Facit till Övningsuppgifter II

MAN 230
30/1 2008

1 a) $2x, 2y, 2z$
b) $\frac{-2x}{(x^2+y^2+z^2)^2}, \frac{-2y}{(x^2+y^2+z^2)^2}, \frac{-2z}{(x^2+y^2+z^2)^2}$
c) $2x, -2y2$
d) $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$
e) $\frac{yz(y+z)}{(x+y+z)^2}, \frac{xz(x+z)}{(x+y+z)^2}, \frac{yx(y+x)}{(x+y+z)^2}$
f) $\frac{1}{2}\cos(\sqrt{x^3+y^3+z^3})\frac{3x^2}{\sqrt{x^3+y^3+z^3}}$
$\frac{1}{2}\cos(\sqrt{x^3+y^3+z^3})\frac{3y^2}{\sqrt{x^3+y^3+z^3}}$
$\frac{1}{2}\cos(\sqrt{x^3+y^3+z^3})\frac{3z^2}{\sqrt{x^3+y^3+z^3}}$
g) $-\frac{2x}{\sqrt{1-(1-x^2-y^2)^2}}, -\frac{2y}{\sqrt{1-(1-x^2-y^2)^2}}$

2 Sätt $u = (x^2+y^2)/2, v = xy$ Vi erhåller då via kedjeregeln och det faktum att om $(x,y) = (1,1)$ gäller att $(u,v) = (1,1)$

$$\frac{\partial F}{\partial x}_{(1,1)} = \frac{\partial F}{\partial u}_{(1,1)}\frac{\partial u}{\partial x}_{(1,1)} + \frac{\partial F}{\partial v}_{(1,1)}\frac{\partial v}{\partial x}_{(1,1)}$$

Vi har att $\frac{\partial F}{\partial u}_{(1,1)} = 2$ och $\frac{\partial F}{\partial v}_{(1,1)} = 3$. Vidare har vi $\frac{\partial u}{\partial x}_{(1,1)} = x = 1$, $\frac{\partial v}{\partial x}_{(1,1)} = y = 1$. Således $\frac{\partial F}{\partial x}_{(1,1)} = 2 \times 1 + 3 \times 1 = 5$. På samma sätte erhålles $\frac{\partial F}{\partial y}_{(1,1)}$

3 Riktningsvektorn är given av $(1/\sqrt{2}, -1/\sqrt{2})$. Vi har

$$\frac{\partial F}{\partial x} = \frac{2x}{(x^2+y^2)^2}$$
$$\frac{\partial F}{\partial y} = \frac{2y}{(x^2+y^2)^2}$$

Således

$$\frac{\partial F}{\partial x}_{(1,0)} = 2$$
$$\frac{\partial F}{\partial y}_{(1,0)} = 0$$

Således lutningen är $(1/\sqrt{2})2 + (-1/\sqrt{2})0 = \sqrt{2}$
Lutningen blir noll om

$$\frac{2x-2y}{(x^2+y^2)^2} = 0$$

d.v.s $x = y$ och eftersom $x + y = 1$ får vi $x = y = 1/2$

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a) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

b) $\begin{pmatrix} 2x & -2y & y & x \end{pmatrix}$

c) $\begin{pmatrix} \frac{1}{1-z} & 0 \frac{x}{(1-z)^2} & \\ 0 & \frac{y}{1-z} & \frac{y}{(1-z)^2} \end{pmatrix}$

d) $\begin{pmatrix} \frac{2(1+v^2-u^2)}{(1+u^2+v^2)^2} & \frac{4uv}{(1+u^2+v^2)^2} \\ \frac{4uv}{(1+u^2+v^2)^2} & \frac{2(1+u^2-v^2)}{(1+u^2+v^2)^2} \\ \frac{4u}{(1+u^2+v^2)^2} & \frac{4v}{(1+u^2+v^2)^2} \end{pmatrix}$

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$$f = r^2 \cos\theta = (x^2+y^2)(x/r) = x\sqrt{x^2+y^2}$$

Således

$$\frac{\partial f}{\partial x} = \frac{x^2}{\sqrt{x^2+y^2}} + \sqrt{x^2+y^2}$$
$$\frac{\partial f}{\partial y} = \frac{xy}{\sqrt{x^2+y^2}}$$

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$$\partial f/\partial x = (\partial f/\partial r)(\partial r/\partial x) + (\partial f/\partial \theta)(\partial \theta/\partial x) + (\partial f/\partial r)(\partial \psi/\partial x)$$
$$\partial f/\partial y = (\partial f/\partial r)(\partial r/\partial y) + (\partial f/\partial \theta)(\partial \theta/\partial y) + (\partial f/\partial r)(\partial \psi/\partial y)$$
$$\partial f/\partial x = (\partial f/\partial r)(\partial r/\partial z) + (\partial f/\partial \theta)(\partial \theta/\partial z) + (\partial f/\partial r)(\partial \psi/\partial z)$$

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a) $z = x^2 + y^2$

b) $z = 2 + 2(x-1) + 2(y-1)$

8 a) $3\sqrt{2}$

b) $-1/4$

c) $\frac{1+(2\log 2-1)e^2}{\sqrt{2}(1-e^2)^2}$