HOMEWORK, COMPLEX ANALYTIC VARIETIES 2016-2017

1, *due Feb 10th*: Do the details in the first step in Demailly's proof of the strong Noetherian property, Theorem 3.22, i.e., prove that is enough to assume that $\mathcal{F} = \mathcal{O}_M$.

2, *due Feb 17th*: Let A be the cusp in $\mathbf{C}^2$, defined by $A = \{z^3 - w^2 = 0\}$. Let $x = (a, b)$ be a point on A. Show that $(w - b)/(z - a)$ is a unit in $\mathcal{O}_{A,x}$ if and only if x is not the origin in $\mathbf{C}^2$.

3, *due Feb 17th*: Show that two equivalent definitions of analytic morphisms given in the very beginning of Section 5 in Demailly are equivalent. Show at least one direction, for the other you may have a look at de Jong - Pfister, Theorem 3.4.21.

4, *due Feb 24th*: Given a complex space X, show that the family $\mathcal{F}$ of irreducible analytic subsets $Z \subset X$, where

- Z is an irreducible component of X,
- Z is an irreducible component of Z'_{sing}, where $Z' \in \mathcal{F}$, *or*
- Z is an irreducible component of a finite intersection $\bigcap Z_j$, where $Z_j \in \mathcal{F}$,

is locally finite.

5, *due March 3rd*: Let $A \subset \Omega \subset \mathbf{C}^n$ be an analytic set. Given an $\mathcal{O}_A$-module $\mathcal{S}$, let $\tilde{\mathcal{S}}$ be the extension of $\mathcal{S}$ to Ω defined by $\tilde{\mathcal{S}}(U) = \mathcal{S}(A \cap U)$. Show that $\mathcal{S}$ is coherent over $\mathcal{O}_A$ if and only if $\tilde{\mathcal{S}}$ is coherent over $\mathcal{O}_\Omega$.

6, *due March 3rd*: In the proof of Proposition 6.1 in Demailly, prove the statement: By construction $\pi(A_k) \subset \Delta'$ is contained in the set B_k defined by some Weierstrass polynomials in the variables $z_{p_k+1}, \ldots, z_p$ and $\operatorname{codim}_{\Delta'} B_k = p - p_k \geq 2$.

7, *due March 10th*: Prove Corollary 6.3 in Demailly. Note that there is a typo in Demailly. The statement should be: If $f_1, \ldots, f_p$ are holomorphic functions on an irreducible complex space X, then all irreducible components of $f_1^{-1}(0) \cap \cdots \cap f_p^{-1}(0)$ have codimension $\leq p$.

8, *due March 10th*: Exercise 11.11 in Demailly.

9, *due March 17th*: Exercise 11.3.c in Demailly.

10, *due March 17th*: Let $\mathcal{F}$ be the sheaf defined in the proof of Theorem 7.5 in Demailly. Show that $\mathcal{F} \neq \tilde{\mathcal{O}}_X$ in general.

Date: March 7, 2017.