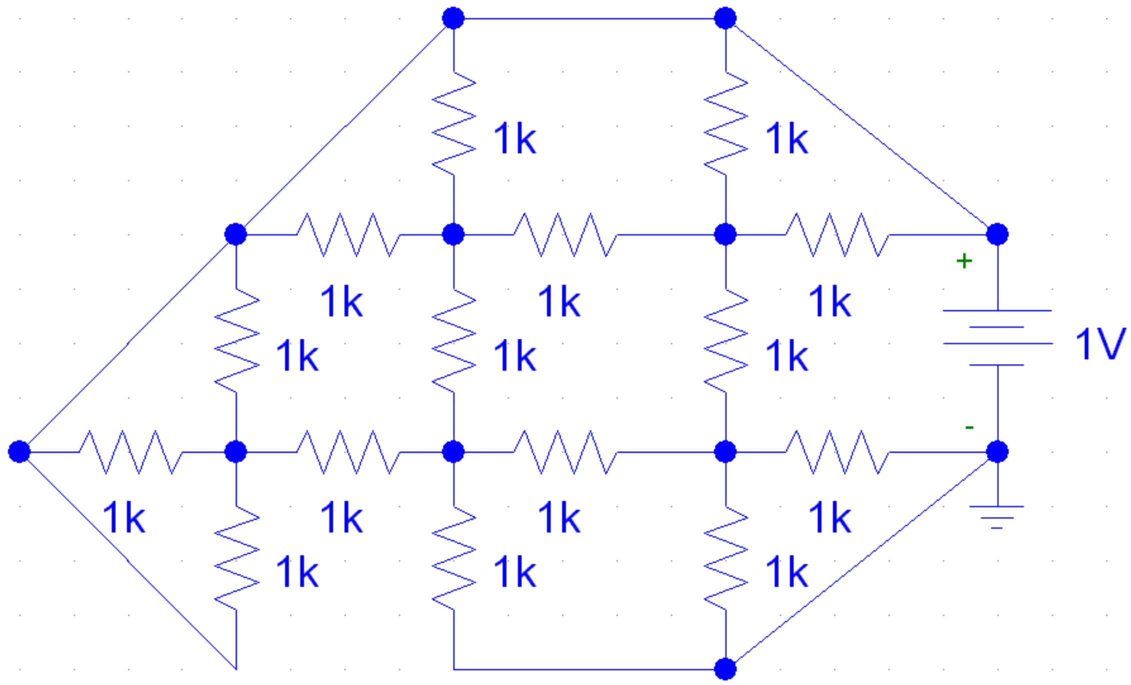


Figure 7, one Pspice realization of figure 6.

Realize the circuit in figure 7 on the board and in Pspice. Measure relevant values.

Notes:

Exercise 1.3.5 For the *Ehrenfest urn model*, there are two urns that together contain N balls. Each second, one of the N balls is chosen at random and moved to the other urn. We form a Markov chain with states the number of balls in one of the urns. For $N = 4$, the resulting transition matrix is

$$\mathbf{P} = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{ccccc} 0 & 1 & 2 & 3 & 4 \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ \frac{1}{4} & 0 & \frac{3}{4} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{3}{4} & 0 & \frac{1}{4} \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \end{array}.$$

Show that the fixed vector $\mathbf{w}$ is the binomial distribution $\mathbf{w} = (\frac{1}{16}, \frac{4}{16}, \frac{6}{16}, \frac{4}{16}, \frac{1}{16})$. Determine the electric network associated with this chain.

Exercise 1.3.8 Consider the Ehrenfest urn model with $N = 4$ (see Exercise 1.3.5). Find the probability, starting at 0, that state 4 is reached before returning to 0.

Realize the circuit in exercise 1.3.8 on the board and in Pspice. Measure relevant values

Notes:

Read the text related to figure 43 in the textbook, reproduced as figure 8 below in the lab instruction.

