Significant Figures

The Lives and Work of Great Mathematicians

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Eric Temple Bell's classical 'Men of Mathematics' was written in 1936. As a young teenager I read it some thirty years later, and it had a profound effect on me, in fact few books have influenced me more, as it made me decide to become a mathematician. I learned that you could be a hero outside sports and military engagement, that in fact that there was nothing more worthy of respect and admiration than intellectual pursuits, and in my case that meant in particular mathematical. Of course the ground has to be prepared for a book to have an impact, and maybe my father had realized that the time was ripe when he suggested that I read it. As indicated above it was a revelation. Later I have been told that Bell's book has great flaws, that he was an opinionated writer, and that he did indeed propagate badly founded myths, and above all, that his book had really very little of mathematical substance, and instead tends to romanticize mathematics (what is wrong with the latter?). And sure enough as I as a mature man and mathematician have returned to Bell, his shortcomings have become apparent to me, yet I still think it is a great book, a book that opens up a new world, can never be that bad.

Ian Stewart is a well-known and successful popularizer of mathematics, in fact so well-known and successful that he belongs to a coterie of a very few, to whom news media turn when something mathematical turns up, confident that they will in the end be able to deliver. Thus what better idea than turning to him to write a new 'Men of Mathematics' geared to a modern readership and dispensing of all the flaws that supposedly mar Bell's effort. However, as a reviewer, I am bound to judge Stewart's book unfairly, a new love has to be remarkable indeed, if it can obliterate an old one. I came to Bell innocent of mathematics and mathematicians, while by now my mind is necessarily jaded with more than half a century of intervening experience. Stewart's efforts will hence be unable to move me in the same way, as those of Bell did once unwittingly.

The approach of Stewart is essentially the same as that of Bell, hence as noted comparisons are inevitable, although there are of course real differences, the first one being the choice of title. While that of Bell is straightforward and 'manly' that of Stewart is a pun, no doubt foisted on him by the publishers, and as all puns of no sustainable value, indeed it needs a subtitle to survive. The format is, however, basically the same, meaning that we have potted biographies, with strong anecdotal elements, supplemented with efforts to convey something of the mathematics. Bell covers more ground treating more than thirty mathematicians, some of them collected together in various single chapters. Stewart is more rigid, one mathematician, one chapter, although I think that his treatment would have benefitted from grouping together the Arab and Indian mathematicians into one chapter, just as Bell grouped together Archimedes with other Ancient Greek mathematicians, although of course Archimedes deserves a chapter by himself. More significantly though, the book by Stewart involves, I estimate, some 700 kB of text, while Bell is allowed more than the double, say about 1.5 MB; thus Bell is allowed to be more leisurely in his treatment, and that makes a difference.

