A Critical review of

Orchestrating a Mathematical theme

The paper under review is a technical paper on pedagogics, whose aim is to contribute to the science of pedagogy, and thus not addressed to the layman. Nevertheless the layman may have a few opinions.

The point of departure is the following claim.

A common claim when mathematics learning is discussed. is that pupils can master mathematical tasks more easily in groups than individually, and that their understanding of the mathematical content can be enhanced through the contributions of others

There is a reference to this claim (Ernest, 1991) as well as various reference for its theoretical underpinnings, dropping names as Vygotski and Piaget. The so called micro-social processes involved in peer interaction have been studied, with references added, and those have stressed that peer interaction can enlarge and enrich children's reasoning. However, that statement, taken as face value, is rather general and vague, and in fact, one would surmise that almost anything *could* enlarge and enrich children's reasoning.

Now the general claim is too far-reaching to be settled definitely by one paper and one study, and this can certainly not be the aim of the authors. Instead it seems that this general claim acts as a guiding vision, rather than something that has been settled, and the paper wants to address one aspect of peer-interaction, namely how *possible variations of thoughts are restricted in interaction, and how such restrictions can help the pupils to gain a deeper understanding of a mathematical content.*

To do this study, four groups of children are observed. To document this observation, one has to resort to video-taping. A complete transcript of such videotaped sessions would be too long to be included, and a synopsis is imperative. The authors provide such a synopsis, on the basis of their pedagogical expertise, a reader who would find their conclusions suspect given the material, could in principle ask for the full transcript, (and/or be moved to perform a similar experiment).

The bulk of the paper is taken up by the synopsis, along with interspersed comments, and the end of the paper is devoted to a final discussion, as well as a presentation of tentative conclusions. The format of the paper thus adhers formally to that of papers in Natural Science, in which there is a presentation of the problem, a discussion of the methods, a report on the experiment, and a concluding section discussing the result. Such a format would be typical for a paper in Biology, but not in Mathematics (which is not a Natural Science). In a Scientific paper the emphasis would be on the methodology, which would be of major interest to a potential reader (who would know of the problem, and feel confident to draw his or her own conclusions).

To start out with the theoretical set up. The general creative process can be summarized by the conflict between enlargement and restriction. This is beautifully illustrated by the process of Evolution, which involves both variation and restriction. Variation, which in Evolution is provided by chance. (Mutation on the level of genes, and random recombinations of genes, caused my sexual mating). And Restriction which is caused by Natural Selection, which is the weeding out of organisms through the interaction with the environment. Taken in general terms the Darwinian Vision is almost tautological. It can be applied to almost anything.[1] This has lead to a variety of various misunderstandings of the concept, from the turn of the century Social Darwinism of the 'survival of the strongest' to various elaborations on the theme of Kiplings 'Just so' stories in 'Psycho-Darwinism' at the end of the millenium. Darwins emphasis was on Natural Selection, he had no understanding of the basic mechanism of variation, which would not be brought in until the discovery of genetics some decades after his death. The beauty of the idea was on 'intrinsic creativity', there was no need for an outside Creator. Darwins temperament was not philosophical, his delight was that of the Naturalist in documenting the various developments. His Vision though provided the Biological Sciences with a wider 'raison d'etre' and has guided it ever since.

Darwinism provides a powerful metaphore for many phenomena outside the realm of biological speciesfication (i.e. based on DNA encoding). Models for the development of the immunesystem, as well as for the development of the brain through the 'testing' of neurological pathways are 'Darwinian' in nature. There is

[1] When I first encountered it I was stunned by the fact that simple beautiful ideas, could also arise in non-mathematical subjects.

even a Darwinian method of programming. In a more general context it has been popular to consider the development and propagation of culture in Darwinian terms[2]

