A stochastic model for packet delay in Voice over IP

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In preparation

VoIP data trace

Internet telephony systems convert a PCM voice signal into a sequence of IP packets. Silence suppression mechanisms detain packetization during silent periods (analog signal below dB threshold). Transmit over the Internet, measure packet arrival times at receiver.

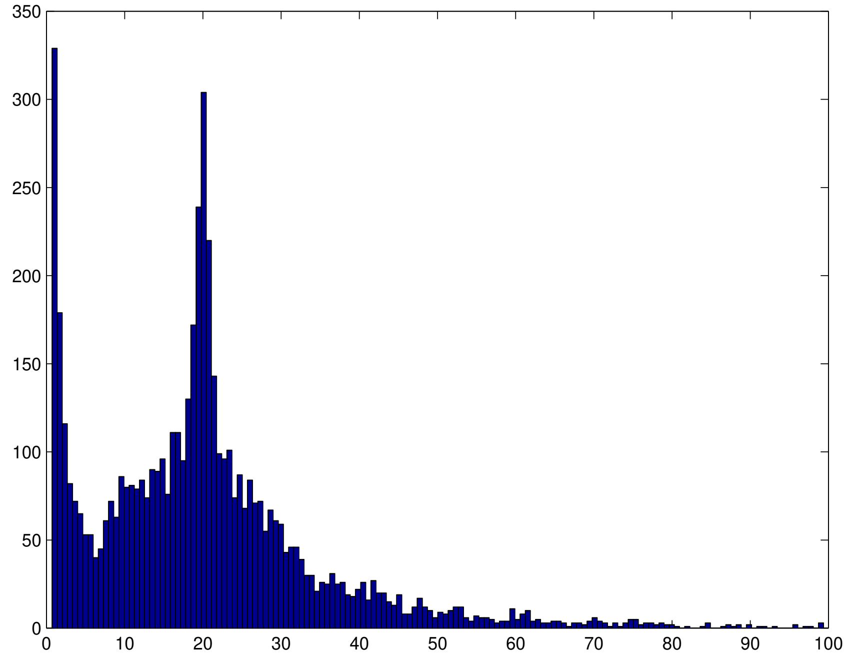


Figure 1: Interarrival times (short), VoIP Argentina-Sweden

Arrival time structure

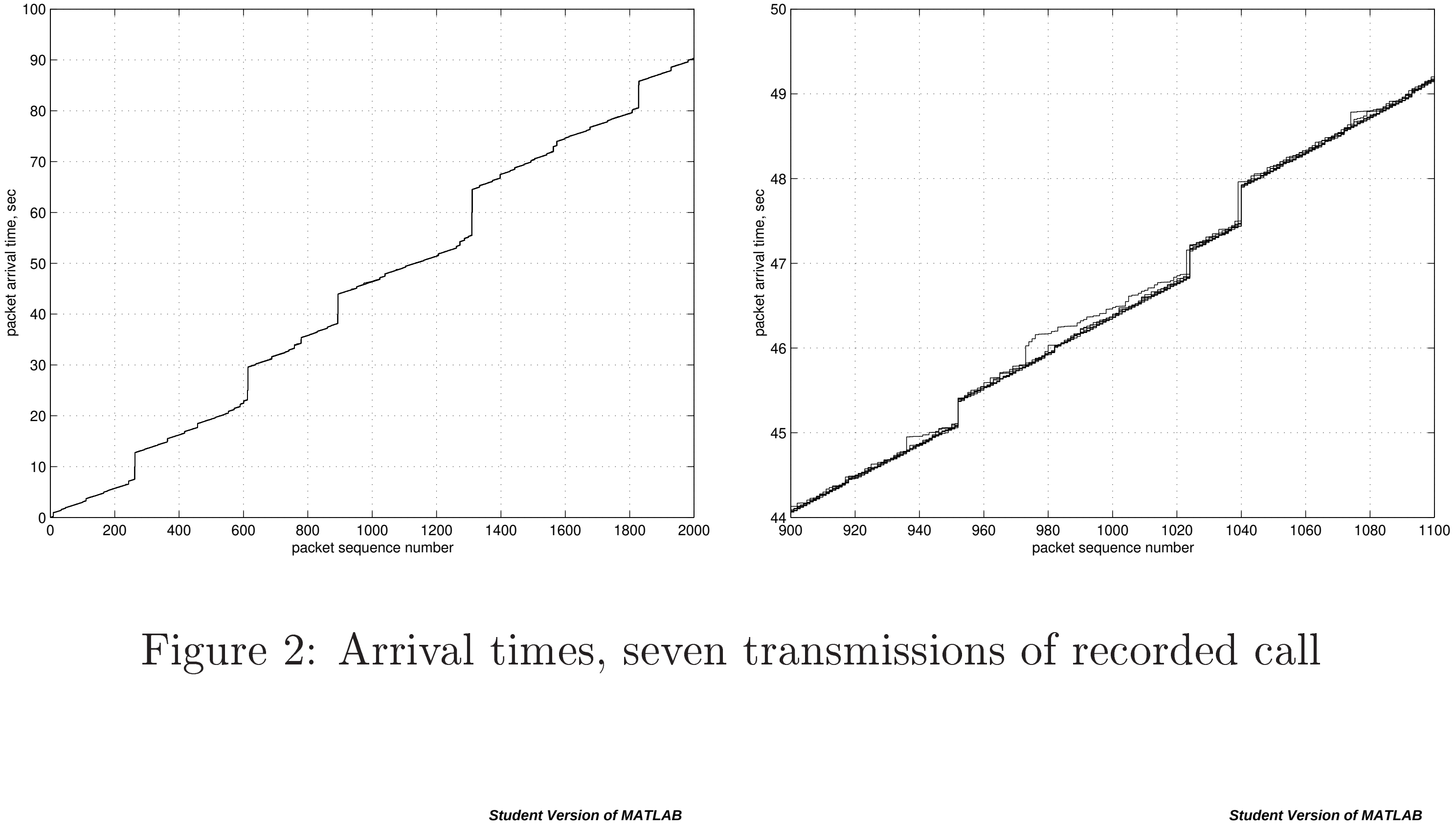
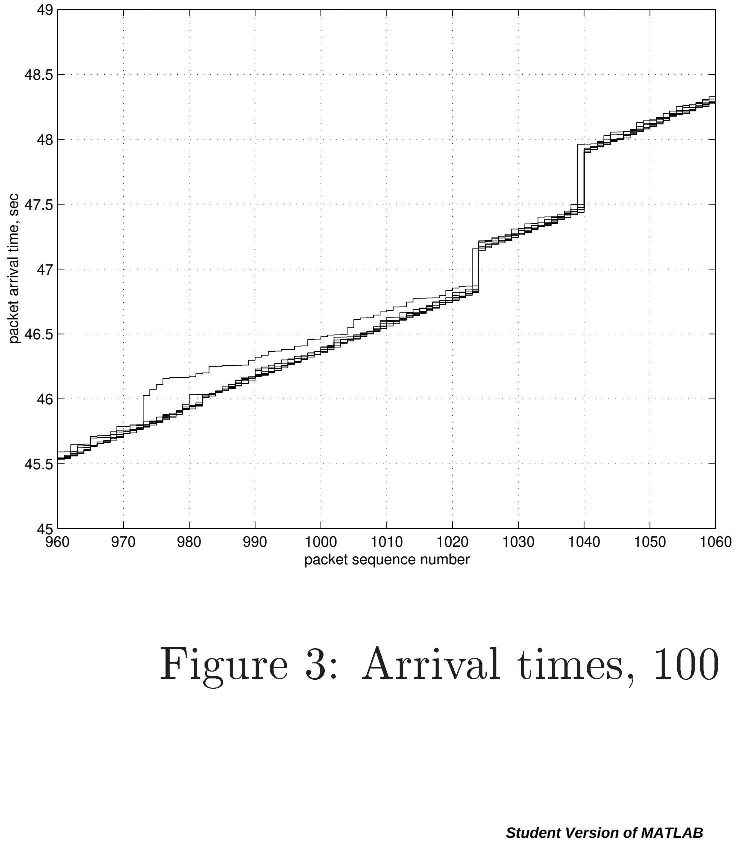