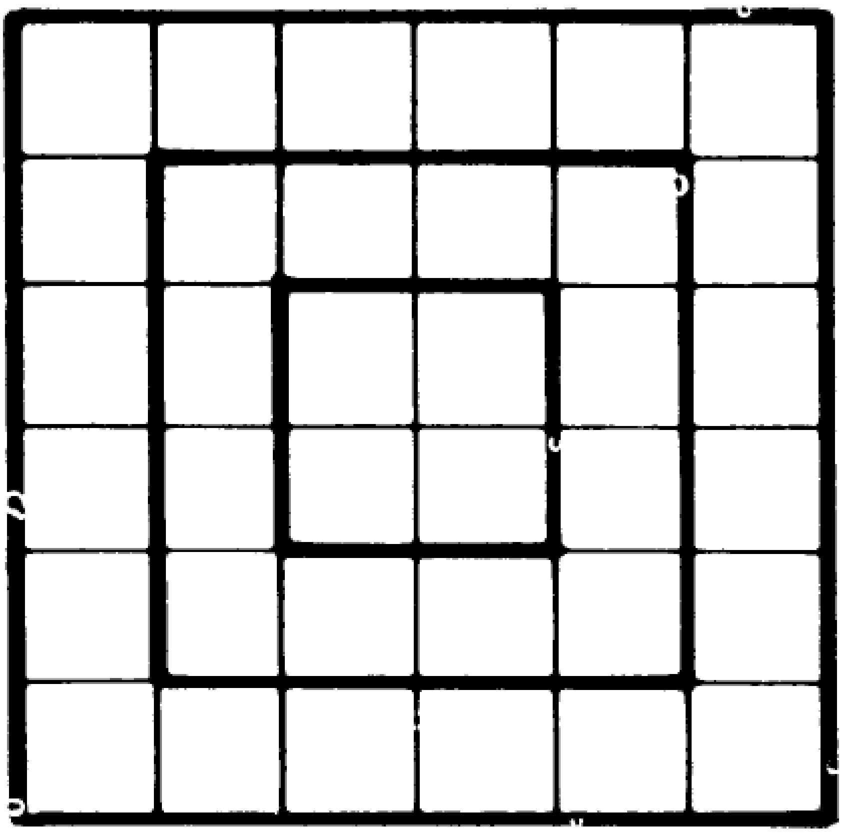


Figure 8. Same as figure 43 in the textbook.

To make the assignment more easy, the problem has been simplified to figure 9.

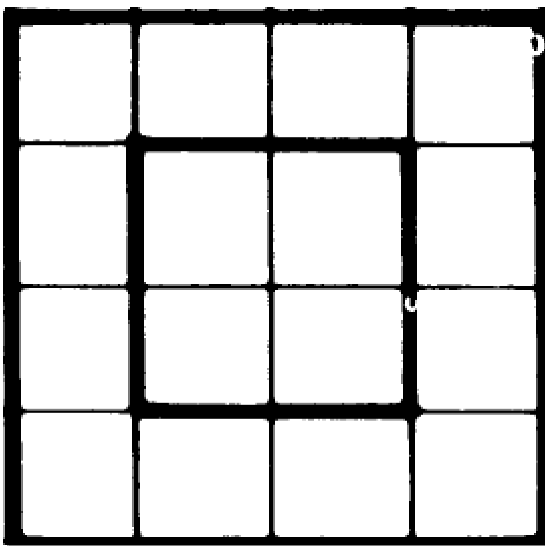


Figure 9. Simplified smaller version of figure 8.

Realize figure 9 as the circuit in figure 10 on the board and in Pspice. Measure the current from the power supply.

