Sannolikhet

En introduktion til den moderna sannolikhetslran

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The book is a Swedish translation of an American one addressing the proverbial layman about the wonderful science of probability. It is a paperback stemming from the early sixties during a time when there were a lot of interesting popular books on science appearing in Sweden and published in cheap paperbacks and hence easily accessible. In my teens I acquired a little library of those the members of which played an important role in my education[1]. Most of those books were translations of Anglo-Saxon sources, but a few were written by Swedes. This publishing tradition was soon abolished, it was apparently not commercially justified, most people found the reading heavy-going, especially when it concerned science.

Now why do I even bother to read this book at all? Is it just because of sentimental reasons, maybe I have already read it in my teens? If so I do not remember what to be found in it, or rather I may have learned from it, but things you truly learn and digest, usually become separated from its source. I have no formal schooling in probability and statistics, apart from an introductory course[2] I read and was examined on successfully on the first opportunity I had starting at University. Thus I know the elementary basics, probably from the age of fifteen or so, and have not really added to it. On the other hand, as has been noted, being such a widely applicable tool, its structure is not very sophisticated, or rather not very involved and complicated, and a little knowledge actually goes a long way. Also some aspects of it belong to general mathematical culture, and is not the sole property of the professionals. It is with this understanding I accepted, not without some misgivings yet nevertheless with a certain alacrity, to contribute to a Swedish cultural journal on the subject of chance. But maybe I made some mistakes, or overlooked something basic, revealing my ignorance, something bound to embarrass an academic. If that is my prime intention, it is by now a little bit late in the day, as I understand that it has already been published. What kind of things am I looking for? The text I a about to delve into is an elementary one, intended for people with no more mathematical sophistication than supplied by an American high-school education [3]. What could I learn which I do not already know? Nothing I hope (although curiosity trumps vanity). One may make correct statements or not, and one may make original and interesting, all four

[1] beyond that of school, and pursued out of genuine curiosity and not, as was generally assumed and dismissed, as a means of improving grades

[2] Ett betyg

[3] indeed one of its purposes is to stimulate the thinking of the reader to pursue the exciting adventure of probability theory and to make a case for it to be included in core curricula replacing what the author thinks of as arcane topics having only a historical interest.

combinations being possible. The best is to make original and correct statements, but this is unlikely. The worst is to make unoriginal and incorrect statements, this is far more likely, and of course very embarrassing. But what about the choice between original and false and conventional and correct? The latter invites no criticism, but maybe a yawn or two, the former is of course irresponsible, but provocative and makes for readability. It is much easier to concoct true statements, it does not require much thinking only some care; than to come up with original statements, the latter being more likely incorrect than correct.

Much of the contents of the book concern basic material which I have known for more than half a century, and can be quickly perused. Then there are some things that are new to me, such as Chebyshevs theorem on confidence intervals in terms of standard deviations (square roots of variance), this, with its admittedly crude estimates, is used to sketch the proof of the central limit theorem, which the author places at the center of probability theory as it connects somehow mathematical probability with reality in terms of frequencies. However, although the statement of the theorem is precise, as mathematical theorems tend to be, it does not say something definitely, as it does not hide, but qualifies its statement in terms of probabilities, and how should those be interpreted? In a way we are in a vicious circle, however, that has little bearing on practice.

Furthermore there is an explanation of the binomial distribution and how it in the limit approaches the Gaussian bell curve[4]. This only works in the symmetric case [5]. Then there is a discussion of the Poisson distribution, without explicitly explaining how it fits into the binomial distribution, but making the important point that what is at stake is not p or the size of the sample N but the value pN.

Now what is the difference between probability theory and statistics? This can be given a very short answer. Statistics is the reverse of probability theory. In the latter we start with a model (or axioms if you prefer) for the probability distribution of a population[6], and from that deduce probabilities for various possible events. It is, so to speak, deductive, and hence entirely mathematical, and you really do not have to be a probabilist to address and even answer the various questions, which in many cases boils down to more or less straightforward combinatorial questions. Statistics is about going the other way. Given events, or samples, in real life what do they tell you about the possible probability model? The truth is in the pudding, so given a pudding how was it baked? If you want you may think of that as inductive reasoning[7] reasoning from the particular to the general. Now this method does not make sense, as the author rightly emphasizes, without some a

[4] The proof sketched exploits the notions of expectant value and variance, I did one from scratch once, and was amused to realize that in the Taylor expansion of $\log(1+x)$ you needed to take into account also the quadratic term.

