## Loving + Hating Mathematics

Challenging the Myths of Mathematical Life

R.Hersh & V.John-Steiner

December 29-31, 2015

The authors set themselves out to explode some myths about mathematicians (as opposed to mathematics). More specifically the following four myths

**1.** Mathematicians are different from other people. In particular they are less complex in their emotions.

2. Mathematics makes for a solitary pursuit.

**3.** Mathematics is a young man's game.

4. Mathematics is an effective filter for higher education.

Now some of those myths are simpler than others. For number two you need only to point at the prevalence of mathematical co-operation and the existence of formal as well as informal mathematical societies and coteries. For number three you only need to point out examples of mathematicians being active at a very old age, some of them producing their best work when common wisdom would held them to be over the hill<sup>1</sup>. Thus the authors digress on those two myths. As to the second discussing in more depth some startlingly successful examples of voluntary and informal groups such as 'Bourbaki' of the French and 'the Anonymous group' of the Hungarians, both blossoming in the interwar years, while formal associations, such as AWM (Association for Women in Mathematics) and IMU (International Mathematical Union) alongside various national societies are different in nature, geared towards mathematicians rather than mathematics itself. And as to the third, it is including in its discussion gender and racial discriminations, topics which are bound to be rather controversial. We will return to those later. Now the chapters dealing with the first myth is obviously directed to the general public, mathematicians do not need to be told that they are not that different from other people and that they have complex emotions, while the three other myths may be more relevant to the mathematical reader than the general one, for whom those two myths are just aspects of the first. Mathematicians may be reminded how much social interaction actually means, in mathematics no one is an island, and also to be given some advice and comfort when it comes to dealing with age, as well as some admonishment when it comes to dealing with ethnic minorities, among which for some strange reason women are counted. As to the fourth this is probably the most controversial and the most intractable, and while the authors may have succeeded in demolishing, or at least seriously put in question, the first three, the jury is still out on the

<sup>&</sup>lt;sup>1</sup> the most spectacular example being the Austrian mathematician Vietoris (1891-2002) publishing a mathematical paper at the age of 103. His wife predeceased him with a few weeks turning a hundred, making them the seventh oldest couple in the world (adding ages). He himself was the oldest Austrian ever documented.

fourth. While the exploding of the first three myths might be a case of charging through open doors, the door is very much shut as to the fourth. Maybe for good reasons.

First are mathematicians really normal people? In the sense of being mathematically competent they are definitely not. The population in general  $^2$  show striking incompetence as to mathematical reasoning, much to the consternation, and to be honest, also the gratification of the mathematical community, which is given a reason to feel superior, even if that sense is not generally acknowledged. But what about emotionally? It has become fashionable in this connection to speak about an autism spectrum, with mathematicians especially, spoken about as Aspberger, meaning to be highly intelligent beings on the extreme of the spectrum (whether left or right, infrared or ultraviolet, is never specified), meaning to be socially awkward, lacking the basic skills for interpersonal communication due to either an implied thwarted emotional development or a congenital defect. Commendably the author couple does not discuss the matter in those terms, there is no reference to Aspberger in the index for one thing, but are content to give some case studies, some of which indeed show deeply disturbed individuals, none of whom could possibly serve as heroic role models for most people. In fact, one of the distinguishing features of the book is its gossipy nature, this certainly makes for readability (although the many arcane references, would make it hard for general readers to appreciate it, but it would be a delight for most mathemati $cians^3$ ), although it can be censored for being superficial and thus evading the serious issues that are after all brought up. Now the senior author is known for his anti-platonist view, claiming that mathematics does not transcend humanity but is along with the arts, and law, just another cultural creation of the human spirit. Now the issue of Platonism is if any a metaphysical issue, and can of course not be settled without some basic metaphysical assumptions, at least as questionable, as the issue itself. It is a highly charged one and a serious consideration would lead us into theology, a path most readers may be reluctant to pursue. So let us be content by making a few more or less trivial remarks. In the sense that mathematics is pursued by human beings it is of course a human activity and subjected to the vicissitudes such are prone to, and which Hersh indeed has described with commendable insight and passion over the years, aspects of which invariably serve as a motivation for the present book and guide its contents. But as far as there is a distinction between mathematics and the human practice of mathematics, to deny such a distinction would lead us into the hall of mirrors of Post-modernist sensibilities, this plays a crucial role in the psychological set-up of a mathematician. Yuri Manin refers to mathematical platonism as being intellectually indefensible (as I have acknowledged above) and psychologically inescapable. Mathematicians are naive Platonists, and if pressed formalists, in the sense that mathematics is but a formal game exploring arbitrary assumptions, which

 $<sup>^2</sup>$  It would be pointless to speak about specific percentages, there are after all a gradual variation of competence, but one figure that makes sense would be that there are only about 5 percent of the general population the members of which show any kind of serious mathematical ability, and the great majority of those, will of course never be mathematicians. In the US alone we are talking about fifteen million people, with about 200'000 in each year.