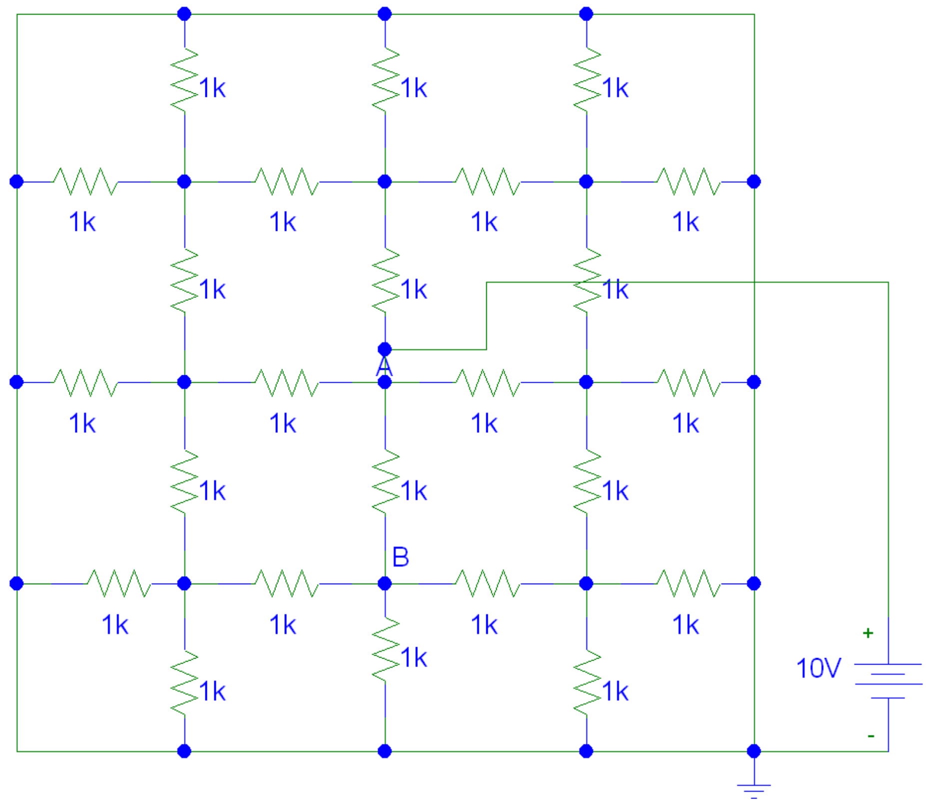


Figure 10, original circuit.

Notes:

Shorten the nodes as in figure 11. Is the current from the power supply changing?

Note differences between the board and Pspice.

Measure current flow in the shorts.

