Models for Extremes of Markov Chains under Asymptotic Independence

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In this talk we consider models for the extremal behaviour of $\{X_i\}_{i\geq 1}$, a stationary first order Markov chain with continuous state space and asymptotically independent adjacent variables. When consecutive pairs are asymptotically independent, as in Gaussian autoregressive processes, modelling procedures based on multivariate extreme value distributions (Smith, Tawn and Coles, 1996) fail to give an accurate representation of the extreme events of the chain. This is because such models imply a stability in the dependence at high levels, whereas in Markov chains having asymptotically independent transitions the degree of dependence between exceedances of x generally decreases as x tends to the upper endpoint of the univariate marginal distribution, with the extremal behaviour of the chain increasingly resembling that of an independent and identically distributed series. We propose a penultimate approach which allows the approximations to change with x and to converge ultimately to classical extreme value limits. This is achieved in two steps: first, by replacing bivariate extreme value distributions with a broader class, introduced by Leford and Tawn (1996, 1997); secondly, through a simulation algorithm that mimics the local behaviour of $\{X_i\}_{i\geq 1}$ as the threshold level increases. Analytical expressions for some measures of the clustering of extremes are also considered.

References

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