<sup>&</sup>lt;sup>3</sup> In an otherwise rather spiteful and superficial review in the SIAM, the author refers to the delight to be felt by pure mathematicians, and hardly surprising this comment is wrenched out of context by the publisher and used on the promotional blurb on its homepage.

incidentally just begs the question, as those formal consequences can be seen as platonist necessities. The game of chess may be a human invention (what else can it be?) but in a legally pursued game, the outcome is not a matter of human discretion<sup>4</sup>. Incidentally the formal view of mathematics is a caricature, and its popularity due to the lack of philosophical sophistication and interest among mathematicians in general<sup>5</sup>. Thus the belief in the platonic nature of mathematics is an overwhelming motivation for mathematicians to pursue mathematics. If it would not be there, the whole activity would be felt as meaningless, not to say perverse. A mathematician really believes that he or she is making discoveries 'out there'. It may be an illusion, and when we come to modern set theory, the sinking feeling that we may be dealing with mirages, also sets onto the mathematician, who prefers to pursue classical mathematics, leaving higher cardinalities to the logicians, who may either be more intrepid than mathematicians or just more cynical.

If we want to pinpoint the mentality of a mathematician, we foremost focus on his or her fondness for pure thinking. The ease with which mathematicians deal with infinity is a case in point, seldom stopping to reflect on its purely cerebral status, with no tangible physical manifestation. The proof of the uncountability of the reals is something a mathematician unthinkingly accepts, seemingly unaware that the infinite process involved in construction of an infinite object, has no counterpart in the tangible physical world, and in particular not something a computer would be able to  $do^6$ . But this ease and, as some philosopher would say, naive acceptance of Platonic realities, is only the more innocent aspect of embracing the cerebral world of mathematics, far more demanding is the other aspect of Platonism, that concepts actually kick back at you. In all serious activities there is an element of testing, of satisfying certain criteria which are experienced as external to you. In the case of the arts, the criteria are subjective according to Popper, while in science they are objective. Theories and explanations have to comply with the facts of the external world, and that testing can be done by anyone, this is the meaning of objectivity in science. Of course the taste of beauty in the arts may be only in the eye of the beholder, thus the subjectivity ascribed to the arts, but from the point of view of the artist, those criteria are external to himself he cannot by will alone change his sense of beauty, thus his work transcends mere wishful thinking, the activity of which founds its purest form in day-dreaming<sup>7</sup>. Mathematics is not a case of day-dreaming, although it otherwise share many of its characteristics, it comes with unforgiving criteria for what is true or not, which can never be ignored. In fact the refusal to ignore constitutes the essence of mathematical work, and as such it requires almost superhuman effort in terms of sustained concentra-

 $<sup>^4</sup>$  similarly Popper concedes that the integers likewise might be a human invention, but not its commutative and associative laws.

<sup>&</sup>lt;sup>5</sup> Hilbert is thought of as the father of formalism. This reflects unfairly on him and misrepresents the spirit in which he pursued mathematics. For him formalism was purely instrumental to deal with nagging questions of consistency, and when it failed he was happy to note it and go on with life i.e. mathematics.

<sup>&</sup>lt;sup>6</sup> Incidentally making Penrose conclude that the human mind is superior to that of a mere computer, and in particular cannot be reduced to a mere algorithm.

<sup>&</sup>lt;sup>7</sup> The observant reader may here find a crack in the Platonist vision, and Hersh indeed speaks about mathematics being external and unforgiving to the individual, but as to the collective a matter of choice, not unlike money, whose reality the individual cannot transcend, but the collective may.

tion and relentless work often of a most frustrating nature. This is the not so innocent consequence of mathematical Platonism. So while there after all may be a huge number of people with a mathematical temperament, there are far fewer who have the stomach for it. Thus mathematical talent is not enough, there is hard work needed as well. Now, admittedly one cannot set up such a clear dichotomy between talent and willingness to work hard, they are of course inseparably intertwined, and somewhat tautologically one may even argue that the latter is part of mathematical talent.

