1 Consider the space of all $n \times n$ matrices, given by the entries x_{ij} and consider the hypersurface given by $\det(x_{ij}) = 0$

a) Compute the dimension of all $n \times n$ matrices!

b) Compute the dimension of all singular $n \times n$ matrices!

c) Define inductively V_{k+1} as the singular locus of V_k and $V_0 = \det(x_{ij}) = 0$ and interpret those in terms of ranks of matrices.

d) Try to compute the dimensions of the various V_k!

2 The set up as in the previous problem, but now restrict yourself to symmetric matrices.

3 Consider the linear space $M(2,2,k)$ of all 2×2, removing the zero matrix and identifying two matrices if they differ by multiplication by λI where $\lambda \in k^*$ and I is the identity matrix.

a) Show that we get a natural 3-dimensional projective space $P(M(2,2,k))$.

b) Show that the singular matrices make up a non-singular quadric in that space.

c) For any line $L \in k^2$ define the subsets $K_L = \{A \in M(2,2,k) : \ker(A) = L\}$ and $I_L = \{A \in M(2,2,k) : \mathrm{im}(A) = L\}$, show that those make up lines in $P(M(2,2,k))$

d) Consider the points $M_L \cap I_L$ for all $L \in P^1(k)$ what is the degree of that variety in $P(M(2,2,k))$?

4 Let $f = ax^2 + bx + c$. Compute the resultant of the polynomials f and f' and interpret it.

5 Let $k = \mathbb{F}_q$ be a finite field. Compute the number of elements of the Grassmannian $G(m,n)$!

6 Consider the Grassmannian $G(k,n)$ and let H be a fixed hyperplane in P^n. Show that we have a decomposition of $G(k,n)$ into two sets one closed consisting of $V \in G(k,n)$ such that $V \subset H$ and one open of $V : \dim(V \cap H) < k$ and use this to establish a recursive formula for the euler numbers of Grassmannian. In particular compute $e(G(5,8))$. Finally what is the relation with this problem and the previous?

7 Consider a pencil of quadrics in P^2. This means giving quadrics of the form $s_0 Q_0 + s_1 Q_1$.

The intersection of $V(Q_0)$ and $V(Q_1)$ are called the base-points of the pencil.

a) Show that there are in general 4 distinct base points.

b) If Q is a quadric passing through the four distinct base points then it belongs to the pencil. (Show that there is a member of the pencil that intersects $V(Q)$ in five points

c) How many singular members does the quadric have? (Show that a quadric is singular iff it is the union of two lines).

d) Compute the euler-number of the fibration. Why does it differ from that of P^2

8 The three quadrics xy, yz and zx pass through three points in P^2. Which? Show that if you perturb those quadrics ever so little their intersections will be empty.