Birth of a Theorem

A Mathematical Adventure

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For every mathematical formula a popular book on science involves the readership goes down by half. This seems to be the received wisdom of the publishing industry. A dozen formulas and you are lucky if your closest friends read it. But here there is a galore of formulas, pages after pages. Cedric Villani really takes the lion by the tail.

This is a book about a Birth, not about a Theorem. A reader may not learn much mathematics, it is not even clear whether he or she learns any mathematics at all, but this is not the intention. It is a book about a Birth, and about how the everyday life of a mathematician really proceeds. As far as I know there are very few if any sustained efforts to write about this everyday aspect. There are of course biographies of mathematicians, very many on Gauss, quite a few about Abel, then you can find books about Lie, Mittag-Leffler, Zariski, Turing, Erdos and a string of other mathematicians most of them totally unknown to the general public. And one should not forget the often maligned 'Men of Mathematics' by Bell, a book which introduced many people to mathematics as a subject engaged in by heroes, the impact of which should not be underestimated. There are also mathematical autobiographies, Halmos, Weil, Grothendieck, Rota, Ulam, Schwarz to name a few, but none of them goes into the 'nitty-gritty' detail that constitutes the prosaic activity of a mathematician. The outward life of a mathematician is seldom dramatic, and as to the extent it is, it is detrimental to the pursuit of mathematics, the most extreme example being Galois. While the outward life typically is unremarkable, the inward life is to a large extent opaque. What does a mathematician do? Maybe even opaque to mathematicians themselves who not seldom cannot recall much of a working day than filling up a waste paper basket. One standard answer is proving Theorems, and the author has taken this literally, with an emphasis on proving. The very theorem is in fact secondary, a precise formulation, as it appears in the published journal, is given in the end of the book, more as an afterthought, with no fanfare, nor any attempt to explain what it states and means. But as already emphasized, the book is not about a theorem, it is about its birth, meaning the sustained effort to prove it.

The understanding of the book is different depending on whether you are a professional mathematician or not. What characterizes a professional mathematician is his constant exposure to incomprehensible mathematics. This makes him resigned and inured and less liable to be upset by something he does not understand than the general reader. Also, paradoxically, the general reader may understand more than the professional mathematician, because he and she may not really understand the depth of their ignorance. Now there are beautiful grand ideas in mathematics which can be appreciated and shared by mathematicians at large, but as we all know the devil is to be found in the details, and the real honest work of a mathematician is to master the details which includes a lot of

sweat and effort, more so than divine inspiration, although intermittent glimpses of one of those epiphanies are the injections without which progress would be permanently stymied. As they say about war, long stretches of boredom interrupted by moments of sheer terror. Yet the real engagement in the details take off a life on its own and become for those involved truly fascinating, while for outsiders it is merely a question of technicalities. At a mathematical talk, the technicalities are usually downplayed, although they provide the meat of the contents, and the so called main ideas are usually presented instead, vaguely and with large movements of arms and hands. In the case of analysis, the technical work means making clever estimates, and there are constant references to those. However, the references are far too sketchy and elliptical in nature to convey any kind of understanding except possible the one of sympathy. But once again, the purpose of the book is not to explain and illuminate the proof of a theorem, but to convey the kind of frustrations such a task invariably involves. Nowhere else than in mathematics can you encounter the same complexity of the structure of logical arguments. To check them there are two options. You explore whether they contradict any known results, and if not, and if on the contrary it spreads light on what is already known, there is a strong incentive to be persuaded, and for most mathematicians that is enough, but for the mathematical collective it is not, you need to carefully go through the logical structure of the proof and check for any local mistakes. Those can pop up in the most unlikely places, as in the celebrated paper by Poincaré, a mistake which necessitated the pulping of entire issue (the costs of which gracefully were underwritten by the great mathematician himself, a deed which now would be inconceivable). Now it is the responsibility of the referee to check correctness of a paper. This is often an onerous task, especially when the paper is long, and often the responsibility will then be shared by many who are portioned out parts of the paper. Most articles do not warrant such attention, and it is quite possible that a large proportion of the articles which are published contains mistakes, and sometimes even serious and unfixable mistakes. The reason they may not be discovered is that the results are not important, so the consequences do not become manifest. On the other hand when really important results are announced, they often turn out to be flawed, probably because they merit more attention, not that they are produced more slovenly, although one may not discount a sizable element of wishful thinking. Any purported proof of the Riemann Hypothesis would go through a most demanding gauntlet, unless immediately rejected for obvious reasons, although by its nature it attracts much fewer crackpots of the trisectional variety than does Fermat.