The authors do not invoke the Darwinian metaphore, and this may be quite wise, as to make a metaphore fruitful you need to make it more precise. It is not clear how variation is produced in a group-discussion, as it would be naive to think of it as produced by chance, some kind of random mutation of ideas.[3] More interestingly though is to study how ideas are restricted. Why are certain ideas rejected? Rejecting an idea, means more than just throwing it out, it also means throwing away all its possible ramifications. Rejecting something involves a big risk, namely the cutting off of an entire branch of inquiry. On the other hand, due to the exponential growth of ideas, you have to restrict yourself. Had Evolution continued unchecked without Natural Selection, there would not have been space for all the various organisms in the Universe.[4]

The metaphore that the authors invoke is instead that of 'Orchestration'. This appears to have arisen in Literary Criticism, and initially refers to the structure of novels. In order to make the metaphore do some work, one really should have some deeper understanding of music, in particular symphonic works. I doubt that the authors possess such technical knowledge, at least they give no indication that they do (and one may wonder whether even the initiator - Bakhtin, does), instead they use it in the rather superficial way of refering to a diversity of voices.

However, personally I would find a discussion of how variation as well as restriction occur more rewarding than the 'orchestrial' description. In fact the authors spend more time on this, than they do in exploring the metaphore of the orchestra.

As to variation, they refer to the concept of *reflective variation* (a standard terminology) as describing how pupils vary the interpretation of a task, or part thereof.[5].

As to restriction, they involve the notion of 'scaffolding'. What this really means is not completly clear from the paper, (nor the origin of the term and what it really has to do with 'scaffolds') but I guess it refers to the explicit presence of a 'guide' an 'expert teacher' who gently steers the thinking away from what would be inappropriate. Thus 'scaffolding' means that there is a Selector. (A God so to speak) In the groups in questions there are no experts, the discussions are allowed to proceed spontaneously. The point seems to be, that some kind of 'scaffolding' can appear naturally (Natural Selection). The crucial question is what kind of constraints are applied. In Scientific inquiry, this is done by empirical testing. What are commonly agreed on tests, that the pupils adhere to?

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[2] More appropriate would be 'Lamarckian', as in culture we have in fact the transmission of acquired characteristics. It should be emphasized that the 'fact of the Evolution' is different from the 'theory of Natural Selection'. Darwin was not the first to present the inconstancy of species, although in retrospect he has been identified with the view, what he did was to suggest a different mechanism than those of his predecessors, a mechanism that has been far more fruitful scientifically, and thus in a 'Darwian' sense, assured his 'survival.

[3] In fact random mutations might not be such a bad model to follow sometimes when you are stuck

[4] Malthus description of the inevitable contrast between arithmetic growth (of resources) and exponential growth (of progeny) did supposedly have a very strong influence on Darwins thinking. In fact it is, from the perspective of the history of ideas, intriguing to note the intellectual influence of the late 18th century school of Economics (Adam Smith etc) on the birth of Evolution. However, it would be not only wrong but downright dangerous to draw the conclusion, as Marxists would be tempted to do, that Evolution (and by extension all scientific theories) are just manifestations of prevalent Economical conditions. Apart from the circularity of the argument, one may point to the example of Lysenko, to illustrate the vulgarism of such opinions. Finally Evolution is a result of checking Economic Growth, but what is denied existence, is often encoded in enhanced complexity of what survives, so in fact it would be tempting to view Evolution as sustained exponential growth in complexity. However, there is nothing in the general nature of Evolution that would ward off the possibility of future degeneration into simplicity. Evolution is a kind of drift, it has no goal, and in particular the development of Human intelligence, wonderful as it may appear to us, is not to be thought of its crowning achievment.

[5] The authors write that *One form of reflective variation is* variation in meaning, however it is not clear, how this differs from the more general concept, excep for the element of 'dynamism'. On the other hand, to the concept of *variation in meaning* is added *elaborative variation* which seems to tack on this dynamic aspect

This is illustrated by the problem of Instruction. It can either be seen as the problem of the Instructor, to replace fallacious ways of thinking with more appropriate. Or to be seen as the problem of the instructees, namely to become aware of on their own, what is appropriate and non-appropriate ways of thinking. In the paper under review, the latter process is obviously the one of study.

The method is simple. A specific task is given to the students, and their reactions to it are documented through video-taping. Their utterances are then interpreted in an intentional way, as a means of trying to get a grip on what they really think. However, the authors are careful, they confine the descriptions to what is observable. But the real interest, nevertheless, lie in 'guessing' what the pupils 'really think'.

