The extremes of the periodogram
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This talk is about joint work with R.A. Davis, P. Kokoszka, G. Samorodnitsky and S.I. Resnick.

In the literature one can often find the vague statement that the periodogram ordinates $I_{n,Z}(\lambda_j)$ of an iid sequence (Z_t) when evaluated at the Fourier frequencies $\lambda_j = 2\pi j/n$, $j = 1, \ldots, [n/2]$ "behave like an iid exponential sequence". If the underlying iid sequence is Gaussian the $I_{n,Z}(\lambda_j)$'s are iid exponential. But does this statement remain valid, if we have non-Gaussian data?

In the course of the talk we consider various functions of the $I_{n,Z}(\lambda_j)$'s. This includes their maximum, the upper order statistics and the point process constructed from those ordinates. Using a large deviation result by U. Einmahl (1989), one can show that these functions have the same limit behaviour as if one replaced the $I_{n,Z}(\lambda_j)$'s with an iid exponential sample. If $EZ_t^2 < \infty$, the empirical distribution function of the $I_{n,Z}(\lambda_j)$'s satisfies a Glivenko–Cantelli result with exponential limit (this was proved by Freedman and Lane (1981), we have a different proof based on Hermann Weyl's (1916) theorem about the uniform distribution of a sequence of real numbers), but the central limit theorem does not seem to work for the corresponding empirical process since higher–order moment characteristics seem to play a rôle. We also mention that weighted sums of the $I_{n,Z}(\lambda_j)$'s (such as the empirical spectral distribution function, MLE– and Whittle–type estimates of the parameters of ARMA processes) depend on the higher–order moment structure of the Z_t's, so that the analogy with an iid exponential sequence may fail.

One has a totally different asymptotic behaviour for functions of the $I_{n,Z}(\lambda_j)$'s when the Z_t's have infinite variance. An indication of this fact is that a finite number of the $I_{n,Z}(\lambda_j)$'s converges to a function of an infinite variance stable process, the empirical distribution based on the $I_{n,Z}(\lambda_j)$'s converges to a random measure (this is a result by Freedman and Lane (1980)) and the extremes have some non–standard limit behaviour as well.

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