Now to the controversial question, whether mathematical talent is innate, and that either you have it or not, or whether it can be acquired. In particular whether suitably nurtured it can be made to develop in the most unexpected places. The former is in the nature of a myth, meaning a conviction based on hearsay, and one subsequently which I suspect the authors are twisting to destroy, in fact they come half-way to do so in a concluding chapter, before holding back. Now, unlike the issue of Platonism, this is an issue that in principle should be empirically, if not resolved, nothing of general nature is definitely empirically resolved, at least illuminated. Now I believe that basic mathematical ability is innate. At least as innate as musical, artistic and literary ability. It also, at least in its early stages, is easy to recognize and judge, more so than artistic and literary talent. It appears spontaneously in different sections of the population and seems to have done so through history. A Newton, a Gauss, and countless other cases did not receive any special instruction before they being discovered. It is part of the mystery of the human mind. Now athletic ability or health in general<sup>8</sup> are seen to differ between individuals and are nowadays attributed to genetic factors. Why this reluctance to give genetic explanations for mental abilities? In one sense there is none, although there is a healthy skepticism as to what could possibly count as explanation. In the other sense it is politically impossible, the prevailing ideology of the equal innate value of humans, and who could oppose such an ideology?, makes for an idealistic, as opposed to materialistic approach. Of course it is not scientifically based<sup>9</sup>, but science itself is ideological, at least the science that pertains to man and his spirit<sup>10</sup>. This digression having gone on for so long can as well run its course. The radical materialistic view ironically leads to Platonism gone rampant. The prevailing idea of intelligence as something that can be preferably implemented on computers (taking the genetic model to its logical conclusion) and thus liable to if not technically unlimited at least to vast improvements beyond the feeble powers of humans testifies to a Platonic conception. It is ironic that mathematicians (like I) who proudly revels in the Platonic nature of mathematics take exception to the idea that machines

<sup>&</sup>lt;sup>8</sup> Those are in some sense correlated but athletes do not necessarily live long, if plain longevity is to be taken as an objective measure. Just as a natural athlete may without training outdo a normal man, people who live long may do so in spite of unhealthy living. But of course when it comes to longevity there is a large element of luck.

<sup>&</sup>lt;sup>9</sup> Incidentally modern genetic research has revealed that humans are, at least in comparison with other mammal species, remarkably genetically homogeneous. Note also the empirically easily ascertained fact that humans irrespectively of race are equally able to acquire the language in which they grow up without a vestige of an accent.

 $<sup>^{10}</sup>$  The British historian and philosopher R.G. Collingwood for that reason took not only a dim view of psychology but even a resentful one.

should be able to do mathematics better than humans (i.e. mathematicians). Logically a platonically developed intelligence should be better equipped than mere humans to deal with Platonic mathematics. Let mathematics take care of itself! But of course it only testifies to the human nature of mathematicians, in particular the sentimentality that goes with it.

Now as noted the fact that so many distinguished mathematicians have come from humble backgrounds, that poverty and deprivations have not been able to thwart the spirit, is both an indication of its innateness and a caution to take the tales of the underprivileged too uncritically. Why should a poor boy be in a more privileged position to evolve a mathematical talent than a materially privileged woman? Against this goes the fact that mathematical ability is not uniform neither geographically nor historically, in order to fruitfully develop it needs a culture already in place. This ties in with the claim that mathematics, like all human activities, cannot exist in isolation. Furthermore there are far more mathematicians today than there were in the Middle Ages or the times of the Classical Greeks. It is all about exploiting a potential. But how large is this potential, does it only involve a limited percentage, say 5 percent, or is it all inclusive? Against the latter speaks teachers experience. The mathematically gifted need very little prompting to show their mettle, just as the athletically proficient stand out easily. It was once noted in a Swedish paper that mathematics at school was too abstract and hence that many mathematically gifted were discouraged. One could as well claim that physical education at school is too physically demanding hence barring many future athletic stars. A certain amount of ability of reasoning abstractly is a hallmark of mathematical affinity. Mathematical instruction can certainly be made more palatable to gifted children, but can it be done so to the less gifted? Just as you can claim that everybody can survive falls from great heights, provided they have sufficient faith in their invulnerability, you can also claim that anyone can be a world master in mathematics provided given the right incentive. But how do you achieve such faith (you have to have a lot of faith in your faith to test it) or what is the right incentive? Incidentally, when it comes to mathematics it is definitely not money.