From Bell I learned that there are three outstanding mathematicians in the history of mankind, namely Archimedes, Newton and Gauss, something I took as the Gospel Truth, and am still unable to seriously question. Nevertheless any serious account of great mathematicians can ill afford to ignore any of them. As to others the author enjoys a great discretion, his principles have been to select mathematicians, who in addition to having done important mathematics also need to have had interesting lives which lend themselves to enchanting dramatizations, and in addition to collectively make up a spread of geographical and temporal diversity, not to forget gender balance. The latter of course makes up a real challenge, surpassed only by that of explaining the mathematics, as the author truly notes, mathematics is not a spectator sport, to appreciate it you have to actively engage in it. To both those challenges we will return. Now given the choice of at least a hundred potential candidates, any choice is bound to reflect the accidental vagaries of the author's individual taste and prejudices. Nevertheless, the intersection with the choice of Bell is rather substantial. Stewart also treats Fermat, Euler, Fourier, Lobachevsky, Galois, Kowalewski, Boole, Riemann and Cantor. Many mathematicians would miss Lagrange, Cauchy, Abel, Jacobi, Hamilton, Cayley as well as many others. Especially the Norwegians might fume at the exclusion of Abel and write a petition. But it would be pointless to quibble, the author has to make rooms for representatives of non-western mathematicians, women and 20th century mathematicians, after all Bell ends with Poincaré and Cantor. To me and many other mathematicians there is only one mathematical tradition, that of the Old Greeks, anything else has only a historical or cultural interest, reminding us that mathematical truths are universal and that many western inventions and insights have actually been prefigured by smart people outside the systematic Western tradition. From a historical and cultural point of view the four non-western mathematicians could, as already noted, been clumped together into one chapter, de-emphasizing the personal interest, as so little is known anyway. To find women mathematician you need to dig deeply, and Stewart only manages to unearth three, namely Byron's daughter Ada Lovelace, Sofia Kovalevskaya and Emmy Noether, where Kovalevskaya was already treated by Bell. The striking scarcity of women in mathematics can easily mislead outsiders to believe that women are actually discouraged by the mathematical establishment. In fact everybody used to be discouraged, male and female alike, the subject traditionally being very competitive. On the other hand mathematicians for all their faults can be lauded for their intellectual honesty. And when they see talent they appreciate it regardless of race or gender or whatever is supposed to be a detriment. Sofia was enthusiastically championed by Mittag-Leffler, who had a reputation of being a grumpy conservative, and Emmy Noether by Hilbert and Klein. And when they fought it was not against other mathematicians but the general establishment be it university officials or influential intellectuals. Strindberg, to take one example of the latter, opposed the appointment of Kovalevskaya on the basis of her being a woman, and thus unsuitable for mathematics. The choice of Ada Lovelace I find a bit questionable, no doubt she was a brilliant woman and very competent, but there are literally thousands of such people, and her accomplishments seem a bit vague and mostly taking form retroactively, yet of course in a discipline crying out for inspiring role models, you can hardly do better, and besides being the daughter of Byron, makes her vey 'sexy'. As to the idea of a programmable computer, indications of which can even be found in the *Nachlaß* of Leibniz, although they had no influence at the time for obvious reasons, this seems to have popped up independently in many minds. Kovaleskaya is, however, hardly a controversial choice, in fact it can be thought of as almost too conventional, she having been the subject of much appreciative attention in later decades. While both Lovelace and Kovalevskaya may run the risk of being overrated, with Noether it is rather the other way around, and to me her choice is not so much one of a woman as a mathematician of the 20th century. In fact the addition of 20th century mathematicians is of course the most exciting aspect of the book, where the author really treads new ground, not just going over well-trodden paths. I have always been curious of a sequel to Bell, where the rich fauna of modern mathematicians would get the same treatment (be it romanticized) as the historical ones. Yet, there are disappointingly very few postbellian choices in the book (and once again for reasons of space). There is Hilbert of course, arguably one of the most influential of 20th century mathematicians, then there is Noether, who earns her place with pride, but why include both Gödel and Turing? Some mathematicians may raise an eye-brow as to the inclusion of Mandelbroit, as being more of a wind-bag than a serious mathematician who can actually prove theorems, not just talk about them. Yet I find it justified, fractal mathematics is accessible and has a wide appeal, and Mandelbroit is after all a phenomenon, showing the great variety of mathematical thinking, and his ideas have had undeniable influence. And no one could quarrel with the inclusion of Ramanujam, although the (romantic?) story has been told many times. (The author does not mind walking along well-trodden paths, and often does so well). But why specifically include Thurston? A choice has to be made among say a dozen equally distinguished candidates, and as the author has wisely decided to write on no living person, the relatively early death of the former apparently singles him out. Most mathematicians may puzzle that Grothendieck is passed over, by many considered the most original and strongest mathematician of the 20th century. And he is dead too, but admittedly recently, and perhaps not at the time the book was conceived.

We now come to the trickiest part of writing the book, namely how to treat mathematics, which is intimately connected to the intended audience. To whom are popular books really addressed? Who reads Stewarts previous books? Is the general public really involved, or is it mostly a case of other academics, in neighboring fields to boot? Thus how much mathematical knowledge can be assumed? Mathematicians are used to not understanding math lectures without being upset, it is part of their professional life, while people in general are annoyed at being told things they do not grasp. On the other hand perhaps the most important audience for popular science, especially mathematics are teenagers. They may not understand everything, but in contradistinction to adults, they may hope that they soon will, and thus what maybe obscure can be thought as exciting and inspiring, giving them a pre-taste of things to come. There are essentially two ways of presenting mathematics to an ignorant audience. The first and obvious is top-down, involving a process of watering-down, cutting out all technicalities and replacing them with very vague notions, The whole thing has something desperate about it, and usually ends up being as incomprehensible to the professional as the layman. This is the approach typically taken by journalists, who are outsiders to the subject, but believe that they have a special gift to make themselves understandable to people in general, mistakingly believing that a major obstacle to understanding is due to arcane terminology. The other approach is bottom up, to take as point of departure a common ground and then work out a typical example having at least one or two essential features crucial to the problem. There will be no completeness, that is impossible, but at least a tangible seed is sown. As noted by the author, mathematics is not a spectator sport, you need to get something to chew on, and here, paradoxically mathematics is at an advantage compared to other sciences, it is actually possible to get some hands-on-experience even in an armchair, all what is needed is imagination, although this can indeed be a tall order. Now, invariably any presentation of mathematics to the public, will contain a bit of both (mathematicians are after all humans, and thus also lazy at times) and sometimes even worse, telling the readers what this is useful for. This may seem a very commendable ambition, but without any details it is bound to become a pointless exercise in vague inanities. No one becomes seriously interested in mathematics if told that this will improve communication between cellular phones, no matter how practical.