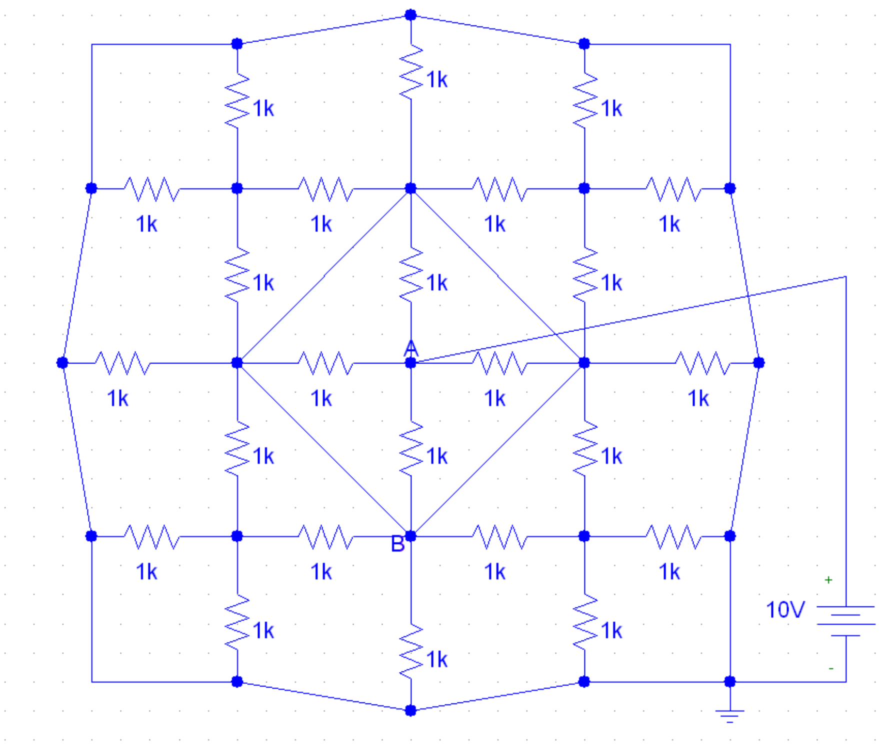


Figure 11, shorten nodes B.

Notes:

Shorten the nodes as in figure 12. Is the current from the power supply changing?

Note differences between the board and Pspice.

Measure current flow in the shorts.

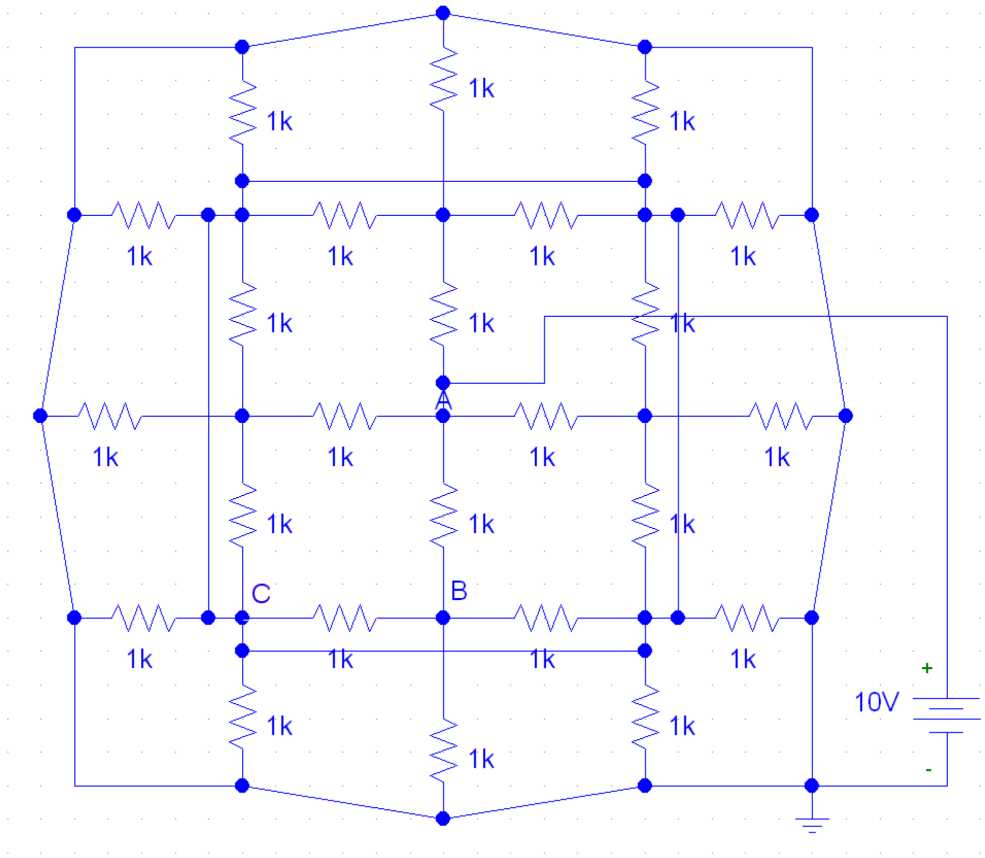


Figure 12, shorten nodes C.

Notes:

Shorten the nodes as in figure 13. Is the current from the power supply changing?

Note differences between the board and Pspice.

Measure current flow in the shorts.