[5] $p = q = \frac{1}{2}$

[6] meaning of course given a definition of how to assign probabiliies of subpopulations, in the spirit of Kolmogorov

[7] The author brings up Francis Bacon as the first proponent of systematic inductive thinking, starting not from models, based on rational design, but empirically from real life data. He then claims that the inductive method did not get into his own until late 18th century. I am a bit skeptical about his description of its history, he makes no mention of Hume and his criticism of the inductive method, which

priori assumptions. One needs to guess a probability distribution and see how it could have given rise to the observed effects. One can of course never rule out any but the most trivial distributions, i.e. those can be falsified by experiments, but one may try to arrange the possible ones with attached probabilities, but of course any such probability computation presupposes a given probability distributions on the alternatives, and if we want pursue it, we get into an infinite regress. What we are dealing with is of course Bayesean inference, whose mathematical underpinning is very elementary, but whose implementations into specific real contexts is problematic. One cannot get something out of nothing, as noted above, and presupposes tacit assumptions, not always brought out into the open. Those questions are not pursued, for obvious reasons, and in particular the 'feud' between those who favor a frequency based approach to statistics, and those adhering to a Bayesean' is not even touched upon, only vaguely hinted to.

The book begins with a discussion of games, which started the whole modern theory of probability through the correspondence between Pascal and Fermat, and ends up with some worked out examples. There are of course ideal games presupposing inexhaustible resources and infinite time, which as with every play with infinities, may lead the unwary to paradoxical conclusions. But if you admit that in practice resources are finite, you may state some simple games and actually come up with definite conclusions. One such example, which can represent the moral of most games, is one in which two people play a succession of fair games and one calculates the probability that either person in the end ends up ruined. If one person has much more resources than the other, in which case it is to be considered as the 'bank' or the gambling institution (be it a casino or a governmental lottery racket), that person (or institution) will most likely prevail. The case will be discussed in an appendix. So when it comes to games. Do not indulge in them, at least not beyond the point when the expected losses will be outweighed by the pleasure the risks entailed. And then of course most activities, especially those in business, may be thought of as games. Then the real challenge is to identify games in which you play at an advantage, either through the odds are stacked in your favor, or in which you can draw on hidden resources. In the first case of a game purely of chance, once the selection is made, you are in business, so to speak. In the second case, steady vigilance is called for.

And then the question stated in a previous footnote: Should there be more of probability in school mathematics as well that everyone to whom such matters enter into their professional lives should be properly instructed. When I entered high school in 1966 there had been a thorough revision of it, in particular its curricula in mathematics and physics, in which in particular probability theory was introduced. My father, who was a high-school teacher in those subjects, was intrigued by the latter, voluntarily inquired into the text-books and formal requirements and had himself tested officially[8]. In the process he enlisted my interest. Having always been interested in figures and statistical tables, as many mathematicians are to whom figures are friends not foes, found it of course conge-

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would imply that it was much in use at his time, and in fact how can it be avoided? The emphasis on rational consideration seems to me to be a step above slavish interpretation of experience, and that step was taken by the Greeks.

[8] Meaning that he got 'university points' which in his case had no practical consequences, he just was intrigued by being seriously challenged.

nial and quickly picked up the basics, which have served me to this day. When it came to university studies I did not pursue it beyond the first introductory stage, as I found it rather boring with its mundane practical applications which did not interest me in the least. Had I instead been subjected to a more sophisticated and mathematical treatment, involving Brownian motion and Statistical Mechanics, no doubt my imagination would have been fired. So the question is how much probability theory do you really need to know? For mathematicians this might be an easier question to answer, in view of the remark above to the effect that probability theory is a very flexible subject with little structure needed to master at least competently[9]. For a more general public it is harder to address. You may with disproportionate effort teach them some simple ways of computations, but those things can be left to the experts [10]. On the other hand what people should really understand is the philosophical aspects of statistics, and that seems to be a subject that even trips the mathematically literate.

And finally who was Warren Weaver (1894-1978)? Trained as a mathematician, he was in later life put in an administrative position at the Rockefeller Foundation of overseeing the giving of grants to scientific applications in general, a statistician obviously thought of as very suitable due to the wide applicability of the subject to most scientific disciplines. This allowed also a platform for being an advocate of science. He also had a spell in AI as being a pioneer when it came to machine translations, his ideas for which I now suppose to be outmoded. Anyway being a fan of Lewis Carroll, he collected many translations of 'Alice in Wonderland' even ranking them, in quantitative terms, as to their success as such.

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[9] This remark is not original with me, but stems from an old friend and fellow student, now regrettably dead since many years - Andreas Wannebo

[10] but this touches a bit too uncomfortably close to the claim that most people have no use for mathematics at all, and that its instruction could profitably be limited to those who will use it in their professional lives