In this context it might be interesting to ask why 'groups'. Would not something similar, albeit on a slightly smaller scale, be going on in a single individual? I guess that the study of groups rather than individuals may be guided by the choice of available methods, although the authors do not say so explicitly, in fact I even suspect that they may not be aware of it. It is much harder to document the 'thought processes', (or rather the symptomes of it), as we as individuals are not used to produce a running commentary on our own thoughts. Although we are all aware of the interior monologue, I suspect it to be much more fragmentary and inarticulate than we are ordinarily aware of. It is only in writing and speaking to others, we make our thoughts articulate (or at least try seriously to do so). The different voices of the participants thus come for free, as those are automatically produced in any social interaction involving transmission of thought and persuasion of argument. What lies behind the production of each individual utterance is of course a mystery, the individual is a black box, and we know him only through his pronouncements, which may, however, not always be seen as oracular. What we have is in fact a cataphony of voices, different statments, which can be the subject of study, and whose object its is to fit into some 'orchestrial framework'.

Given the context of the study, it seems to me to be premature to claim that students in groups can perform better than individually. In particular that the collective orchestration of voices, leading to a variation of the mathematical theme, can enable them to exceed their initial understanding, in a way they would have been unable to do on their own. It has once been claimed by a well-known Swedish mathematician, that in group work, the norm to consider is not the integral norm, but the sup-norm. In other words that the performance of a group is only as good as that of its 'best' member. All the pupils do not add up, the contributions are not independant, but most subsumed in the most dynamic of its members.

Like all sweeping statements, this is naturally open to all kinds of obvious objections. To say that cultural progress is a collective endeavour is of course a 'truism'. No single mathematician, no matter the depth of his genius, would have been able to create all of our mathematics. Different people think differen thoughts, and even the proverbial blind hen, may think of something no one else has ever done. (The random production of variation). We also know empirically from joint work in mathematics, that it is not just an addition of individual contributions, but an interweaving of insights, providing ladders, transcending the capabilities of either author. (To give an example. A mathematical reasoning, involves a chain of deductions and insight, the removal of any link, breakes the chain. If various links are supplied from different authors, no part of the link would have been conceived individually.)

However, in the study the authors set up, it is not clear whether the 'best' students in each group may not have arrived at the solutions on their own, maybe even faster, not being distracted by other voices. But it is difficult to check this proposition, as controlled experiments are notoriously difficult to envison, let alone execute, in complex circumstances. The tacit task of the author is not really to investigate this question, but only to describe the processes that lead to a solution, as they are more transparent, or at least amendable to observation, in a group setting. But the basic view of the authors color their interpretations. They make little effort to identify the various voices systematically, as to decide whether each group member contributes equally, or whether some group members are more active and 'constructive' than others. Instead they view the group as an entity, and consider the individuals, as opposed to individual statements, as irrelevant. This may also explain their tendency to view the result as a confirmation of the general claim that group discussions are beenficial to students.[6] To make such claims scientifically sound one needs to

[6] That it is, is of course a kind of truism again. If the performance of the group as a whole, is reflected on each of its members, the 'weaker' of them will naturally benefit. This makes sense in a strictly 'examinational context'. The pertinent question is whether they also individually have benefited. Even if they have taken a peripheral part, the illusion of active participation may nevertheless be beneficial, as the 'instruction' they

study the growth of variation, as the group enlarges[7], as well as studying the efficiency of restriction.[8].

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received, may be seen as self-generated. Also, it is a common claim, that explaining something to somebody, enhances your own understanding, even if not that of the ostensible subject of instruction. Thus the good, leading student of the group, may paradoxically very well benefit the most.

[7] This is subtle, as too big a group radically changes its character

[8] It is not only a case of effective restriction, but also relevant. Could it be that restriction could work worse in a group situation, due to some kind of perverse peer pressure? Also the same could apply to variation. Due to the 'sociology' of a group, individual members may be more or less inhibited to suggest ideas