Extreme value theory for M/G/1 type queueing models with subexponential increments

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Abstract

For reflected random walks $\{L_n\}$ with heavy–tailed (subexponential) distribution F of the increments $X_1, X_2, \ldots$, it has recently been shown (the speaker [1] and Heath, Resnick & Samorodnitsky [6], independently) that $\max_{1,\ldots,n} L_k$ has the same asymptotic properties as $\max_{1,\ldots,n} X_k$.

Behind this result is a general heavy–tailed heuristics, predicting a large value to occur as consequence of one big jump. Therefore, it seems reasonable to expect a similar extreme value behaviour in a rather wide class of models with heavy–tailed jumps. For example, if $\{(J_n, L_n)\}$ is an ergodic Markov chain with state space $E \times \mathbb{R}$, stationary distribution $(\pi_i)_{i\in E}$ for $\{J_n\}$ and

$$\mathbb{P}(L_1 > x \mid J_0 = i, L_0 = y) \ \sim\ c_i\overline{F}(x)$$

for some subexponential F and all i, y, it should typically hold that

$$\mathbb{P}\left(\max_{1,\ldots,n} L_k > x\right) \ \sim\ n\overline{F}(x)\sum_{i\in E} c_i\,. \tag{1}$$

However, making this rigorous has proved remarkably difficult (one example, storage processes, has been treated in [1]).

In the present talk, based on joint work [3] with Jakob Riishede Møller, we show how to establish (1) for M/G/1 type queueing models. This class is treated extensively in Neuts [8] and incorporates many examples of relevance in telecommunications; here heavy tails is currently of considerable interest because of recent statistical studies indicating their presence say in session lengths on the Internet, package

lengths in communications networks etc. Mathematically, the process $\{(J_n, L_n)\}$ can be viewed as an reflected Markov additive process with modifications at the boundary $L = 0$ (L represents the level and J the Markov component).

The approach is to involve maxima over regenerative cycles, as done extensively by other authors ([5], [7], [9], [1], [6], [2]). The problem is the boundary modification since with simple reflection, the desired result can be deduced from [4]. To this end, random walk comparisons are used.

Apart from extremal behaviour, an important corollary of the tail asymptotics for the cycle maximum is tail estimates for the stationary distribution of L. Again, this is of relevance in telecommunications since such asymptotics is used to quantify buffer overflow probabilities etc.

References

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