Asymptotic Minimax Risk Bounds in Univariate Extreme Value Statistics

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Hall and Welsh (1984) established optimal rates of convergence for estimators of the extreme value index based on i.i.d. random variables. Essentially, they proved that with

$$\mathcal{F} = \mathcal{F}(\beta_1, \varepsilon, A, \rho) := \Big\{ F \text{ d.f.} \;\Big|\; F^{-1}(1-t) = t^{-\beta} \exp\Big(\int_t^1 \eta(s)/s \, ds \Big), \\ |\beta - \beta_1| \le \varepsilon, \; |\eta(t)| \le At^\rho \;\; \forall 0 < t \le 1 \Big\}$$

one has for all sequences of estimators $\hat{\beta}_n$ for $\beta = \beta(F)$ and all $u > 0$

$$\liminf_{n\to\infty} \sup_{F\in\mathcal{F}} P_F^n \Big\{ |\hat{\beta}_n - \beta| > \delta_n u \Big\} > 0 \tag{1}$$

where

$$\delta_n := n^{-\rho/(2\rho+1)}$$

and P_F^n denotes the distribution of n i.i.d. r.v.'s with d.f. F (cf. also Drees (1998)). It is the main aim of the talk to derive a strictly positive lower bound for the left-hand side of (1), i.e., a bound for the asymptotic minimax risk of the standardized estimation error $\delta_n^{-1}(\hat{\beta}_n - \beta)$ under the 0-1-loss function $1_{[-u,u]^c}$.

The main tool for the proof is the theory of weak convergence of experiments. Define local alternatives

$$\begin{aligned} F^{-1}_{n,\beta_0,h}(1-t) &:= t^{-\beta_0} \exp\Big(\int_t^1 \frac{\delta_n h(\delta_n^{-1/\rho} s)}{s} \, ds \Big) \\ &= t^{-(\beta_0 + \delta_n h(0))} \exp\Big(\int_t^1 \frac{\delta_n (h(\delta_n^{-1/\rho} s) - h(0))}{s} \, ds \Big) \end{aligned}$$

of the Pareto quantile function $F^{-1}_{n,\beta_0,0}(1-t) = t^{-\beta_0}$, $|\beta_0 - \beta_1| \le \varepsilon$, where

$$h \in \mathcal{H}^{(n)}_{A,\rho} \uparrow \mathcal{H}_{A,\rho} := \Big\{ h \in L_2[0,\infty) \;\Big|\; |h(t) - h(0)| \le At^\rho \;\; \forall t > 0 \Big\},$$

and denote the pertaining probability measures by $P_{n,\beta_0,h}$. Then the sequence of local experiments

$$E_n = \Big(I\!R^n, I\!B^n, (P^n_{n,\beta_0,h})_{h\in\mathcal{H}^{(n)}_{A,\rho}} \Big)$$

converges weakly to the Gaussian experiment

$$E = \Big(C[0,\infty), \mathcal{C}[0,\infty), \Big(\mathcal{L}\Big(W + \frac{1}{\beta_0}\int_0^\bullet h(s)\,ds \Big)\Big)_{h\in\mathcal{H}_{A,\rho}} \Big)$$

where W is a standard Brownian motion. Hence, by LeCam's minimax theorem, minimax risk bounds for estimators of the local parameter $h(0)$ of interest in the limit experiment, which can be calculated using the methods introduced by Donoho and Liu (1991) and Donoho (1994), yield asymptotic minimax risk bounds for E_n and thus lower bounds for the left-hand side of (1).

References

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