Uppgifter för Räkneövning Torsdag 6/4

1 Write down the partitions of 6 in standard notation

2 Show that for any two permutation σ, τ the products $\sigma\tau$ and $\tau\sigma$ are conjugate.

3 List the types of permutations in S_7 which are derangements

4 Decide whether the permutation

$$(12)(345)(678)$$

is odd or even

5 In the permutation above find out how many permutations in S_8 commute with it.

6 Write down the partitions of 6 which correspond to the cycle structure of even permutations

7 Two permutations of cycle structure $[5]$ in S_5 are always conjugate. But are they always conjugate with respect to an even permutation?

8 A permutation of a deck of cards is effected by cutting it into two piles and then transposing them. Call such a permutation a cut, and there are obviously fifty-two different cuts. When are cuts even and odd permutations? Try to show that the cuts generate all permutations.

9 As above but restrict yourself to a deck of six cards [123456] one example of a cut is [1234][56] leading to [56][1234]. Write down all six cuts and their cycle structures.

10 Get an approximation of $n!$ by considering $\ln n! = \sum_{k=1}^{n} \ln(k)$ and comparing this sum with the integral $\int_1^n \ln(t)dt$