SOME EXTREME VALUE BEHAVIOR ASSOCIATED WITH THE HOPEFIELD MODEL

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The Hopefield model is one of the most central and easiest models of a neural network. One of its primary functions is to retrieve a pattern stored in a memory in response to the presentation of an incomplete or noisy version. As a mathematical object it has been the subject of intense study over the past decade. The purpose of this talk is to uncover some extreme value behavior associated with this model.

Suppose $M(N)$ patterns are to be stored in a Hopefield model with N neurons. Each pattern ξ_i, $i = 1, ..., N$, has an associated Hamiltonian $H(\xi_i)$. When studying the Hopefield model one assumes that $\xi_i, i = 1, ..., N$ are independent $N-$vectors of random independent signs $+1, -1$, each taken with probability $1/2$. Typically a random pattern σ presented to the network will settle down after a number of iterations of the Hopefield dynamics to one of the stored patterns. We will describe the extreme value behavior of the Hamiltonians $H(\xi_i)$, $i = 1, .., M(N)$ as both N and $M(N)$ converge to infinity. These $M(N)$ Hamiltonians are exchangeable random variables. It is shown using a strong approximation result of Zaitsev (1987) that any finite subset of size k these Hamiltonians after an appropriate transformation acts very closely like k independent standard normal random variables. This allows us to derive the asymptotic extreme value behavior of these $M(N)$ Hamiltonians as N converges to infinity.