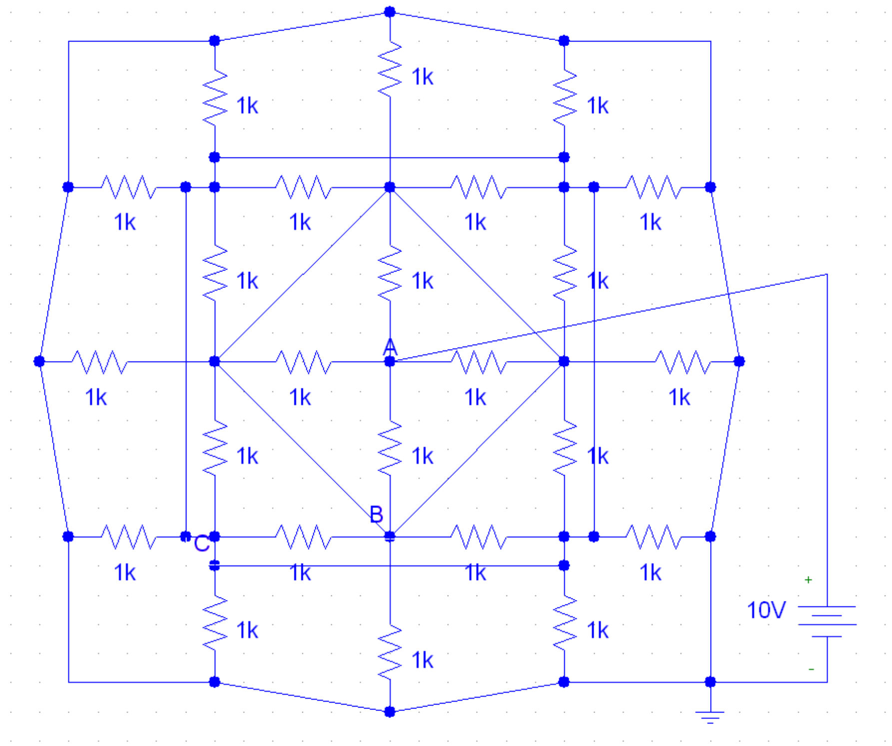


Figure 13, shorten nodes B and C.

Notes:

Now realize that, if the total current from the power supply is of interest, the circuit in figure 10 can be simplified as is figure 14 and 15. From figure 15.4 the current can be calculated directly.

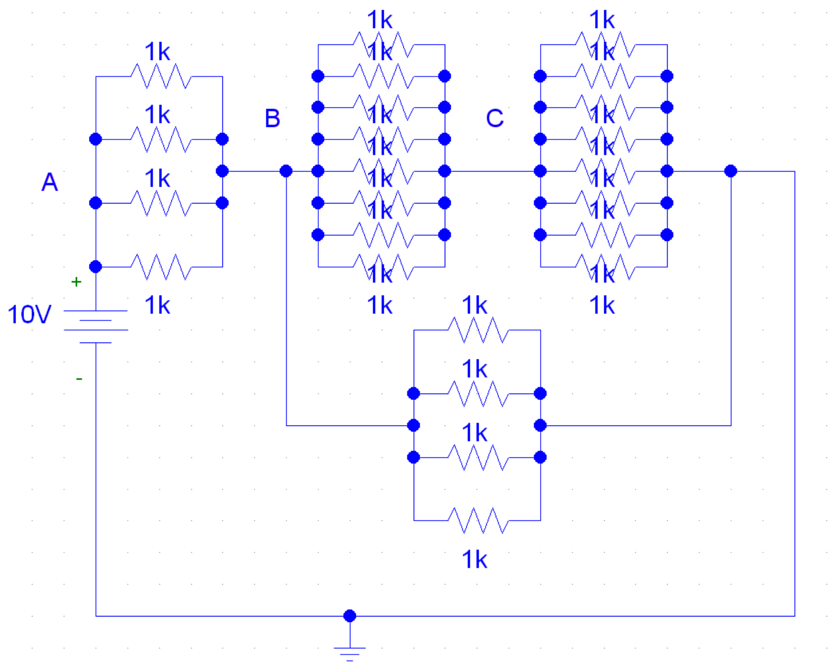


Figure 14, another view of figure 10.