Figure 2: Arrival times, seven transmissions of recorded call

Arrival time structure, cont'n



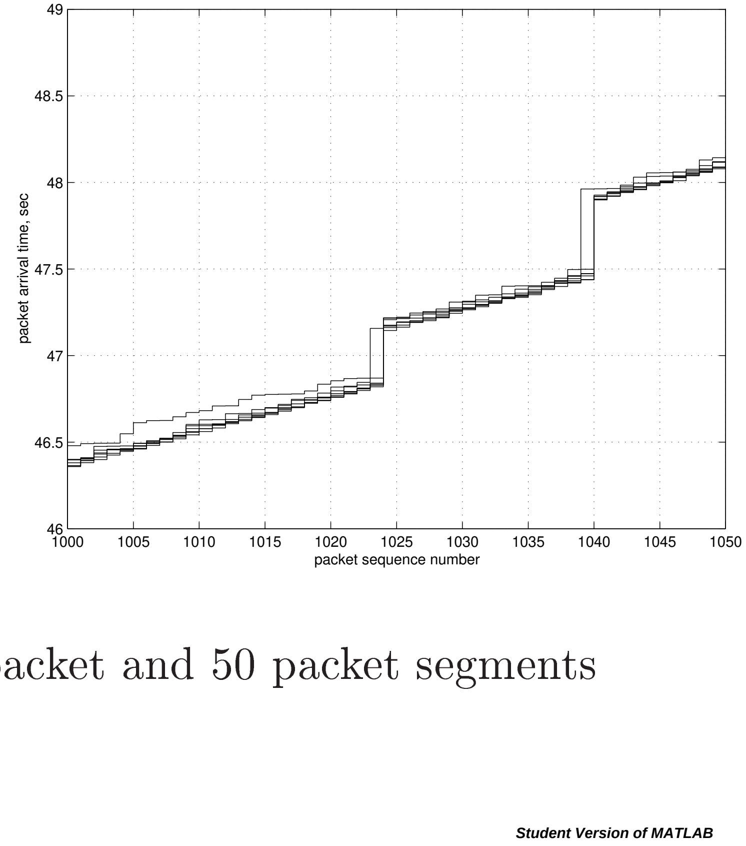


Figure 3: Arrival times, 100 packet and 50 packet segments

Notation and basic model assumptions

- Packetizer: 160 byte IP packets, $8000/160 = 50$ per second, 20 ms packetization interval;
- Time units: packet k arriving at time t_k means clock time $20 \cdot t_k$ ms;
- Silence suppression: Silent period of length X_k between packets no $k-1$ and k;
- Departure time packet k: $k - 1 + S_k$, $S_k = \sum_{i=1}^{k} X_i$;
- Transmission time if packet k could be isolated from rest of stream: Y_k;
- Sequences (X_k) and (Y_k) are i.i.d. and independent of each other;
- Typical(?) statistics: $\alpha = P(X_k > 0) \approx 3\%$, $\mathrm{E}(X_k | X_k > 0) \approx 25$ time units ≈ 0.5 sec;

Consequences of the model assumptions

Put

$$T_k = \text{arrival time at receiver of packet } k, \quad k \geq 1.$$

Delivery of packet k from the sending unit begins at time

$$k - 1 + S_k \qquad S_k = \sum_{i=1}^{k} X_i.$$

The packet spends the time Y_k propagating through the network and arrives at the receiver at time $k - 1 + S_k + Y_k$, unless it overtakes one or several of the previously transmitted packets numbered 1 to $k - 1$ and in this way is forced to adapt to the slower pace of packets ahead in line. Since voice packets must arrive in the same order as they are sent these considerations show that the actual arrival times are given by

$$T_1 = S_1 + Y_1, \quad T_k = \max(T_{k-1}, k - 1 + S_k + Y_k), \quad k \geq 2.$$

It follows directly that the observed packet interarrival times are obtained from

$$U_k = T_k - T_{k-1} = \max(0, k - 1 + S_k + Y_k - T_{k-1}) \quad k \geq 2.$$

Analysis of the model, asymptotic moments

As $k \to \infty$

$$E(U_k) \to 1 + E(X_1)$$

$$\mathrm{Var}(U_k) + \mathrm{Cov}(U_k, T_{k-1}) \to \mathrm{Var}(X_1)$$

This can be derived from

$$T_k = T_0 + \max(Y_1 + S_1, 1 + Y_2 + S_2, \ldots, k - 1 + Y_k + S_k), \quad k \geq 1,$$

and the alternative representation

$$T_k = X_1 + \max(Y_1, 1 + T'_{k-1})$$

where on the right side

$$T'_{k-1} = \max(Y_2 + S_2 - X_1, 1 + Y_2 + S_2 - X_1, \ldots, k - 2 + Y_k + S_k - X_1)$$

has the same marginal distribution as T_{k-1} but is independent of X_1 and Y_1.

Steady state analysis, conditional on silencer

Given a fixed realization $s_k = \sum_{i=1}^k x_i$, $k \geq 1$, of the silence suppression intervals the following steady state distributions appear: The observed delays

$$V_k = \text{arrival time - departure time} = T_k - k + 1 - s_k, \quad k \geq 1$$

form a Markov chain with real positive states and asymptotic distribution

$$P(V_\infty \leq x) = \prod_{i=0}^{\infty} F(x + i + s_i), \quad x \geq 0, \qquad F(y) = P(Y_k \leq y).$$

The sequence of interarrival times (U_k) has the asymptotic distribution

$$P(U_\infty \leq x) = 1 - \int_0^\infty \prod_{i=1}^{\infty} F(y - x + i + s_i)\, dF(y) \quad x \geq 0,$$

in particular, a pointmass in zero of size

$$P(U_\infty = 0) = 1 - \int_0^\infty \prod_{i=1}^{\infty} F(y + i + s_i)\, dF(y).$$

Statistical problem

The silence suppression intervals (X_k) are created during conversion from PCM voice to IP packets, i.e. before transmission. This leads to a two-step (naive) statistical procedure:

1. Observe at sender departure times for streaming IP packet sequence: $d_k = k - 1 + s_k = \sum_{i=1}^{k-1}(1 + x_i) + x_k$, $k \geq 1$ (measured in 20 ms slots);

2. Observe packet delay at receiver: $V_k = T_k - d_k$. Find empirical distribution $\bar{F}_V(x)$ corresponding to steady state V_∞. Since

$$P(V_\infty \leq x) = \prod_{i=0}^{\infty} F(x + i + s_i), \quad x \geq 0, \qquad F(y) = P(Y_k \leq y)$$

estimate distribution function of Y by

$$\hat{F}_Y(y) \approx \bar{F}_V(y) / F_V(y + 1 + s_1).$$