There is a large part of the population that has no aptitude for math, and consequently in many cases a downright hate for it. I recall a Russian pedagogue writing that one has to respect such pupils, math is not for everyone, just as music and sports are not. Maybe you can only go so far when it comes to generate mathematical interest and nurture ability. Thus you can ask the question, is mathematics really necessary? Why should it be part of school curriculum? Is this only a case of a fossilized tradition, on par with the study of Latin? One thinks of generations of hapless school children cluelessly sweating over Euclid<sup>11</sup>, or pointlessly memorizing the formula for the solution to the quadratic equation<sup>12</sup> never having to use it in the future. The point is that there is even less incentive to use mathematics now, with all those digital aids you no longer even need to do arithmetic in your head (which for most children is the gateway to mathematics), than in the past. Why not scrap mathematics altogether and let it be an option for the very few who take to it directly, just as music is an option or chess and go may be offered on the side. It is

<sup>&</sup>lt;sup>11</sup> There is a poem by the late 18th century Swedish poet Bellman bemoaning his fruitless studies

 $<sup>^{12}</sup>$  I have never memorized the formula, I repeatedly derive it every time I solve a quadratic equation by completing the square, a beautiful method which you can never forget once you have seen it.

true that modern civilization with its invasive technologies is contingent upon mathematics more than any other, and much of our technological and natural environment does make much more sense with a little mathematical understanding, yet as to the first there will certainly be enough people willing and able to provide the necessary expertise, and as to the latter just a little education may be more harmful than none, just as half-truths are worse than no information at all. Now against this populistic view, which seems to be gaining momentum, at least there is a large reservoir of potential sympathizers, one may put up a few counterarguments.

First the idea of mathematics as a filter. At least to a mathematician, or those mathematically inclined, failure of basic competence is viewed as a cognitive defect, on par with dementia. Can you really trust a doctor who when taking your pulse over fifteen seconds innocently asks you for confirmation how to compute it in beats per minute?<sup>13</sup> For a mathematician it is inconceivable that you can perform any kind of complicated mental operations possessing no mathematical ability. Without it how can you reason logically for one thing? It is important that we are not speaking about mathematical knowledge per se, but only the ability to think conceptually and abstractly in a clear and logical way. This does not require any study, as C.S.Peirce rightly observed, mathematics is not the study of necessary consequences per se, but one in which necessary consequences are instinctively drawn, hence a formal study of logic is not necessary to think logically<sup>14</sup>. Thus, although mathematicians have no problem with the fact that most of mathematical knowledge may be irrelevant for the pursuit of most occupations, it is much harder to accept and understand that the inability to learn mathematics can be compatible with a general mental competence.

Old habits die hard, and sometimes for very good reasons. The claim that mathematics is no inescapable component of general intelligence is not one to accept uncritically, although there is strong social pressure to do so. The general public may have some respect for mathematics, but a reluctant one, and would shed few tears if the basis of such respect was removed<sup>15</sup>. An argument for this view is that mathematics is so specialized and esoteric that the specific applications a mathematician in general may wield are far too narrow, and that consequently that the mental set-up of such an individual must be correspondingly narrow, not to say irrelevant. Once again based on a confusion between mathematical knowledge and ability (as far as those can be distinguished). However, one of the aims of this book is to counter this caricature. In fact there is after all a notable correlation between mathematical ability and other cerebral abilities, although admittedly it varies from individual to individual, with better mathematicians in general displaying a wider spectrum. Gauss as a young man wavered between following a career of the study-

 $<sup>^{13}</sup>$  This happened to me once.

<sup>&</sup>lt;sup>14</sup> It is very striking that in introductory courses in philosophy, people with a mathematical bent take to logic formalization instantly as something they always seem to have known, while people lacking this are as perplexed as they are by the quadratic formula.

