The Mathematics of Great Amateurs

J.L.Coolidge

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According to Goethe the amateur differs from the professional by trying to do things that go beyond his capacity, and maybe even being impossible. The amateur is supposed to love what he does, while to the professional it maybe a chore. To the amateur what he does is a form a phantasy, while the professional deals with the real world and hence his vision may be less extensive and more narrowed, or if you prefer, more focused.

The author would not classify Fermat or Cayley as amateurs, although they derived their income from non-mathematical work, in both cases being part of the legal profession. But he does classify Pascal as one, although Bell includes him among the Great Men of Mathematics. In fact this points to another spurious line of demarcation, an amateur of mathematics might be highly gifted as a mathematician, in fact more so than most professional mathematicians, and Pascal is clearly a point in case here, having an inborn talent far superior to that of most mathematicians. But Pascal fails, in the eye of the author, to have been a professional, because his contributions lay elsewhere and his real passion and strength of intellect were not devoted to mathematics, which he mostly thought of as a distraction from the pains of the real existential questions which tortured him. Anyway, the general distinctions the author makes are not really convincing, the real value is to be found in the different case studies, of which there are sixteen, and each of them illuminating the dichtonomy between amateur and professional from different points of view.

To start from the beginning. Plato held mathematics in very high regard, as actually is common with most philosophers, the real ones as well as the fakes (such as the modern French variety), but the relationship was not reciprocated, as is quite common when to comes love for mathematics. I use to refer to Plato as the patron saint of mathematicians, but for all his appreciation no mathematical result of any significance comes from his mind. He could appreciate mathematics but not do it, which is also the case with music. Most people can emotionally appreciate music but not create it, for this something else is required, and it is not just a matter of motivation and hard work. The same in mathematics. So here we have one distinction, the professional mathematician can deliver, the amateur cannot. This distinction applies to Plato as opposed to Euclid, but it does not apply to Pascal. As to Renaissance artists they were of a very different temperament from the modern ones, and the theory of perspectives, which mathematically is one of studying projections from three-dimensions to two, is as just noted a purely mathematical theory. The author brings out three Renaissance artists of importance, namely Pietro (dei Franceschi), da Vinci and Dürer (one could also include Brunuleschi, who is traditionally credited with having discovered the theory of perspective in the modern tradition[1]).

[1] The principles are so easy so it is hard not to believe that the discovery must not have made repeatedly in human history, especially by the Greek where the subject of conic sections is a study of projections; but

Of those three Pietro is the most serious mathematician, enunciating principles of solving mathematical problems connected with it in a synthetic way, making the occasional mistake, which is also considered the hallmark of an amateur. It is worth noting that the mathematical principles become quite pronounced due to the prevalent architecture where right angles and straight lines predominant, giving rise to a wealth of vanishing points which with their rays of lines dominate standard perspective drawings. A favorite subject of the mathematically minded artist is the floor alternatively tiled with black and white tiles[2] When it comes to foreshortening of the human body, this is of course as well mathematically determined, yet with the absence of straight lines and right angles, it is much harder to ascertain, yet an artist worth his salt (i.e. a professional) is expected to master it, and they way he does that is not that easy to make explicit as in the case of the proceedings of a Pietro. Now da Vinci comes out as the weakest of the three. An artist of remarkable skill and accomplishment he was also no mean scientist and engineer, manifesting his skill of observation with the ability of drawing, thus single out core principles[3]. Da Vinci being a man of unlimited curiosity it is inevitable that he would encounter mathematics and he did some of things of substance, yet in the end so much of his mathematical activity degenerated into mere doodling being fascinated by lunes, i.e. figures made out of circular arcs, for which it was possible to compute exact areas (due to hidden cancellations). Thus, although blessed with a certain mathematical facility, he never really understood what mathematics was about, thus being insensitive to its inner structure and what constitute an important mathematical statement. But he were many things, and it would be ungenerous to fault him for not being a mathematician as well. Once again we have here another aspect of the distinction between amateurism and professionalism as it applies to mathematics. This is also illustrated by Dürer, who certainly was fascinated by mathematics, and in the chapter his obsessions with helics is discussed, as well as the practice of representing three-dimensional objects by their projections onto two planes, typically intersecting perpendicularly. Dürer apparently believed that conic sections must be egg-shapes having more curvature closer to the apex. This is a natural belief, I recall I succumbed to it myself when I was first alerted to conic sections, based on visual imagination. And here we have another difference between a mathematical amateur and a professional, the latter is not content with relying on visual senses alone, and mathematics is if anything, a systematic attempt at transcending sensual imagination.

Of all mathematical amateurs the contributions of Napier are by far the most basic and influential. It is of course impossible to believe that had not he lived and invented them, somebody else would have invented them soon enough, so central they are in mathematical thought. Yet if inventing the logarithms may not be such a feat of mathematical originality, it is for that very reason an extremely important one. Thus amateur as he may have been,

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maybe the crucial factor is whether it would have been considered interesting and relevant, the subject of painting been more ritualized expression than scientific exploration.

