Platonism can mean many things even in mathematics, thus many of the divisions, not seldom quite bitter, among mathematicians as to the proper philosophy of mathematics are more apparent than real resulting from answers to different questions, not different answers to the same question. (Divisions among philosophers are of course legion, and in fact they appear to thrive on them as in their absence there would be very little of at least academic philosophy). But what are the salient features of Platonism especially in mathematics?

First and foremost Platonism is part of metaphysics. This by itself makes it suspect among scientists because one of the dicisive components in the success of modern science is to divorce itself from metaphysics. (On what one may not speak one should remain silent). The ultimate stand against metaphysics was formulated in the Positivist program of the Vienna Circle in the 20's and 30's. Their aim was to weed out nonsense statements and questions to which category all the metaphysical ones belonged in order to retain the hard and empirically founded ones. This was a culmination of a tendency in philosophy to become more pragmatic, not to inquire into the roots of things but to explore their consequences (often their practical ones). Among its proponents one should mention the Americans C.S.Peirce and W.James (who actually was a mediocre philosopher but a brilliant psychologist and as such had a lot of influence) and above all the foremost philosopher of science during the 20th century - K.Popper, known for his falsiability criterion. Now the latter kept himself aloof from the positivists, who tried to claim him as their own. The point is as the British philosopher Collingwood remarked that you cannot evade metaphysical standpoints. The very act of rejecting metaphysics is a metaphysical stand. Popper accused the positivists of being so called 'uncritical rationalists'. The very basis of their program could not be empirically based and should thus be rejected as metaphysical. In short the belief in rationalism must be based on faith, science cannot be scientifically proved. Anyone who claims that he can rationally prove everything is making some tacit assumptions he cannot include in his discourse. Popper did thus not reject metaphysics but admitted to certain metaphysical beliefs himself, such as the existence of an external reality, that this reality is rationally ordered as to be amenable to our rational study (Pierce talked about a congruence between mind and cosmos), and that although we can never achieve certainty and thus all our beliefs are provisional and liable to be refuted in the future (thus Russells quip that it is not the case that we believe in religion and know in science as is generally believed, but the other way around), they are nevertheless approximations to a fundamental Truth to which they are converging asymptotically.

Thus among the metaphysical questions we can mention such as Solipsism, Free Will, the existence of God (whatever that is), the duality of mind and matter. And in this context the essence of Platonism is that abstract entities exist, that the material reality we know is but a manifestation of an abstract underlying one, to which we miracolously have some connection to through the immortality of our souls. This view has obvious theistic overtunes, which would make most modern people somewhat embarrassed. And indeed it is quite common that people express views which are essentially Platonic and excuse themselves for that very reason, yet as I will argue below, it is not quite as outlandish as one may think if one takes it too literally.

Now one cannot argue about metaphysics in the same way we argue in science or mathematics. In the metaphysical realm science, poetry, religion somehow merge, and we argue through evocation and metaphor. The task to 'prove' Platonism through a rational argument is as fruitless as to disprove Solipsism, propose Free Will (if the world is deterministic maybe we are destined to come up with irrefutable arguments for free will?) or God; thus in order to make it a proper subject of discourse, we should not get fixated on what Popper terms 'What is' questions but on more pragmatic 'How' questions, following the pragmatic lead of seeking clarification rather than faith.

The notion of a (timeless?) 'essence' is a very natural one, a problem arises with it only if we are trying to formulate that 'essence' explicitly rather than leave it implicit. Formulations are like material manifestations of ideas. To take a few examples. Human language is one such. It has been argued very convincingly that the capacity for language learning is innate and that all human languages share common features, that they are in fact just various manifestations of a universal blue-print all subject to a universal grammar. The difficulty to explicitly formulate such a universal grammar is formidable and efforts to do so have so far failed. This does not mean that there is none, only that it is probably too abstract to be properly pinned down. Thus we can only learn language in the concrete, i.e. by picking up a particular language, we cannot learn language in the abstract. (This is something that is obscured in modern pedagogy in which the learning of principles are extolled at the expense at the learning of concrete facts. The point is that the learning of seemingly meaningless facts is the clue to grasp something far more evasive, just as the learning of meaningless sounds is the clue to learning a language and with it the power of thought such a skill brings with it.) In the same way mathematical ideas are 'materially' embodied in formulations of theorems and proofs, and we only understand something by catching this evascence through a lot of verbiage. But can we not cut out this 'verbiage' and leave the underlying meaningful idea bare? This is like trying to cut away the right-hand part of piece of plywood in order only to have one part left with no right-hand part at all.

Finally thoughts are, at least for the philosopher, the most intimate things, more intimate than proverbial chairs and tables. All reflections start in thought. Thoughts are about things and are complex entities indeed. A thought cannot as thought be dissolved into simpler parts, but of course a thought can be thought about just as anything else, and then rendered into a thing itself, and as such be analyzed into its constituencies. But then of course it is then no longer a thought and in addition somehow external to you, even if it was once yours. Materialism is about constructing the world bottom-up, including your own thoughts (as things) involved in the process. Now can thoughts be communicated and reconstructed? One may think of thoughts as containing two parts, a private, incommunicable one concerned with what is usually refered to as 'quale', and an objective communicable one. The mysterious 'quale' make up consciousness, and many philosophers argue that it will for ever be beyond scientific understanding even if in principle it should be an emergent feature of neurological activity. The communicable part will likewise be neurologically encoded, but such are just manifestations, and the thought itself somehow exist beyond its various manifestations, just as the thoughts expressed in a book do not reside in the various printed copies of it. In Platonism there is an understanding that the objective component of thoughts are directly related to immutable Platonic forms. (Thus while the 'redness' of a rose may just be a subjective impression and as such incomparable with that of others, more abstract notions such as equality and numbers can be shared.)