<sup>&</sup>lt;sup>15</sup> I remember how our lecturer in Christianity at high-school comforted some students by saying that many distinguished men of power knew no mathematics. I recall being on the verge of saying 'and see the sorry state of the world as a result' or words to that effect, but held my tongue being (at the time) shy and well-behaved.

ing of classical languages and mathematics, and in the heyday of the Tripos of Cambridge University during the 19th century those placing high in mathematics usually did so in the classics as well. The authors quote a list of suggested components of intelligence, supposedly being independent, but to me showing a high degree of interdependence. I would say that a good visual and spatial imagination are crucial in many areas of mathematics, much more so than musical ability. However, a penetrating discussion of the various abilities of mathematics, and how they may vary from mathematician to mathematician, and from discipline to discipline would be very interesting. Mandelbroit claimed that there were mathematicians of the eye and of the ear, the former, like him, visual (and visionary?) and intuitive, the latter formal and sequential, with good memory for details, and to be associated with music and algebra. This may be fanciful but could serve as a point of departure, but it clearly would go beyond the set bounds of the present book. In this context it may be relevant to make a short digression to laud mathematicians on an aspect they are seldom given credit for, namely intellectual honesty, something they may share with practical engineers dealing with machinery (and hence also computer programmers) and intimately connected with a platonic conception of mathematics. This is due to the fact, already alluded to, that the pursuit of mathematics are constrained by facts that cannot be ignored. Thinking has to comply with the facts, with truth, there simply is no room for wishful thinking, This also makes it easier for a mathematician to concede being in the wrong, as this is the way things are, and should not be taken personally unlike in other spheres of thought. It is also said that in the academic world of Nazi-Germany the mathematicians and the theoretical physicists were the least likely to be seduced by the propaganda, while those in the humanities were the most likely, in spite of being devoted to the human spirit. If true this is both a source of pride for mathematicians as being very disturbing. Finally, one should not succumb to the temptation to idealize. Just as there are many striking instances of great mathematicians with a wide culture, you may find that your colleague next door is a rather unremarkable fellow, with limited views and a constrained imagination, and of course closer to the norm than the intermittent exception you only know of because of being an exception.

But there is also a stronger case for mathematics. A more idealistic one. The essence of an education, in particular a liberal education, is to widen your mind and pursue things for their intrinsic interest, even if they have no immediate vocational applications being more geared towards individual intellectual enrichment. Now education is more and more seen as a right on par with clean water and medical services and the progress and wealth of a society is more and more seen in terms of how large a section of its population receives an education, invariably seen as a university one because of the added prestige. But is not mathematics part of the supreme achievements of human culture and to be ignorant of it means an intellectual deprivation. If people demand the right of an education should they not accept what it entails? In fact most of what is taught is not useful. History and geography are such examples, and indeed people in general show a shocking ignorance, although such subjects are seen as much more congenial and few people admit to history phobias (the most irksome aspect being the learning of dates). It would go too far afield to discuss why many esoteric subjects are indeed so much more easily accepted than mathematics. To take the natural aversion of mathematics seriously and respect it rather than trying to convert people, would logically lead to a division of education along classical lines of having one highly theoretical track for those for whom such pursuits come natural, and one more vocational where people would not bothered by being exposed to esoteric subjects of no practical interest to them. Such a development, logical as may appear, has generally been resisted on the basis of people being of equal worth, and thus deserving of equal treatment. And we are back to square one again.

Incidentally is there a much wider mathematical interest than what traditional school education succeeds in discovering and nurturing? The wide appeal to various puzzles and games, such as sudokus and chess seems to indicate that there is a dormant and potential interest among the members of the public waiting to be tapped. But for one thing, even a relatively small percentage translates into large numbers when put in a populous context. Even if mathematical competence is rare, we have seen that we can expect millions and millions of Americans having it, enough to support large publishing industry. Even people who regularly read, even if only best-sellers, may not amount to more than a fraction of the adult U.S. population. Of course you see people everywhere doing sudokus on the subway, but hardly anyone who reads a popular math book. Still it would be interesting to take part of actual statistics. Then there is another thing, is a fondness for puzzles and chess proof of mathematical ability? Obviously both require systematic and logical thinking, superficially at least not that different from the pursuit of mathematics, yet there seems to be a profound difference. Professional mathematicians are not particularly interested in sodukos (or cross-word puzzles for that matter, although the latter seems to have played a significant role when British intelligence was recruiting) nor especially good at  $chess^{16}$ , the exceptions, such as Lasker and Euwe, being striking enough to be listed, although of course neither of those were world-class mathematicians, if highly competent ones. I think that the interest of mathematics goes much deeper than a proclivity for games. It could be that some mathematicians are attracted to mathematics for its games aspects, not for the deeper existential. This would tie up with the discussion above on different kinds of mathematicians.

