The Bootstrap Methodology in Statistical Extremes

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The bootstrap methodology (Efron, 1979) is used here in different contexts, using either complete data if possible or the few top order statistics available, in order to choose properly the threshold suitable for semi-parametric estimation of the tail index. On the basis of a suitable estimation of the bootstrap mean square error of estimators (Hall, 1990) we provide computational methods for the choice of the threshold, merely by an adequate manipulation of the available data. Two different bootstrap estimators of the threshold, and corresponding estimators of the tail index, are compared by means of simulation techniques. Whenever working in the domain of attraction of Gumbel's limit law for the maxima, with a tail function of the type $1-F(x) \approx A\, x^p \exp\{-x^q\}$, for large x, $p \in \mathbb{R}$, $q > 0$, it is known that the rate of convergence of $X_{n:n}^q$, suitably normalized, towards the limiting law is faster than that of $X_{n:n}$ (Gomes, 1984). In this paper we also study the impact on semi-parametric estimators of the null tail index, like the moment's estimator (Dekkers, Einmahl and de Haan, 1989), of a preliminary adequate estimation of q, followed by the power transformation of the data, previously to the estimation (either bootstraped or not) of the tail index.

References.

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