# Limit distributions of $l_\alpha$-norms of vectors of positive i. i. d. random variables

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**Abstract**

The normal distribution and the extreme value distributions appear as the endpoints of a range of limit distributions obtained from $l_\alpha$-norms of $n$-vectors of positive i. i. d. random variables.
Let $X_{1n} = (X_1, \dots X_n)$ be an $n$-dimensional vector of positive i. i. d. random variables $X_1, \dots, X_n$ and let $\|X_{1n}\|_\alpha$ its $l_\alpha$-(quasi-)norm:

$$\|X_{1n}\|_\alpha = \begin{cases} (\sum_{i=1}^n X_i^\alpha)^{1/\alpha}, & 0 < \alpha < \infty \\ \max\{X_1, \dots, X_n\}, & \alpha = \infty \end{cases} .$$

Assume $X_1 \sim F$ belongs to the domain of attraction of the Weibull distribution $\Psi_\gamma(x)$, and let $\alpha(n) = (1 - F^{\leftarrow}(1 - 1/n))^{-1}$. Then, for any $c > 0$, there exist constants $a_c(n)$ and $b_c(n)$, $n \in \mathbb{N}$, such that the limit distribution

$$F_c(x) = \lim_{n\to\infty} \mathbb{P}\Big( \frac{\|X_{1n}\|_{c\alpha(n)} - b_c(n)}{a_c(n)} < x \Big)$$

exists and, furthermore, $F_c$ converges to $\Psi_\gamma$ as $c \to \infty$ and to the normal distribution as $c \to 0$.
A similar assertion holds if $X_1$ belongs to the domain of attraction of a Fréchet distribution and if $\alpha(n) \equiv 1$. Comparable results are expected to hold if $X_1 \sim F$ belongs to the domain of attraction of the Gumbel distribution and if $\alpha(n) = n^{-1} b(n) / \int_{b(n)}^{x_F} 1 - F(t)\,\mathrm{d}t$ where $x_F$ is the endpoint of $F$ and $b(n) = F^{\leftarrow}(1 - 1/n)$.