Most working mathematicians are naive Platonists. By naive Platonism means a belief in the objective and independant existence of basic mathematical concepts, such as the integers. (Pierce suggests that the integers make up a more basic concept than logic itself and hence every attempt to base integers on logic is bound to be very contrived and ultimately fail.) Even while idealized mathematical concepts like points and lines as well as infinite sets may not be physically realized they do obey laws of their own which we can only ignore at our peril. Furthermore although mathematics is very varied the different parts are beautifully interlocked and confirm each other. Mathematics presents a very solid edifice to those who get to grips with it. It is this external and immutable aspect of mathematics that makes your devotion to it meaningful. If mathematics would just be a convention the magic would disappear and there would be no point in dealing with it. Furthermore not only can mathematics model the real world, the physical world can also provide a model for mathematics. The modern super-computer is a good example of this, making the spirit into flesh. Abtuse mathematical reasoning can actually be computationally checked turning mathematics into an empirical science (or at least some important parts of it) and thus mathematical hypothesises can be subject to falsification.

Philosophers have tried to come up with some explanations of the remarkable solidity of mathematics. The standard solution is to see mathematics as a formal game consisting of long chains of tautologies. This is a very mechanized view of mathematics, exploiting the fact that mathematics (like all sciences to some extent) involve very intricate reasoning, far more than would ever enter into metaphysical speculations. So intricate indeed that they often go beyond the grasp of an individual, who may be able to understand each logical step but not the conclusion itself which may come as a total suprise. Elementary examples of such extended and externalized reasonings are calculations. The result of a calculation is like an experiment, it is independent of the wishes of the one who performs them (an unending source of frustration not only to matematicians). Formalizing mathematics deprives it of all meaning, but in the process a new meaning occur, namely that of formalized systems and their consistencies (the consistency of an axiomatic system to be viewed as a formal definition of truth). Such questions can be encoded into numbers and hence into questions of numbers, i.e. number theory. It is now well-known to a wider circle than mathematicians that this was exactly what Gödel did, drawing his conclusions using an elaborate diagonal principle, mixing the formal with the meaningful metatheory of the formal. The philosophical verdict seems to be that Platonism constitutes the belief that any mathematical system that does not contain contradictions exist. This is a rather mechanical view of Platonism, and Platonism in mathematics should also involve another ingredient - intuition. It should be kept in mind that both Gödel and Hilbert were mathematical platonists, and the formal approach to mathematics that Hilbert initiated, was not a reflection of an austere conception of mathematics, but was simply a tool to once and for all set mathematics on a firm basis so that the real project of discovery could start unimpeded by doubts and unintended pitfalls. Hilberts project foundered on the rock of Gödels theorem, the negative conclusion of which has been taken as an indication that mathematics cannot be formalized (much to the dismay of many logicians) and hence strengthening the case for platonism.

One should make a distinction between mathematics and the practice of mathematics. The latter is undeniably a human activity subject to the wonders and pitfalls of such. The historical development of mathematics, the coming and going of its fashions, what is important in mathematics and what is beautiful, all of those are very human concerns, and it would be rash to attribute those to any eternal Platonic truths. But nevertheless intuition provides, maybe more in mathematics than in any other human endeavor, a mystery that is inexplicable. A reductive explanation of intuition is that it is merely interpolative, that it is a result of an extensive experience allowing the conscious suppression of intermediate reasoning. This is the kind of intuition that any professional mathematician is capable of, and would in my mind more be of the nature ot so called 'tacit knowledge' to be found in skilled craftsmen. What I am thinking of are the great leaps of thoughts that have been taken in mathematics by its most inspired practioneers and which have revolutionized the subject. Those cannot be explaiuned merely in terms of interpolation and experience, although both are necessary. The idea that those transcendent intuitions somehow are directly linked to a Platonic realm is of course a very attractive one, or at least a very romantic one. It all of course hinges on what 'is linked to' really means, and a reasonable Platonic stand would be to assert that this could be made more precise without involving superstition.

Now creative activity in general, not only in mathematics is hard to explain, and it might be the case that it will turn out to be impossible. If such limits of human understanding would be the case, the rejection of the dualism of mind and matter would always remain a metaphysical stand and never be subject to an empirically based demonstration.

The most well-thought out and articulate case against Platonism in mathematics has been presented by Reuben Hersh. He claims that mathematics is objective as far as the individual is concerned but subjective as far as mankind is considered. Such a distinction ties in with Poppers claim that sociology cannot be based on psychology, that in fact societies have a continuity that goes further back in time than the evolutionary development of the human brain, and that hence that the equipment with which a newborn is given at birth, is not limited to the biological. (Incidentally it would make a neurological explanation of mathematics not only infeasible but also impossible in principle.) Examples of such phenomena are many. Language is one, morality another, neither of which does not makes sense in an extra-human setting. As individuals we have to comply to the usage of language, we cannot set it ourselves. What is considered as beautiful language is a social convention not a personal choice. The same thing obviously goes for morality as well. Yet in all the examples Hersh proposes serious problems occur when you try to probe too deeply. Those does not occur in mathematics. In mathematics there is a remarkable consensus as to what is correct or not, and in disputes the losing party do not experience defeat but enlightment, the issue being felt as impersonal. Also, unlike other human cultural pursuits, mathematics shows a remarkable consistency over anthropological borders. In spite of much nonsense about ethno mathematics, 1+1=2 in all cultures.