Now it is easy to criticize the mathematical presentations, but very hard to come up with better alternatives. Let me just in passing note that it is a pity in the context of Gauss not to mention the Gauss map and its connection to curvature. Maybe paradoxically the most accessible mathematics is connected with logic. To explain what Boole, Turing and Gödel did is not so terribly difficult as far as mathematics is concerned, maybe because it is not really mathematics, because for all its hype it has had very little influence on mainstream mathematics. Stewart professes not to be a Platonist, although like all practicing mathematician he is well aware that the sense of mathematics transcending human thought lies at the center of all mathematical activity, which would be sensed as meaningless if otherwise. The Russian mathematician Y.Manin has expressed it as Platonism being intellectually indefensible, but psychologically inescapable. Now the working mathematician's Platonism is socially disparaged as naive, but such an attitude I find a half-measure, there is more to Platonism than that, in fact it imbues all of Western Science in its ambition to look for hidden explanations which are not sensually manifest. In my opinion, most of the criticism of Platonism, is in the nature of setting up a straw man. In fact the author shows remarkable naivety in proposing that Gödel's theorem shows that mathematics is not the bedrock science in which everything is either true and false that everybody has naively assumed. Gödel was a notorious Platonist, and his theorem is in fact a celebration of Platonism, and not to understand that is to fail to distinguish between epistemology and ontology, a basic fact of elementary philosophy, in fact the very motivation for its pursuit. True, in post-modernism thought, there is nothing but epistemology, but we can leave the post-modernists to stew in their own juices. Yet I personally have to admit that the higher cardinalities of Cantor makes me seriously doubt the Platonic nature of mathematics. I think that at least the psychological explanation for that is that in mainstream mathematics, everything fit together in a web, and one may prove theorems in many different ways. While in Cantorian set-theory, there seems to be, just as in fiction, only one channel, while in real life we can take in things through a multitude of senses which are all coherent and reinforce each other, just as in mathematics. There seems to be only one way of proving the uncountability of reals, namely through the diagonal argument, so does it have any meaning beyond that trick? In main-stream mathematics, as Dieudonné remarked, the uncountability is merely a negative fact. Without it, modern measure theory would not exist. Also the possibility of countable models of the reals, points to this almost purely epistemological nature of the concept. Yet of course, if we could imagine minds that could actually go through an infinite set of objects (in finite time?), uncountability would take on a more tangible meaning. In fact at the core of the proof of Gödel's theorem, this is a feat we are asked to imagine.

When it comes to the potted biographies there should be no insurmountable obstacles. However, I must admit that with few exceptions, I find these biographies merely passable, in some cases even excruciatingly bad. I believe that part of the problem is the same that plagues modern education, namely the easy access to Internet and the concomitant attention to take short cuts. Bell had to read and digest texts before putting them on paper. I do not doubt that the author has done the same, but in some cases, his output is somewhat suspicious with irrelevant details. This points to the most serious criticism of the book, namely the haste with which it has been put together. There are many trivial, yet irritating mistakes, we all know that Cantor died 1918, as the caption of the introductory photo informs those readers, who do not already know. But why write in the ensuing text that Cantor died in 1917? The author has not been well served by his publisher, which points to a more general problem. In the past any serious publisher had excellent editors whose charge was not only to spot typos and such mistakes, but also to check facts. And any half-decent editor would have sent the version of the Boole chapter back to the author, just as I would have done to a student, had he or she produced the same. In fact when it comes to the biography of Boole, one gets the impression that the author accidentally had submitted two preliminary drafts, which have been printed back to back. (That the customary picture, this time is the identical one for Archimedes, does not help matters.) The author has the habit of first presenting the subject in media res, and than presenting a fuller chronological biography, in the Boolean chapter, this is only confusing. Similarly in the chapter on Gödel, a disproportional part of the text is given over to the shooting of Schlick, the leader of the positivists of the Vienna Circle, by a disgruntled student, a fact that has almost nothing to do with Gödel at all. And even more puzzling, why write that at the take over by Hitler in 1933, Gödel became interested in logic (as if there would be a connection), when on top of that his epoch-mating proof was published already in 1931? And the piece on Thurston is almost as much a piece on Perelman, who would have been a prime subject for the author, but, alas, he is not dead yet.

