**Hemuppgift II**

*8/5*

**1** Calculate the coefficient of $xyz$ in $(x+2y+3z)^3$

**2** Compute the euler-number $\phi(999)$

**3** Show that the eighteen powers of two $\{2, 4, 8, 16, 13 \dots 1\}$ are all distinct, thus show that the map $n \mapsto 2^n$ defines a permutation of the eighteen non-zero objects of $\mathbb{Z}_{19}$.

a) Write down the cycle representation of this
b) Determine whether the permutation is even or odd
c) Compute the number of permutation of $S_{18}$ which commutes with it.
d) Find the inverse of 13 in $\mathbb{Z}_{19}$ and the two square-roots of 7

**4** Show that 17 divides $n = 10^8 + 1$ and compute the last two digits of $m$ where $n = 17 \times m$ without performing any long division.

**5** Find the last digit of $2^{100}$ and try to find the last two.

**6** Try to find three distinct primes $p_1, p_2, p_3$ such that if $n = p_1 p_2 p_3$ we have that if $(a, n) = 1$ then $a^{n-1} = 1(n)$. Such numbers are refered to as pseudo-primes or Carmichel primes.

**7** Let $p$ be a prime and $N$ the sum of all quadratic residues. Compute $N$

**8** Compute the number of words of length 12 in the alphabet $\{a, b\}$ such that the combination $\dots bb \dots$ is prohibited.