Clustering and dependence in extreme market returns

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Abstract

Several recent studies have applied concepts from extreme value theory to stock returns and foreign exchange rates. However these studies assume that stock returns form an IID or weakly dependent sequence verifying a local condition which excludes "clustering" of extremes (Leadbetter's D' condition). This hypothesis is not realistic in the case of stock returns, which exhibit the well known phenomenon of "volatility clustering". After reviewing of the various approaches used to estimate tail behavior of market returns and comparing their results, we propose to extend these studies to take into account the dependence using the concept of the *extremal index* of a stochastic process.

We discuss the notion of extremal index both from a probabilistic and a statistical point of view, discuss its interpretation in financial terms and its relation to the phenomenon of volatility clustering in financial time series.

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These concepts are applied to several data sets of stock, index and futures prices, both at daily and intraday frequencies. We propose an empirical method for detecting to what extent a non-trivial value of the extremal index is related to dependence in the time series under study.The results are compared with Monte Carlo simulations of ARCH processes for which the extremal index can be calculated theoretically.

Finally, we discuss implications for Value-at-Risk modeling. We show how the methods previously considered for calculating Value-at-Risk using extreme value techniques should be modified in order to account for the dependence in market returns and propose a simple method for doing so.

Keywords: extreme value theory, risk management, heavy tails, conditional heteroskedasticity, volatility, Value-at-Risk.

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