EXTREMES OF INTEGER-VALUED MOVING AVERAGE MODELS

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May 28, 1998

Abstract

We study the extremal behaviour of a wide class of integer-valued moving average sequences obtained as analogues of classic moving average (MA) models. The integer-valued moving average (INMA) model is defined by

$$X_n = \sum_{i=-\infty}^{\infty} \beta_i * Z_{n-i}, \tag{1}$$

where $\{Z_n\}$ is an integer-valued sequence of independent and identically distributed random variables, and $*$ represents binomial thinning defined by Steutel and van Haarn (1979) as

$$\beta * Z = \sum_{i=1}^{Z} B_i(\beta),$$

where $\{B_i(\beta)\}$ is an independent and identically distributed sequence of Bernoulli random variables, $P\{B_i = 1\} = \beta$. Some particular cases of (1) were introduced by Al-Osh and Alzaid (1988) and separately by McKenzie (1988). We consider a wide class of sequences which includes the models introduced by these authors. The class is defined considering several dependence structures in the thinning operations involved in the terms $\beta_i * Z_n,\ i \in \mathbb{Z}$, for each n. The properties of the class are fully described by Brännäs and Hall (1998) where estimation problems are also considered.

Since the INMA model resembles the MA model replacing scalar multiplication by random thinning, the study of the extremal behaviour of the sequences was mostly motivated by the works of Rootzén (1986) and Chernick et al. (1991). Two situations are considered:

1. In the first the marginal distribution of the sequence has an exponentially decaying tail of the form

   $$1 - F_Z(z) \sim K[z]^{\xi}(1+\lambda)^{-[z]} \quad \text{when } z \to \infty, \quad \xi \in \mathbb{R},\ K, \lambda > 0.$$

2. In the second the marginal distribution of the sequence has a regularly varying tail.

For the first case we follow the steps of Rootzén (1986) introducing the necessary changes to account for the discrete nature of the data and the random properties of thinning. Curiously we obtain analogous results as if the INMA model was a kind of MA model through a magic looking-glass. The main result is that the maximum term of the sequence has a limiting behaviour of the form

$$\begin{cases} \limsup_{n\to\infty} P\{M_n - \check{b}_n \leq x\} \leq e^{-(1+\check{\lambda})^{-x}} \\ \liminf_{n\to\infty} P\{M_n - \check{b}_n \leq x\} \geq e^{-(1+\check{\lambda})^{-(x-1)}} \end{cases},$$

for an appropriate choice of the constants $\check{b}_n, \check{\lambda}$. Just as it happened with the MA model, the sequence behaves as if it was independent and identically distributed.

For the second case we start the analysis of the limiting distribution of the maximum term as in Chernick et al. (1991) and proceed with some new relations implied by the thinning operation. Even more surprising than in the first case, we conclude that, for whatever dependence structure considered in the thinning operations, the sequence has the same limiting distribution for the maximum term given by Chernick et al. (1991), which means that the sequence has an extremal index given by

$$\theta = \frac{1}{\sum_{i=-\infty}^{\infty} \beta_i^{\alpha}},$$

where α is the index of regular variation.

The results are illustrated by a simulation study.

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