Intensive course on stochastic traffic modeling

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Ingemar Kaj
ikaj@math.uu.se
Department of Mathematics
Uppsala University

UPPSALA
UNIVERSITET

Outline of talks

1. Introductory models
2. Packet delay in Voice over IP
3. Multiaccess systems, Slotted Aloha

3/4. CSMA/CD, Collision resolution

4/5. Congestion control, TCP, mean-field approach

Introductory models

- Standard models
  - Discrete and continuous time Markov models
  - Jackson networks
  - Non-Markov service systems, M/G/1, M/G/∞
  - Load-throughput analysis, Performance measures
- Client-server web model (Jackson)
- Reliable data transfer (Renewal)
- Hand-off termination probability (Renewal)
- Switching, Head-of-line blocking (Markov)

Acknowledgement based reliable transfer

Stop-and-Wait, Go-Back-N, Selective Repeat

Send window of N packets, wait for ACKs. If all packets acknowledged within the round (round-trip-time) continue with next round. If not, (re)transmission is resumed T_0 rounds later. The protocols vary in their retransmission policies. Assume packets are lost independently with probability p. Let

$$K = \# \text{ rounds until a packet is lost} \in \mathrm{Ge}_+(p)$$

and associate a renewal process N_t with the successive intervals $U_i = K_i + T_0$. The reward of cycle i is e.g. $R_i = K_i - 1$ (retransmit all packets in the first loss round of the cycle). Then

$$\mathrm{Throughput}_{\mathrm{GBN}} = \lim_{t\to\infty} \frac{1}{t} \sum_{i=1}^{N_t} R_i = \frac{E(R_1)}{E(U_1)} = \frac{N(1-p)^N}{1+(1-(1-p)^N)T_0}.$$

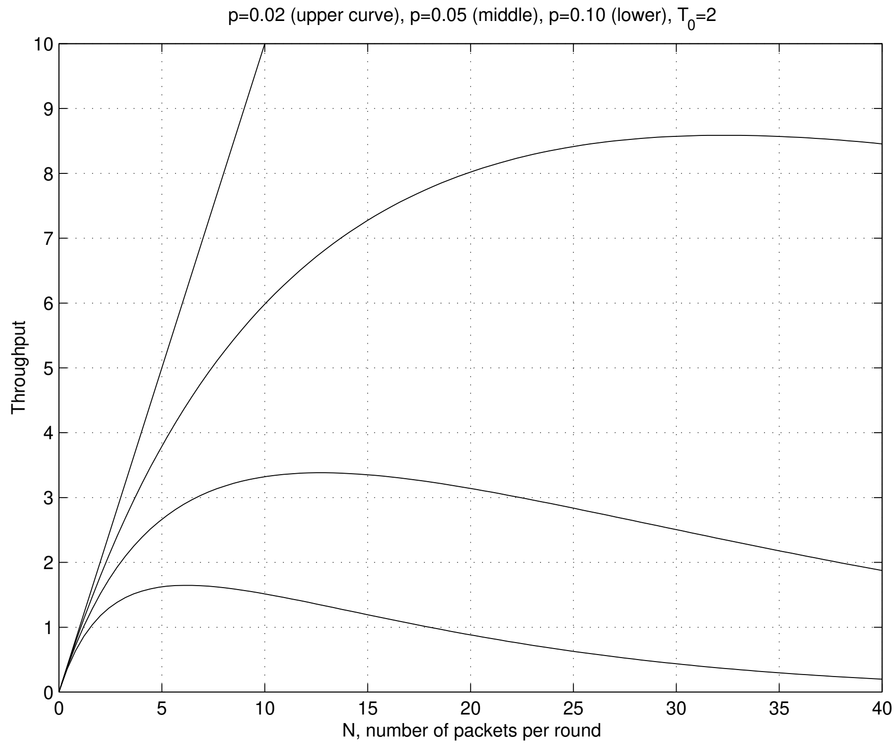

Figure 1: Load-throughput relation for simplified Go-Back-N

Hand-off termination probability

$$M = \#\text{hand-off transitions until first forced termination} \in Ge_+(p_f)$$

$$P_{\text{term}} = P(N_S \geq M), \quad N_t = \text{intercell renewal process}\ , \quad S = \text{call holding time} \in \text{Exp}(\mu)$$

$$P_{\text{term}} = \frac{p_f}{\mu\, E(U_2)} \frac{1 - E(e^{-\mu U_2})}{1 - (1 - p_f) E(e^{-\mu U_2})}, \qquad (U_i)_{i \geq 1} \quad \text{intercell times.}$$

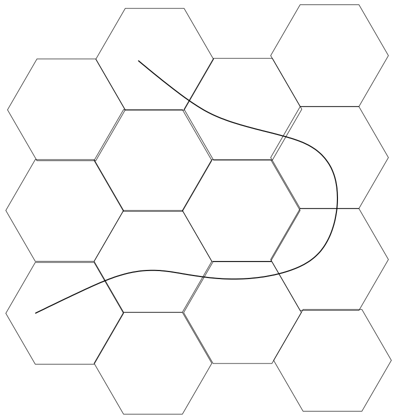

Figure 2: Mobile phone unit requesting new channels during call

Crossbar model, head of line blocking

- m input ports, m output ports;
- Slotted time;
- A cell appears at each input port with probability p in each slot;
- Cells are equally likely to be destined to any output;
- Usual independence assumptions;
- If two or more cells are contending for the same output, one randomly chosen cell is transmitted, the other remains backlogged at input;
- New cell arriving to an input occupied by backlogged user is lost;

Analysis of loss crossbar model

Put

$K_n^j = \#$ backlogged input lines in slot n with cells destined for output j

$I_n^j = \#$ new cells arriving in slot n destined for output line j.

The switch throughput, i.e. the fraction of successfully transmitted cells per slot, is

$$T_n^{(m)} = \frac{1}{m} \sum_{j=1}^{m} 1_{\{K_n^j + I_{n+1}^j \geq 1\}}.$$

Key observation:

Given the K_n^j's, the family $\{I_{n+1}^j\}$ has the multinomial distribution Multnom$(Z_n, \frac{1}{m}, \ldots, \frac{1}{m})$, where $Z_n = \sum_{j=1}^m I_n^j$ is binomial Bin$(m - \sum_{j=1}^m K_n^j, p)$ distributed.

Result: HOL-blocking results in the throughput reduction

$$E(T_\infty^{(m)}) \geq \frac{1 + p - \sqrt{1 - 2p + 3p^2}}{2 - p}; \quad \text{As } p \to 1 \quad E(T_\infty^{(m)}) \geq 2 - \sqrt{2} = 0.5858...$$