To return to the human encounter with mathematics. The title of the book would seem to refer to the emotional ambivalence people, especially mathematicians of course, have towards mathematics. That it is a matter of love and hate, the former being the first formed emotion, while the second would only gradually develop, due to the inhuman nature of mathematics. As the authors want to bring home to the reader, you can be very emotional about mathematics, but mathematics, unlike art, provides no way of expressing your emotions, which makes for frustration. Reading the book you could easily get the impression that either you love or hate mathematics. If you are a mathematician, you love the subject from day one more or less and pursue it ever happily after, while if mathematics is not your thing, you simply hate it. The truth is of course that no people are having more problems with mathematics than mathematicians, and while some of those may be seen as stimulating challenges, most of them are merely tedious and frustrating, and it takes a peculiar kind of determination to overcome them. The first aspect of mathematics as a filter, of being a supreme criterion of intelligence and mental power, with which

<sup>&</sup>lt;sup>16</sup> I have it from the testimony of reasonably capable chess-players among my mathematical colleagues (Vikram Mehta and Vasil Tsanov) that Serre and Faltings are lousy players

many mathematicians grow up with, and which may stimulate their competitive spirits, especially in their youth, as referred to by the Hardian phrase 'A young man's game', may invariably lead to specialization and concomitant sterility. One always hears about the writer's block, but there is also a mathematician's bloc, which could be much more serious. The competitive aspect of mathematics, which has its advantages as being an anti-dote against laziness and a whip to increase effort, the latter necessary for any serious work<sup>17</sup>, may in the end lead to burn-out and deep disenchantment. The second aspect of mathematics, of being a culture in which to revel, does to some extent ameliorate the dangers of competition, unless in sports, winning is not all, you can pursue a meaningful mathematical life without being the best<sup>18</sup>. Still one has to be realistic, and a suitable balance has to be struck, mathematics pursued without competition may be idyllic, but as a consequence it runs the risk of complacency.

Finally is mathematics an inhuman subject towards we all harbor some deep-rooted aversion to as human beings? Some cautionary tales seems to indicate that. On one hand that too deep an immersion may lead to madness, and the authors bring out the case of Bloch who cold-bloodedly killed members of his family, or the notorious una-bomber. Those cases are not so convincing though, what came first madness or mathematics? In addition none of them were very distinguished mathematicians, even if Bloch might have achieved attention even if not a murderer. Different it is with Grothendieck, whom the authors devote many pages to. Here we have a unquestionable mathematical genius, among many seen as the pinnacle of 20th century mathematics, whom one cannot accuse of clinical madness. Here we have a case of almost unparalleled immersion, which did ensue in a deep disenchantment with which many of us can sympathize.

Finally the sensitive issue of women and mathematics. The statistics is damning and to an outsider it clearly indicates oppression. But is the matter so simple? Mathematicians in general show no aversion to women in mathematics, on the contrary they are as individuals more than willing to bend backwards to accommodate what is perceived as female talent, many explicit examples can be given, but for obvious reasons should not. No people regret the low incidence of female mathematicians more than male mathematicians, more sincerely so than female mathematicians I would say<sup>19</sup>. I have alluded above to the intellectual honesty of mathematicians, people like Mittag-Leffler, Weierstraß and Hilbert promoted individuals such as Sofia Kovalevskaja and Emmy Noether, not primarily because of being feminists, but that they acknowledged genuine talent wherever they discovered it, even such a decided bigot as R.L. Moore (one of the bogey men of the book), could not help appreciate his star student Mary (married Rudin) just as the rhetorical anti-semite Shafarevich took great care of his Jewish students. Thus the lack of prevalence of women in mathematics is certainly not due to personal opposition, as testified by many (admittedly successful) women mathematicians. But if so what is it due to?

 $<sup>^{17}</sup>$  It is thus not surprising that those carrots and whips associated with competition are being enthusiastically endorsed by academic bureaucrats.

<sup>&</sup>lt;sup>18</sup> Initially my attitude towards mathematics was one of competition, thus it was an eye-opener when after having failed an advanced university exam while in high-school, to start afresh and realize that the subject was intrinsically interesting, and not only providing an arena in which to show-off your provess.