Now the spectre of Bell haunting this review, it could be of some interest to compare the treatments of Bell and Stewart on a single character, and let us single out Euler, as being, in the words of the author, the most significant mathematician virtually unknown to the general public (and hence a must in any book of this kind). Now to treat Euler properly, Stewart tells us, you need to write a book, and even then it is a challenge, and proceeds to give him ten pages, while Bell uses sixteen, so even from the outset the author is handicapped. The approach is strikingly similar, both start out with a breezy survey in media res, before settling down to a more systematic account. From Bell we learn about the importance of Academies in the 18th century and the freedom to pursue pure research with no demand on useful applications (although Euler did, as with som many other things, involve him in such as well, in particularly navigation) while Stewart regales us of a tale involving Euler's failed attempts to provide water works to the Kings garden (but he does inform us, somewhat tersely and dutifully, about Euler's practical accomplishments as well). Then Bell proceeds with a lengthy account of his years first at St-Petersburg, then the rather unhappy interlude in Berlin, where Fredrick the Great did not appreciate him fully (as Stewart evokes) and his happy return to Russia into the open arms of Catherine, another Great (there is also another earlier Catherine, maybe not so great, associated to the St-Petersburg Academy). It is remarkable the kind of prestige distinguished scientists enjoyed by the Men and Women of power at the time, I doubt that even Einstein did it in modern times. Bell amuses his readers with the story of how Euler fibbed off Diderot by a bogus proof of the existence of God. A story that is apocryphal, as Diderot was actually a rather accomplished amateure mathematician; in fact there is actually a forgotten book by Coolidge which treats great amateures in mathematics (in addition to Diderot and others also Napier and Pascal), with a much more technical attention to the mathematics involved, which may account for its obscurity. The account by Stewart is necessarily terser and in particular he wisely dispenses with the anecdote above. On the other hand he has picked up that Euler married a certain Katarina Gsell, the daughter of a local artist, on his first Russian sojourn. Mathematically Stewart deals with the celebrated formula $e^{i\theta} = \cos\theta + i\sin\theta$ in particular $e^{i\pi} = -1$ by many considered the most beautiful formula in mathematics. Bell discusses it in another context and quotes a statement to the effect that to a mathematician this is as obvious as 1+1=2, something which must be rather discouraging for most readers. Stewart remarks wisely that it might have been mind-boggling at the time of Euker, but nowadays it is more in the nature of a convention. He proceeds to infinite sums, especially Euler's summation of inverted squares, something that had baffled his contemporaries. As expected Stewart does not indicate how it was done, only that it was not rigorous if correct. Furthermore Eulers constant is brought up and listed with 17 digits, and there is a brief discussion of Fermat's 'little theorem' without a proof (Bell makes a big point of it in his chapter on Fermat and offers it as an intelligence test to the reader, knowing that they will fail) as well as mentioning the theorem of quadratic reciprocity and sums of squares. His importance as a writer of text-books is mentioned, especially his way of standardizing notations such as π, e and i etc (actually π was introduced by a British mathematician Jones in the early 18th century, whose son, the celebrated judge and linguist resident of India, discovered the common Indo-European roots of Hindi and European languages, and hence can be thought of the father of modern liguistics and allow us the kind of digression so common in the books under review). He then proceeds to discuss the famed bridges of Königsberg and a combinatorial problem touching on Latin squares and its relations to modern sudokous. Then there is a final more or less dutiful page of listing Euler's accomplishments in mechanics, concluding the chapter with a quote by Laplace admonishing us all to read Euler, because he is the master of us all. Bell on the other had does not have any explicit treatment of Euler's mathematics, there is a passing example of Eulers' unorthodox ways of dealing with infinite series, but most of it consists of a general description of his work in mechanics intergrated in the general biographical narrative. Thus Bell does not make the same separation between biography and mathematics as does Stewart. He concludes with the last words of Euler, namely 'I die' and thereby ceasing to breath and calculate (in that order?).

Finally in a concluding section devoted to a general survey of 'mathematical people' the author puts the standard question whether mathematics is a question of nature or nurture. While the second cannot be discounted, it is far from sufficient. And does not the example of Gauss show even its lack of necessity? If we are to understand the author right. Here I am, like most mathematicians, inclined to agree with him, especially when I stretch his conclusion beyond that of his possible intentions.

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