

The Mathematical Experience

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I got the book in the early 80's. I may or may not have read it before I heard Borel's praise of it at IAS in the spring of 1982. Anyway my initial reading is almost forty years back in time. It made an impression on me, a book presenting mathematics from a new angle, personal and philosophical of course, but most of all a view from the inside, which may have been what impressed Borel. Somehow it made it easy to identify with the authors.

The book contains some mathematics addressed to non-mathematicians but not in a didactic manner but more like the authors putting themselves into the position of encountering what was well-known to them as if it was new to them. Thus combining the fresh eye with the support of experience. But most of all the book leads up to a polemic of sorts against the tradition of mathematical philosophy and its categories of Platonism, Formalism and Intuitionism. Mathematics is mathematics and as such it exists. It is characterized by two facts, which may be seen as incompatible. On one hand it is a human invention and thus on par with any other humanistic venture such as the Arts and Law, but on the other hand it shares with science the objectivity of its study, what it brings forth is replicable and there is a remarkable consensus as to its results, which are often attained independently. The objects which have been invented are mysteriously beyond our subjective command they have properties we can only find out about by hard work and ingenuity, others which we cannot, and then of course many properties we never suspected. An engineer can invent a machine but he discovers that it works. In mathematics you can ask meaningful questions about your constructions and invented concepts, and even if you cannot answer them others might; but to demand the same of say fictional characters would be absurd.

Hersh, who is the main author and whose spirit permeates the book, takes exception to Platonism, seeing it as mere superstition positing an ideal realm of forms beyond space and time to which individuals may have secret access through a mysterious intuition. On the other hand he shares the working mathematicians conviction that what he is doing has the appearance of reality and kicks back at you. In fact he even writes that the different ways of viewing mathematics, platonically, formalistically, intuitionistically, are but limited viewpoints, in order to get a real feel for mathematics you must integrate many view points, just as planar projections of a cube looks like entirely different figures, but once we integrate them to a 3-dimensional object do they make sense. This is if anything a Platonic metaphor. He also takes exception to formalism in its view of mathematics as a game with no intrinsic meaning where we cannot even ask questions about truth. The formal aspects of mathematics has very little to do with the way mathematics is practiced as a living growing subject. I am fond of thinking of the reduction to a deductive game as similar to the presentation of a picture pixel by pixel. Finally intuitionism he find pedantic and its denial of the excluded middle robbing the ontological background of mathematics. Although Hersh shares many of the experiences of mathematical reality he refuses to

assign to it any extra-human existence, instead he finds the idea of Popper's three worlds congenial, and assigns it in World 3.

I read the book a long time ago, and even if I had forgotten most of the details, in fact to the extent that I would have been hard pressed to recall any of the contents, it no doubt influenced me deeply and it helped me to form many of my ideas of mathematics, which I may have thought original to myself. My intellectual maturity is now much greater than it was then and much of the arguments and references make much more sense now than they could have done back then. For one thing I do not think I encountered Popper until my mid-forties when he had just died, but in the book he is mentioned several times, and as seen above with specific references to his philosophy. Did I never finish the book, or did the name and the references mean nothing to me back then?

The book is presented a little bit like a scrap book. There is no general narrative, but different topics are loosely put together. This makes for a lightness of touch, your attention span is never tried and there is a variety of styles. Although the book is addressed to the general intelligent reader it is, as noted, never didactic. Another reason for that is that much of the text is in the nature of essays, and an essay is always written in order to find out what you really think allowing the reader to eavesdrop on you.

Reuben Hersh once told me that he had for many years loose scraps of writing which he did not really know what to do with and then Davis came along and helped to bring it all in order as well as providing texts himself; yet it is hard to think of it as anything else than a book by Hersh, as confirmed by the overture in the beginning, giving the reasons for writing the book, and the final discussion about the philosophy of mathematics is definitely Hersh, but that does not mean that Davis would have disagreed,

No book is perfect, and this one has a few defects. The presentation of non-Euclidean geometry leaves quite a few things to improve. In one incomplete table they give the analogues to Pythagoras theorem and the area of a circle in terms of its radius for the Euclidean and non-Euclidean cases without involving the analogies with the trigonometric functions (as in the elliptic case) and the hyperbolic analogues in (hyperbolic case). But of course this does in no way impair the general impression.

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