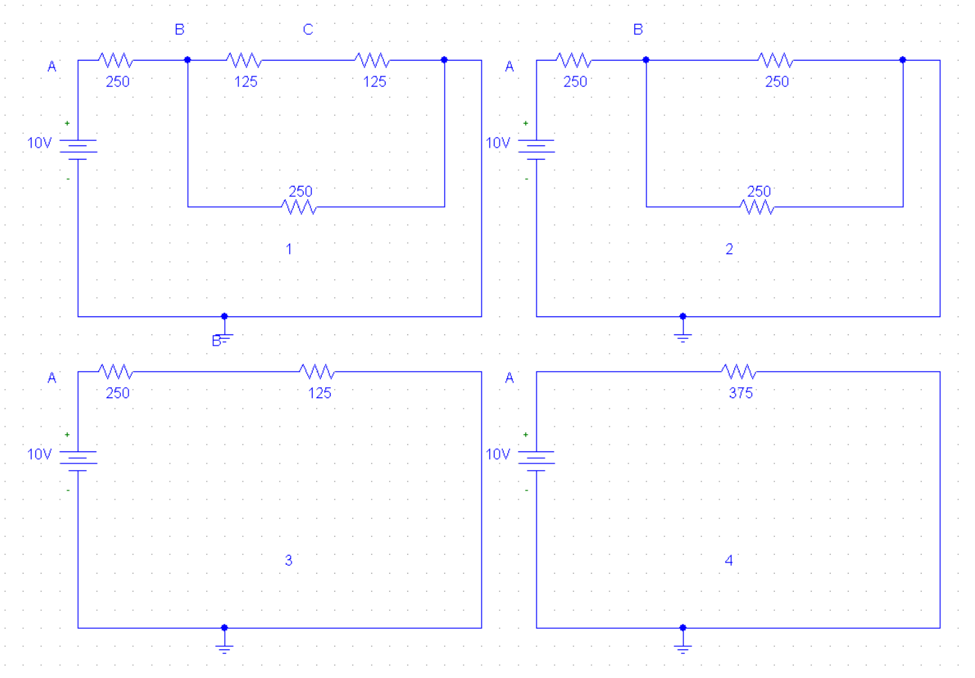


Figure 15, simplification of figure 14 in steps 1-4.

Notes:

Current = ………………..

Add another short as in figure 16. How does the current change and why?

Measure current flow in the new short.