<sup>&</sup>lt;sup>19</sup> As a young lonely boy I longed for nothing more than then meeting a mathematically gifted girl.

Some institutional fault, or that women, forbidding thought, are genetically inferior? Now the lack of women is due not so much to active  $barring^{20}$  as the fact that so few women seem to choose mathematics. Why should women need more encouragement than men? Or do men get more encouragement automatically? As I have brought up before, many outstanding mathematicians have come from very humble backgrounds, in which there were no expectation whatsoever that they should rise above their stations. The example of Gauss should suffice. Admittedly the examples of the very great are so erratic that no statistical inferences are meaningful. But is it not true that the prejudices against the poor go deeper than gender and racial?<sup>21</sup> A statement like women are genetically inferior to men mathematically does of course not make any sense, or at least not any unique sense. For one thing it is very problematic to assign numbers to mathematical potential, especially without reference to achievement, although psychologists would love to do  $so^{22}$ . But let us for the sake of argument assume that we can assign a number to mathematical potential (this is a tall order indeed, but implicit in the kind of discussions we are about to comment upon), which will be distributed, but not necessarily normally. The average score for women may very well be higher than that of men, but the scores of the latter may be showing a larger spread, and hence there could be a much higher incidence of male mathematicians say at the top hundred or five hundred. Would that be so bad? This is in essence and idea suggested by the Harvard President Sumner, and which led to such an uproar that he had to step down. It can very well be that he was a bastard, and that his suggestion was made in a malevolent spirit, but the suggestion as such is innocuous. Let us digress a bit. Originally successful Olympic athletes tended to be White, even in running meets, now in the latter meets both on short and long distance the medalists tend to be Black. Formerly they did not have the opportunity, now they have. Should we say that Blacks (never mind the great racial diversity amongst them, among the many Negro tribes, there is one light-skinned) are genetically superior to Whites, at least when it comes to running (or that they are deprived are more fruitful alternatives to achieve recognition, but it is hard to see Bolt as a loss to mathematics). White people could still be better on the average, with the very top of the spectrum dominated by Blacks. Who would care?

 $<sup>^{20}</sup>$  The authors bring up the case of a Berkeley woman who was denied tenure at the department. The reviewing process for tenure, or any other evaluation of mathematicians, is far from fool-proof, for one thing it is not part of mathematics but the social practice of mathematics, and as such open to all kinds of prejudices personal or cultural, of which gender prejudices can of course never be ruled out. In that particular case I have no doubt that she was treated unfairly, but how can we be so sure that it was because she was a woman? Of course to bring this issue into the debate was smart, only then would she have had any chance of redress, But what if she had been a man? What then could (s)he have brought up? Who would have cared? Shit happens.

 $<sup>^{21}</sup>$  A South-African colleague, of impeccable liberal instincts, has informed me that the new South-Africa entailed the liberation of a Black elite of a few millions, but the lot of the great majority remains materially at least unchanged. That class divisions go deeper than mere racial.

<sup>&</sup>lt;sup>22</sup> There is in the book a reference to that women do not show any difference than men on cognitive tests on mathematical ability. This is a remarkably sweeping statement, implying that there is a psychological test, which supposedly in a limited time can give a clearer indication of an individuals potential as a mathematician, than a lifetime career developing that potential. How can such a test be tested?

That men are genetically superior to women when it comes to sports is something that has had to be accepted. They compete in different divisions, this is a hard fact of life which no amount of mental pep-talk can change. Now among primates there is a rather pronounced physical dimorphism, more so then among many other mammal species. Why should there not be a corresponding difference in the mental sphere (not necessarily to the advantage of men, as we note there are very many different mental capabilities as the authors have reminded us). If you take an idealistic philosophical point of view, such would be anathema (much can be said for philosophical idealism), but from a materialistic Darwinian perspective, which ostensibly dominates the present scientific ideology, any opposition would be a case of sentimentalism<sup>23</sup>. Could it be that women are wiser and more human and thus shy away from mathematical obsessions? Any number of hypotheses can be concocted.

January 1-3, 2016 Ulf Persson: Prof.em, Chalmers U.of Tech., Göteborg Sweden ulfp@chalmers.se

 $<sup>^{23}\,</sup>$  Wallace exempted the human brain from evolution, Darwin had no such computcions