So after a long struggle, ironing out inconsistencies, filling up gaps, they submit it to a prestigious journal, in fact the same in which Poincaré was engaged, known as 'Acta Matematica' founded by that great busybody - the Swedish mathematician Mittag-Leffler - who knew everyone who mattered. Much to their dismay the paper is not accepted. It is a long paper, and long papers take out a lot of remaining space, making many prospective settlers on its pages homeless, so it is a bit hard to have them accepted. The results really have to be striking to justify the space it will usurp. No mistakes are found, yet the various referees are not convinced that the proof is watertight. A big disappointment but the author and his co-worker grit their teeth, do a complete overhaul, and then finally getting it in sufficient shape to convince and submit it a second time, and now with success.

The Theorem is born.

The account is a little of a collage. Many bits and pieces are glued on the board. Not only excerpts from e-mails, but also snatches of dreams, reflections on music that has made a deep impact on the author, frustrations from the lack of good bread and decent cheese in the vicinity of the institute at Princeton as well as vignettes from child care, transatlantic moves, attendances at congresses, job-offers (such as the directorship of the Institute Poincaré) and other vicissitudes of everyday life, which play such an important part of it when lived, but which in retrospect recedes into the background, and from the point of view of history disappears into total oblivion.

A Theorem is born, and afterwards the author can bask in the sweet joys that comes from recognition and celebration. The Fields medal at Hyderabad clearly constitute a high-point in the career of a very successful mathematician. Most mathematicians do not even dream about it, but the author did, thus proving that it is not the medal itself which constitute an encouragement and motivation for further hard work, but the prospect of getting it, and in this way, albeit a bit cruelly, it serves to stimulate and provoke hard and dedicated work, for many more than actually receive it. No big thing initially, but as the years have gone by, striking recipients have provided the medal with mystique and grandeur, and now it looms large and actually provides the dramatic *raison d'etre* of the ICM.

In what way does someone such as the author differ from other mortals? Genius is a word that comes to mind, some people are just more gifted, and that is a fact. Mathematics as a social phenomenon is in fact a celebration of genius. On the other hand pedagogues claim that that is not what matters but education. The myth of genius is not only not true but harmful. It is education that matters. Anyone can be a hotch-notch mathematician, it is just a matter of the right training. What else can yo expect from educational people? But it is hardly realistic. Any teacher with classroom experience (and educators rarely have it, and if so they are very successful in repressing it) notice that some pupils take to mathematics immediately, other struggle and never seem to catch the point. One can speak of a kind of autism. Autism on the other hand, we are assured, is not the fault of education, so parents should not feel guilty, it is genetic and biological. But could it not be that the majority of people, for no fault of their own, nor of the educational system, are mathematically autistic? Some mathematicians are socially autistic (at least in some mild sense). Is that progenital or acquired by mathematical training? If so, maybe mathematical training is not such a good thing and should be administered sparingly. But who should be sacrificed? This should indicate that the issue is not as simple as the educators would like to believe. Then there is the issue of luck. Maybe some people, and I am now speaking about professional mathematicians, are lucky to hit upon a gold mine. A subject is rich, gives a lot of connections, and seems to fire the imagination. What strikes the reader is that the successful mathematicians are in generally much more devoted, they work harder. Longer hours and intensive hours, never really letting go. But that requires devotion, and how do you get it? You have some good ideas generating more good ideas and you find yourself in a benevolent circle with positive feedback. Could be, but the frustration of the good mathematicians are just as great, if not greater, than the less good ones. Still they persevere? Has the poison penetrated deeper into their minds? Are they

addicts unable to kick the habit? One can speculate. You are not supposed to burn your feet walking on glowing pieces of coal, provided you are convinced you will not burn. The same is supposed to be true if you jump from a high building. Non-falsifiable claims.

So why do people do mathematics? Maybe most general readers will not be convinced, professional mathematicians on the other hand may be reassured. And maybe that is good enough.

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