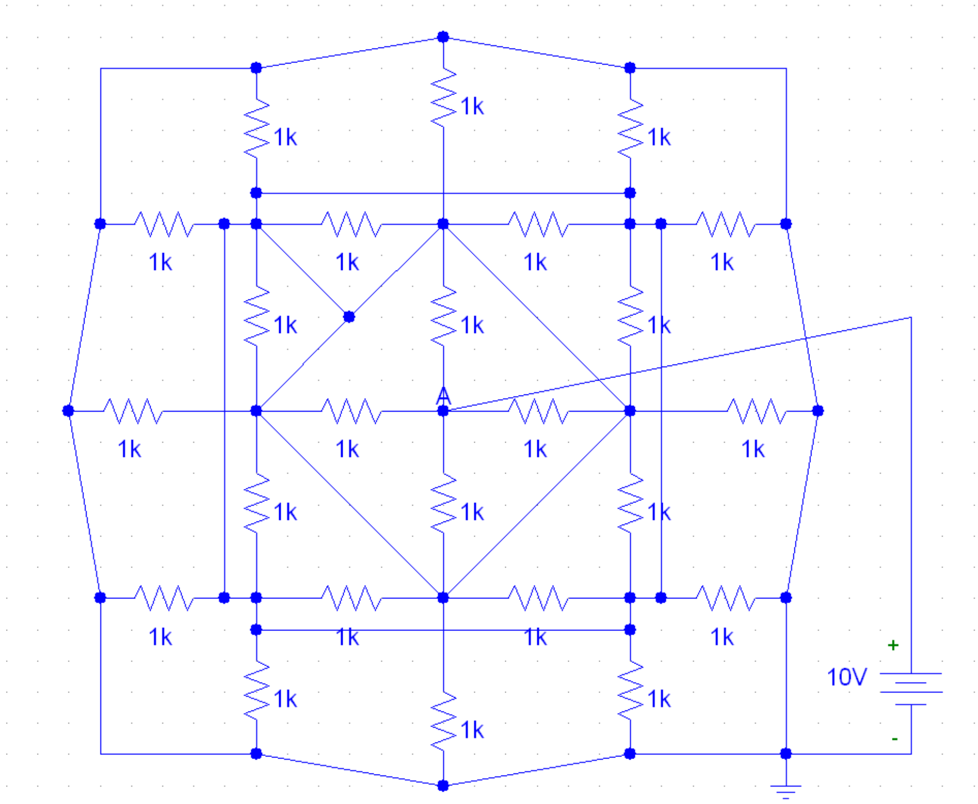


Figure 16, shorten circuit.

Notes:

Verify that the circuits in figure 16 and 17 are equal, and make a simplification as in figure14 and 15.

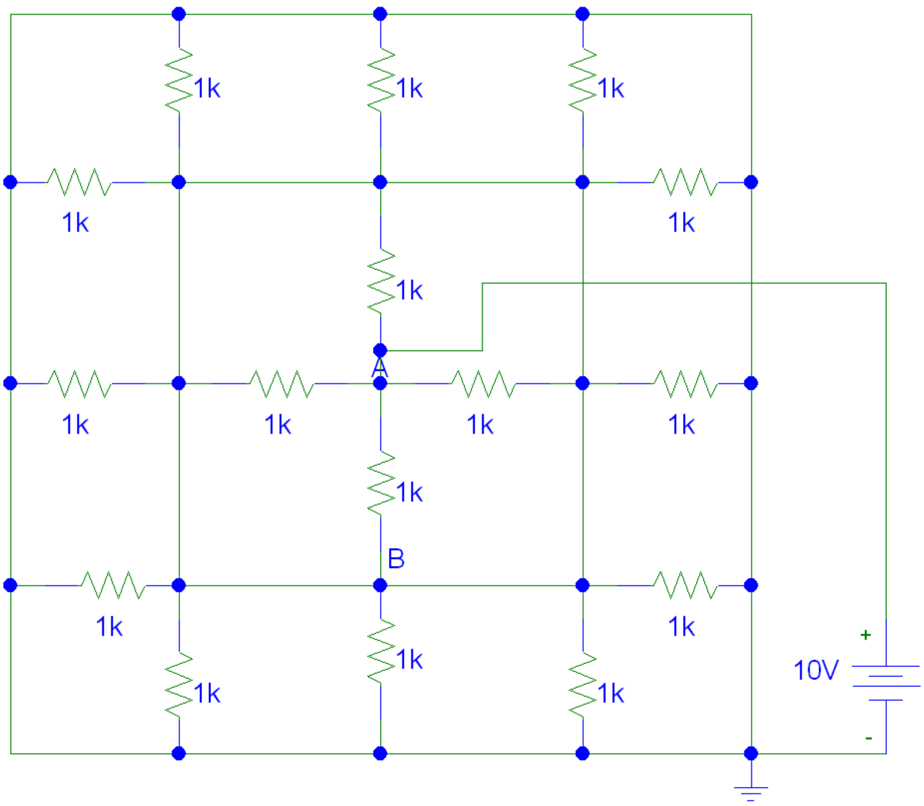


Figure 17, practical shorten circuit.

Notes: