Exercises

Linear and Multilinear Algebra

September 4 2009

due September 18 2009

`Starred exercises (*) are somewhat more involved`

1 The map $x \mapsto ax$ defines for every a a permutation. Determine its cyclestructure.

2 Let G be a group and the map

$$G \to Aut(G)$$

given by conjugation that is
$a \mapsto (x \mapsto axa^{-1})$ also known as an inner automorphism

a) Show that the map is a homomorphism
b) Find its kernel
c) Show that the image of G makes up a normal subgroup to $Aut(G)$

The quotient $Aut(G)/G$ is usually known as the group of outer automorphisms

3 List all the conjugacy classes of S_6 and give the number of elements in each. Do the same for $GL(2,5)$.

4 Determine the automorphism groups for the following groups
a) A cyclic group Z_n
b) A dihedral group D_{2n} note the difference between n odd or even.
c) A non-commutative group $(Z_{p/q})$ of order pq, with p,q primes and $p|q-1$

5 Show that if $Z_{p/q}$ as above occurs as a subgroup in the symmetric group S_n then $n \geq q(> p)$. Is it sharp, i.e. can we find it inside S_q?

6 Look at the group of invertertible 2×2 matrices. Give a simple condition on such a matrix A for it to be conjugate with his invers.

7 Let G be a group, Z contained in its center and assume that G/Z is cyclic. Show that G is abelian.

8 * Find all non-commutative groups of order p^3 where p is a prime. In particular investigate the case $p = 2$ i.e. find all non-commutative groups of order 8

Hint: Show that for such a group, the center is identical with the commutator both being cyclic of order p and the quotient G/Z is isomorphic with $Z_p \oplus Z_p$