[2] In fact the standard perspective drawings along with the matching architecture is so ingrained in our cognitive facilities that it can produce very strong illusions of people changing sizes as they move in a space set up with a false perspective.

[3] An intelligent drawing is far more instructive than a photograph, although the latter may be more objective and detailed, but it is this very lack of discrimination that damns it for the purposes.

and maybe not remarkably mathematically gifted, the contributions he made surpasses those of what most professional mathematicians can ever dream of. The most interesting thing about his discovery was that natural logarithms came first, it was only Briggs (a more professional mathematician?) who turned them into base ten, so much more convenient for numerical work. Napier's logarithms, of course commanded most of his time, as it involved lengthy computations of tables, yet it was subservient to other activities, none of which has merited the attention of posterity, in contradistinction from Pascal, whose work on philosophy and theology (which also occupied Napier) is far more known than his mathematical.

Pascal has many elegant theorems on his conscience, many of which seem to be lost, yet what is known as Pascal's theorem, concerning a characterization of those hexagons that can be inscribed in a circle and discovered at the tender age of sixteen, almost makes him a Mozart of mathematics and did open up a new avenue of exploration, be it that the first tender footsteps had already been taken by Pappus in Hellenistic times. But Pascal was no Mozart, because Mozart was not just a musical prodigy but a professional from the start, which saved him for music. Pascal was not a professional, maybe his mind was too large and impatient, and he never fulfilled his initial promise, in spite of scattered flashes of isolated brilliancy. In fact his most enduring accomplishment as far as mathematics is concerned is his, together with Fermat, lying the foundations of probability theory, for which no talent of that of either Pascal or Fermat is really required.

Pascal is of course very well-known, and the mathematics presented in the book, his stands apart. But Antoine Arnauld, a close friend of Pascal, may be less known. He corresponded with both Descartes and Leibniz although at different stages of his life obviously, but seems to have been more interested in philosophy than mathematics, and he was also above else a combative theologian yet believing that truth can be learned not only on authority (the Bible) but on observation and reasoning. The author judges his contributions mostly for their pedagogical value. One was a treatise on reasoning, i.e. logic, and he makes a distinction between analysis and synthesis, ideas very close to that of Plato, although he makes no reference to him. To prove that A descends from B you can either list all the children of B, and their children and so on, and then search for A in the list. Or you can look at the parents of A, and their parents, and look whether B will appears in the list. But he also produced a more mathematical work, a treatise on geometry, inspired by an initial attempt of Pascal, on whom he thought he could improve, aghast at the confusion due to an unnatural order in the presentation. Thus the author lauds him as taken the first steps to move away from Euclid's canonical presentation. Another obscure amateure that has been unearthed is the Dutch lawyer and statesman Jan de Witt. His main contribution was to study curves kinematically, and the author delves into some detail (this is not the book for the mathematically faint-hearted). Finally he also dabbled in annuities, a very practical subject. This made him look into the mathematics of mortality, a somewhat morbid subject, and thus the probabilities connected with it. Brouncker an Irosh peer profited from his intimacy with Wallis. He worked out a continued fraction expansion of $\frac{4}{\pi}$ but the author notes that for somebody with such material and moral priviliges, and with such connections to the best scientists at the time, his contributions are in fact very modest. However, the French marquise l'Hospital was of another calibre, and every beginning student in mathematics encounter him through his epynomus rule. As a curiosity can be mentioned the curve $y^5 = x^3$ which plays a role in modern singularity theory, but which looks quite smooth to the naked eye, at least when one only considers the real part (and what else can you look at literally?). Then there is the great 18th century naturalist Buffon, whose monumental *Histoire naturelle* overwhelms with its wide erudition. Buffon with such an appetite would naturally turn to mathematics as well. His most well-known mathematical work is of course his needles thrown at a floor with equally spaced boards giving an approximation of π. Thus his interest was chiefly probabilistic, as testified by his clear analysis of the St-Petersburg paradox and the nature of expectation in indefinite games. His compatriot and contemporary Diderot took to mathematics for the same reason, namely his omnivorous taste as an Encyclopaedist. He was clever and did some good things in his youth, working on involutes and pendela. Bell makes fun of him as a matehatical ignorabus, in connection with an anecdote involving him, Euler and Catherine the Great, the later annoyed by his atheism. The story is obviously false, Diderot was as noted quite accomplished as a mathematican. Finally the philosopher Bolzano is treated, and that is yet another name made familiar to mathematical beginners thanks to Bolzano-Weierstrass.

May 19, (October 17) 2017 **Ulf Persson:** *Prof.em, Chalmers U.of Tech., Göteborg Sweden* ulfp@chalmers.se