EULER CHARACTERISTICS AND EXCEEDENCE PROBABILITIES

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For half a century Rice's formula and its variants (which tell one the mean number of times a smooth random curve crosses a fixed level) have been used to approximate the exceedence probability of actually crossing the level. The approximations are generally very good for high levels, and for almost two decades there have been rigorous results explaining why this should be the case.

In dealing with random surfaces, or higher dimensional random processes, curve crossing counts have, for almost two decades, been replaced with an application of the Euler characteristic. Furthermore, mean Euler characteristics have been used, with great success, but without rigorous justification, to approximate exceedence probabilities also in this setting.

After introducing Euler characteristics, I will attempt to explain WHY these applications have been so successful, and also relate them to the currently popular "tube" approximation technique. One of the consequences of the discussion will be to show that Euler characteristic approximations almost always beat tube techniques when both are applicable.